**SOLVED BY: AYOUB**

**MTH-501 ASSIGNMENT 2 (2022)**

**……………………………………………………..**

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**QUESTION:**

Find the eigenvalues and eigenvectors of the following matrix?

|  |  |  |
| --- | --- | --- |
| 1 | −1 | 4 |
| 𝐴 = [3 | 2 | −1] |
| 2 | 1 | −1 |

**SOLUTION:**

First we find eigen values and eigen vectors Eigen values:

(𝐴 − 𝐼𝜆) = 0

1 −1 4 1 0 0

([3 2 −1] − [0 1 0] 𝜆) = 0

2 1 −1 0 0 1

1 − 𝜆 −1 4

[ 3 2 − 𝜆 −1

2 1 −1 − 𝜆

] = 0

𝐶1 − (𝜆 − 1)𝐶2 𝑎𝑛𝑑 𝐶3 + 4𝐶2

|  |  |  |
| --- | --- | --- |
| 0 | −1 | 0 |
| [𝜆2 − 3𝜆 + 5 | 2 − 𝜆 | 7 − 4𝜆] = 0 |
| 3 − 𝜆 | 1 | 3 − 𝜆 |

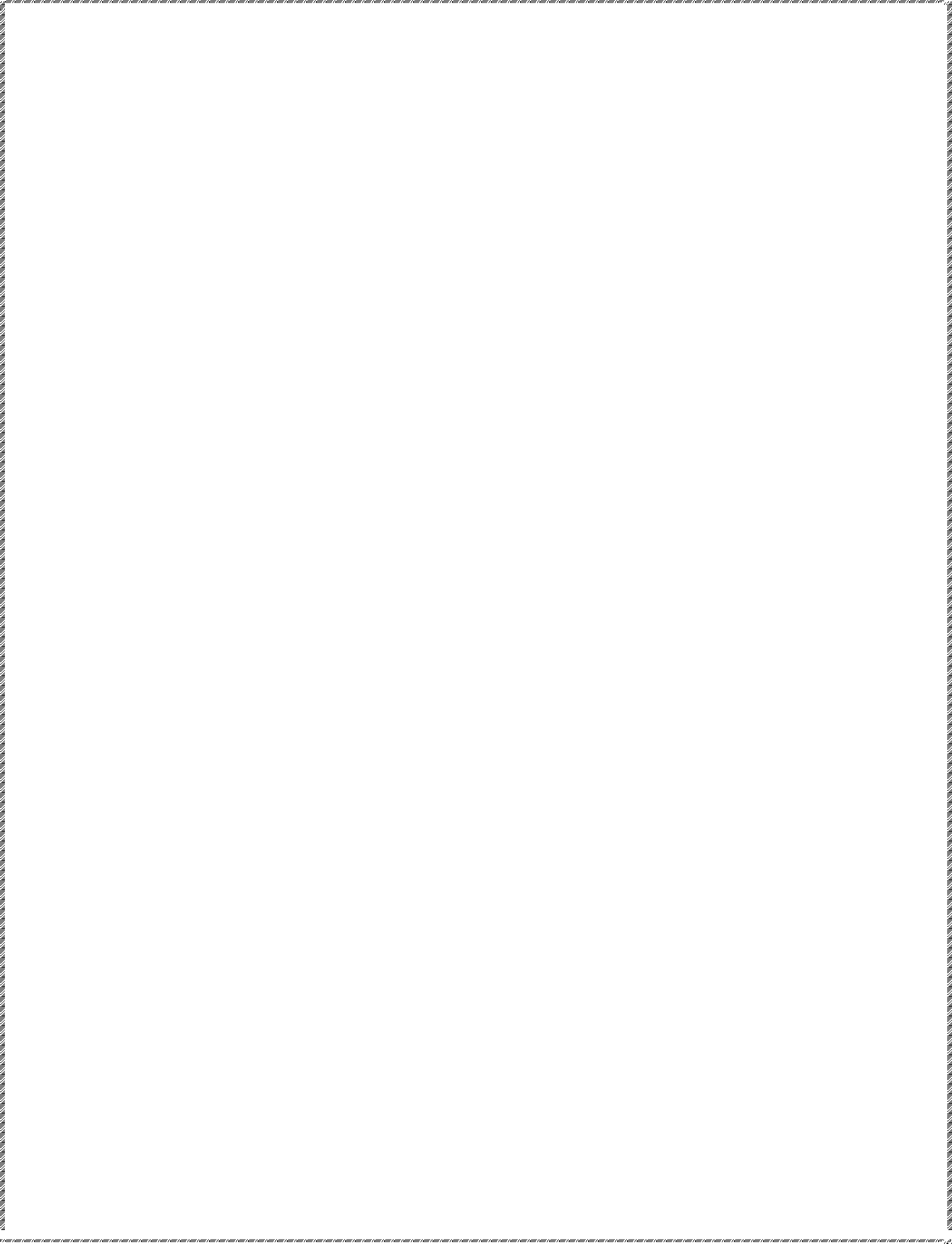
(0)(−1)1+1 |2 − 𝜆 7 − 4𝜆| + (−1)(−1)1+2 |𝜆2 − 3𝜆 + 5 7 − 4𝜆| +

1 3 − 𝜆

(0)(−1)1+3 |𝜆2 − 3𝜆 + 5 2 − 𝜆| = 0

3 − 𝜆 1

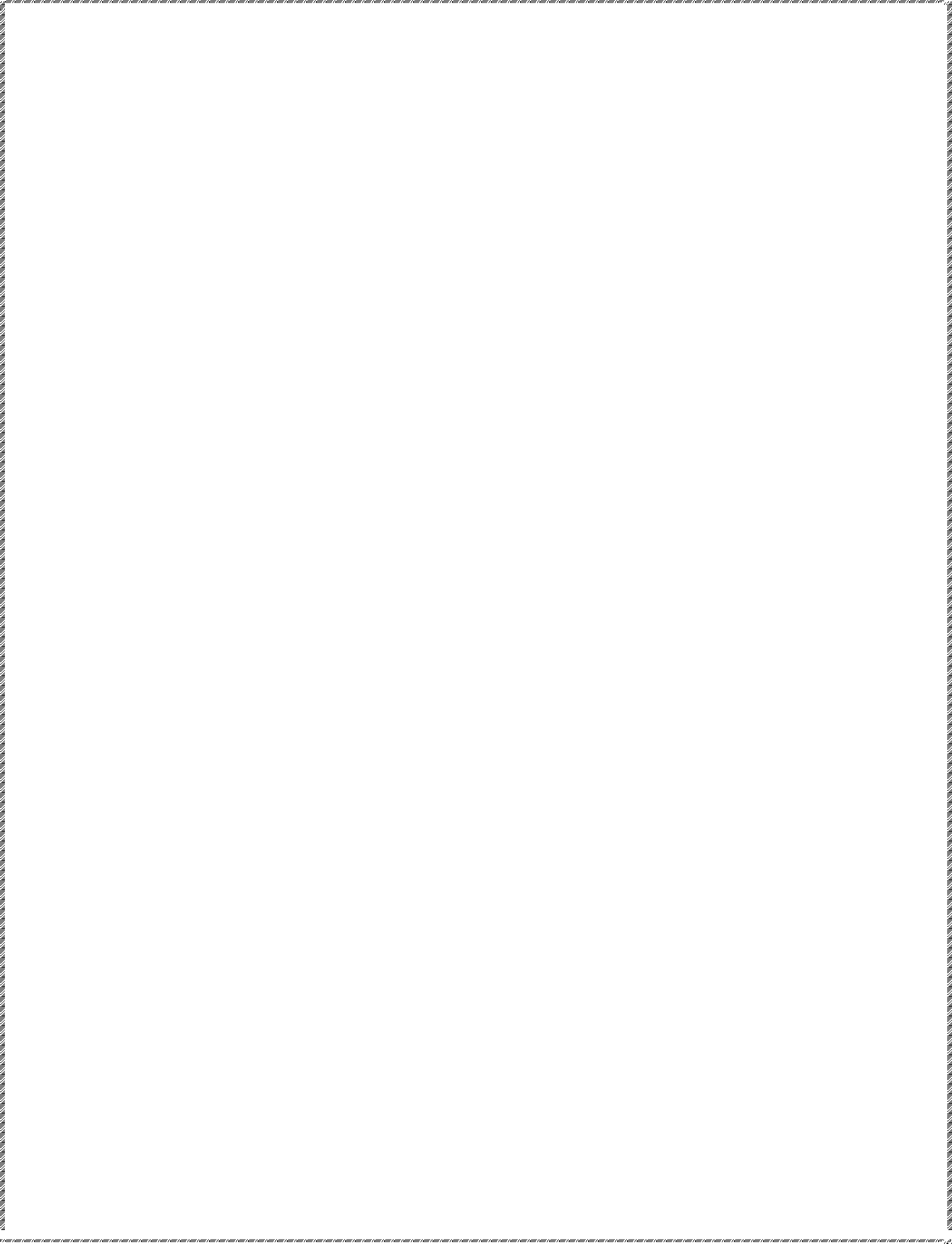
3 − 𝜆 3 − 𝜆



(−1)(−1)3 |𝜆2 − 3𝜆 + 5 7 − 4𝜆| = 0

3 − 𝜆 3 − 𝜆

|𝜆2 − 3𝜆 + 5 7 − 4𝜆| = 0 3 − 𝜆 3 − 𝜆

(𝜆2 − 3𝜆 + 5). (3 − 𝜆) − (3 − 𝜆). (7 − 4𝜆)=0

−𝜆3 + 2𝜆2 + 5𝜆 − 6 = 0

−(𝜆 − 3)(𝜆 − 1)(𝜆 + 2) = 0 (𝜆 − 3)(𝜆 − 1)(𝜆 + 2) = 0

𝜆 − 3 = 0 , 𝜆 − 1 = 0 , 𝜆 + 2 = 0

𝜆 = 3 , 𝜆 = 1 , 𝜆 = −2

Now we find the Eigen Vectors When 𝜆 = 3

|  |  |  |  |
| --- | --- | --- | --- |
| 1 − 3 | −1 | | 4 |
| = [ 3 | 2 − 3 | | −1 ] |
| 2 | 1 | | −1 − 3 |
| −2 | −1 | 4 | |
| = [ 3 | −1 | −1] | |
| 2 | 1 | −4 | |

Apply row operation

|  |  |  |
| --- | --- | --- |
| 1 | 0 | −1 |
| = [0 | 1 | −2] |
| 0 | 0 | 0 |

To find the null space, solve the matrix equation

1 0 −1 𝑥1 0

[0 1 −2] [𝑥2] = [0]

0 0 0 𝑥3 0

If we take 𝑥3 = 𝑡, 𝑡ℎ𝑒𝑛 𝑥1 = 𝑡, 𝑥2 = 2𝑡

𝑡 1

Thus,𝑋⃗→ = [2𝑡] = [2] 𝑡

𝑡 1

This is null space

1

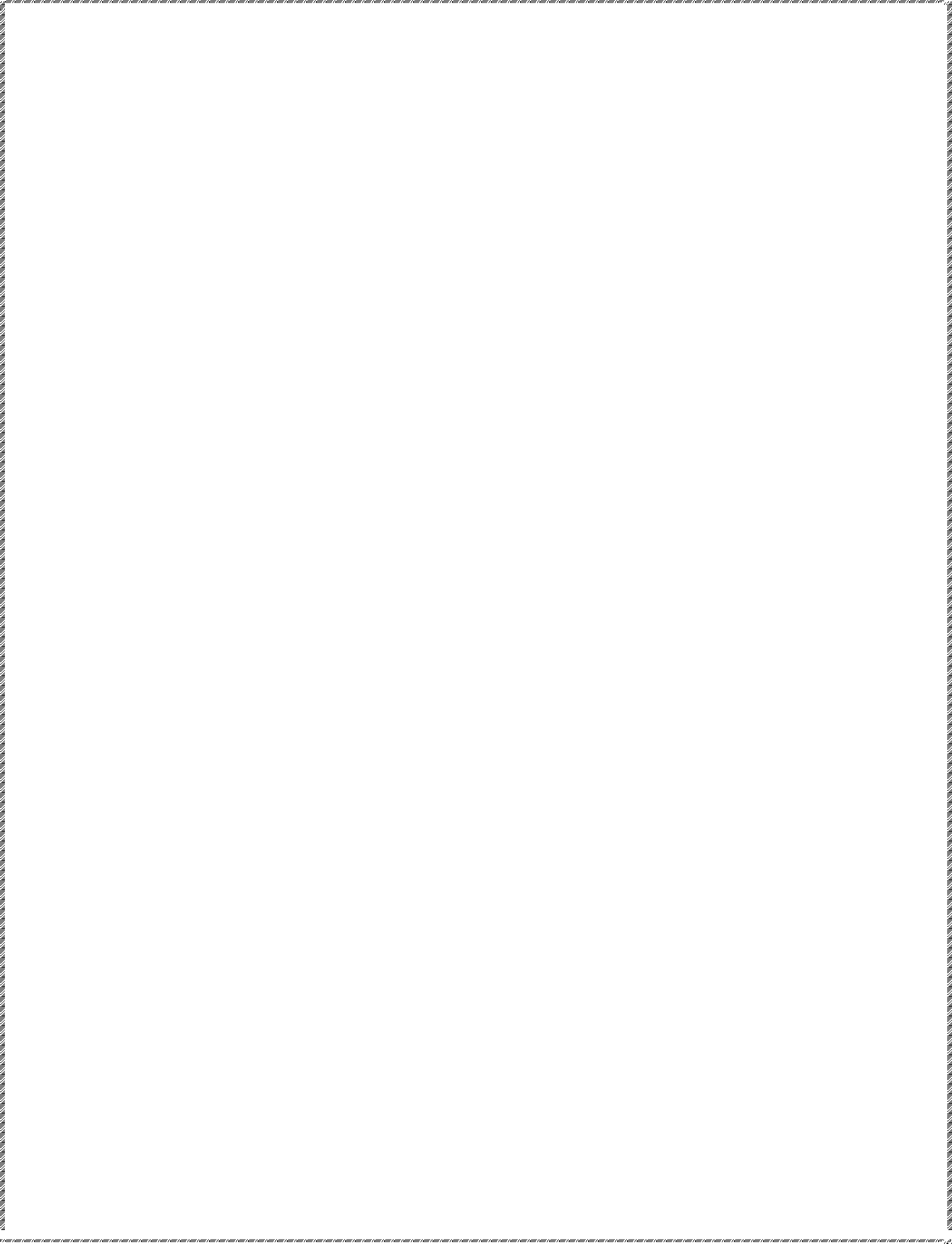
Null space {[2]}

1

When 𝜆 = 1

1 − 1 −1 4

= [ 3 2 − 1 −1 ]

2 1 −1 − 1

|  |  |  |
| --- | --- | --- |
| 0 | −1 | 4 |
| = [3 | 1 | −1] |
| 2 | 1 | −2 |

Apply row operation

|  |  |  |
| --- | --- | --- |
| 1 | 0 | 1 |
| = [0 | 1 | −4] |
| 0 | 0 | 0 |

To find the null space, solve the matrix equation

1 0 1 𝑥1 0

[0 1 −4] [𝑥2] = [0]

0 0 0 𝑥3 0

If we take 𝑥3 = 𝑡, 𝑡ℎ𝑒𝑛 𝑥1 = −𝑡, 𝑥2 = 4𝑡

−𝑡 −1

Thus,𝑋⃗→ = [4𝑡] = [ 4 ] 𝑡

𝑡 1

−1

The null space {[ 4 ]}

1

When 𝜆 = −2

1 − (−2) −1 4

= [ 3 2 − (−2) −1 ]

2 1 −(−2) − 1

|  |  |  |
| --- | --- | --- |
| 3 | −1 | 4 |
| = [3 | 4 | −1] |
| 2 | 1 | 1 |

Apply row operation

|  |  |  |
| --- | --- | --- |
| 1 | 0 | 1 |
| = [0 | 1 | −1] |
| 0 | 0 | 0 |

To find the null space, solve the matrix equation

1 0 1 𝑥1 0

[0 1 −1] [𝑥2] = [0]

0 0 0 𝑥3 0

If we take 𝑥3 = 𝑡, 𝑡ℎ𝑒𝑛 𝑥1 = −𝑡, 𝑥2 = 𝑡

−𝑡 −1

Thus,𝑋⃗→ = [ 𝑡 ] = [ 1 ] 𝑡

𝑡 1

−1

The null space {[ 1 ]}

1

# So, eigen value are

3, 1, -2

# Eigen vectors

1 −1 −1

{[2] , [ 4 ] , [ 1 ]}

1 1 1

**Solution:**

First we find eigen values and eigen vectors Eigen values:

(𝐴 − 𝐼𝜆) = 0

2 − 𝜆 0 0

[ 1 2 − 𝜆 1 ] = 0

−1 0 1 − 𝜆

(2 − 𝜆)(−1)1+1 |2 − 𝜆

0

1

1 − 𝜆

| + (0)(−1)1+2 | 1

−1

1

1 − 𝜆

| + (0)(−1)1+3 | 1

2 − 𝜆| = 0

−1 0

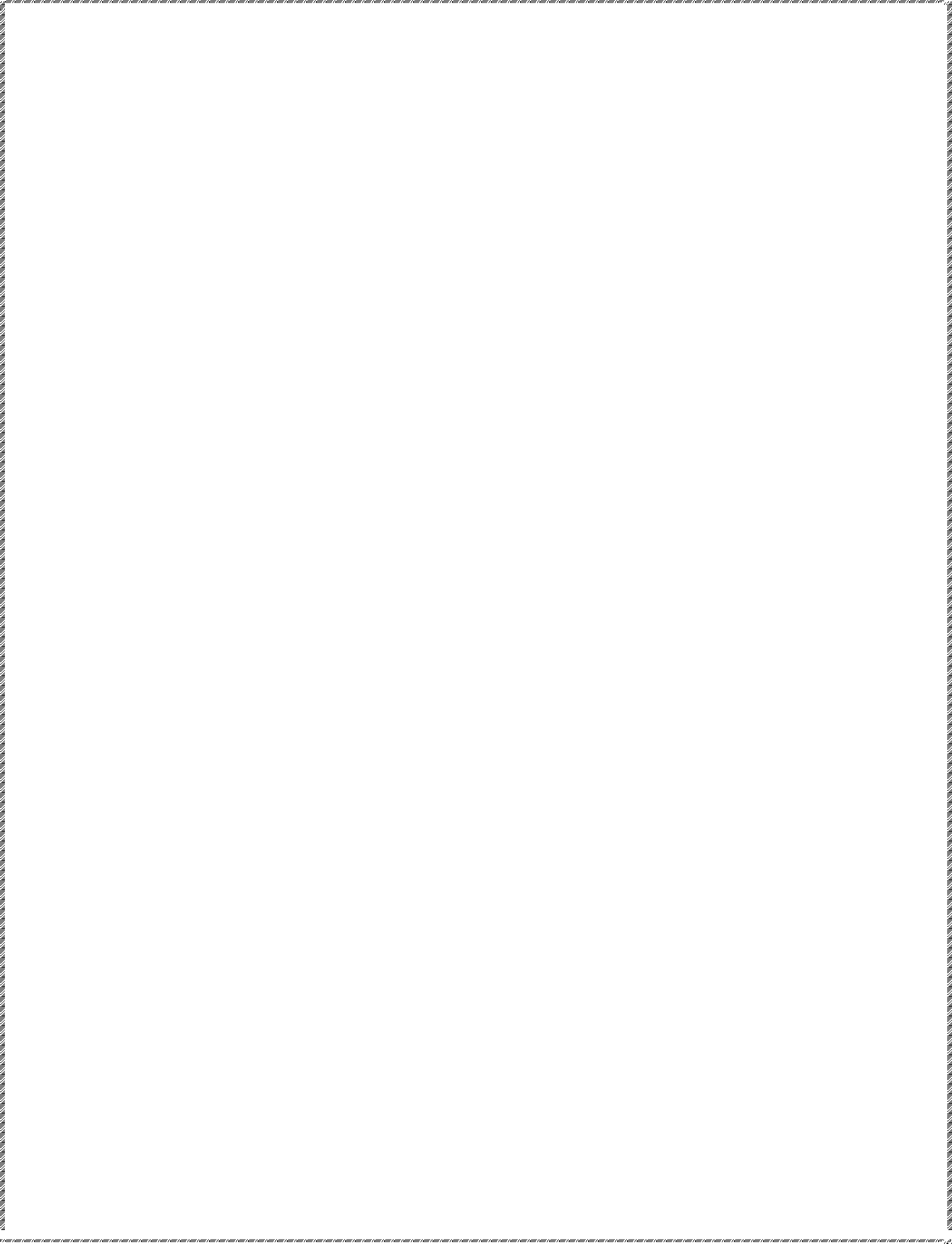
|  |  |  |
| --- | --- | --- |
| **Question: 2** |  | **Marks: 10** |
| Q # 2: Diagonalize the following matrix. |  |  |
| 2 | 0 | 0 |
| 𝐴 = [ 1 | 2 | 1] |
| −1 | 0 | 1 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 2 | 0 | 0 | 1 | 0 | 0 |
| ([ 1 | 2 | 1] − | [0 | 1 | 0] 𝜆) = 0 |
| −1 | 0 | 1 | 0 | 0 | 1 |

(2 − 𝜆)(−1)2 2 − 𝜆 1

| 0 1 − 𝜆

| = 0



(2 − 𝜆)(2 − 𝜆)(1 − 𝜆) = 0

2 − 𝜆 = 0 , 2 − 𝜆 = 0 , 1 − 𝜆 = 0

2 = 𝜆 , 2 = 𝜆 , 1 = 𝜆

⇒ 𝜆 = 2 , 𝜆 = 2 , 𝜆 = 1

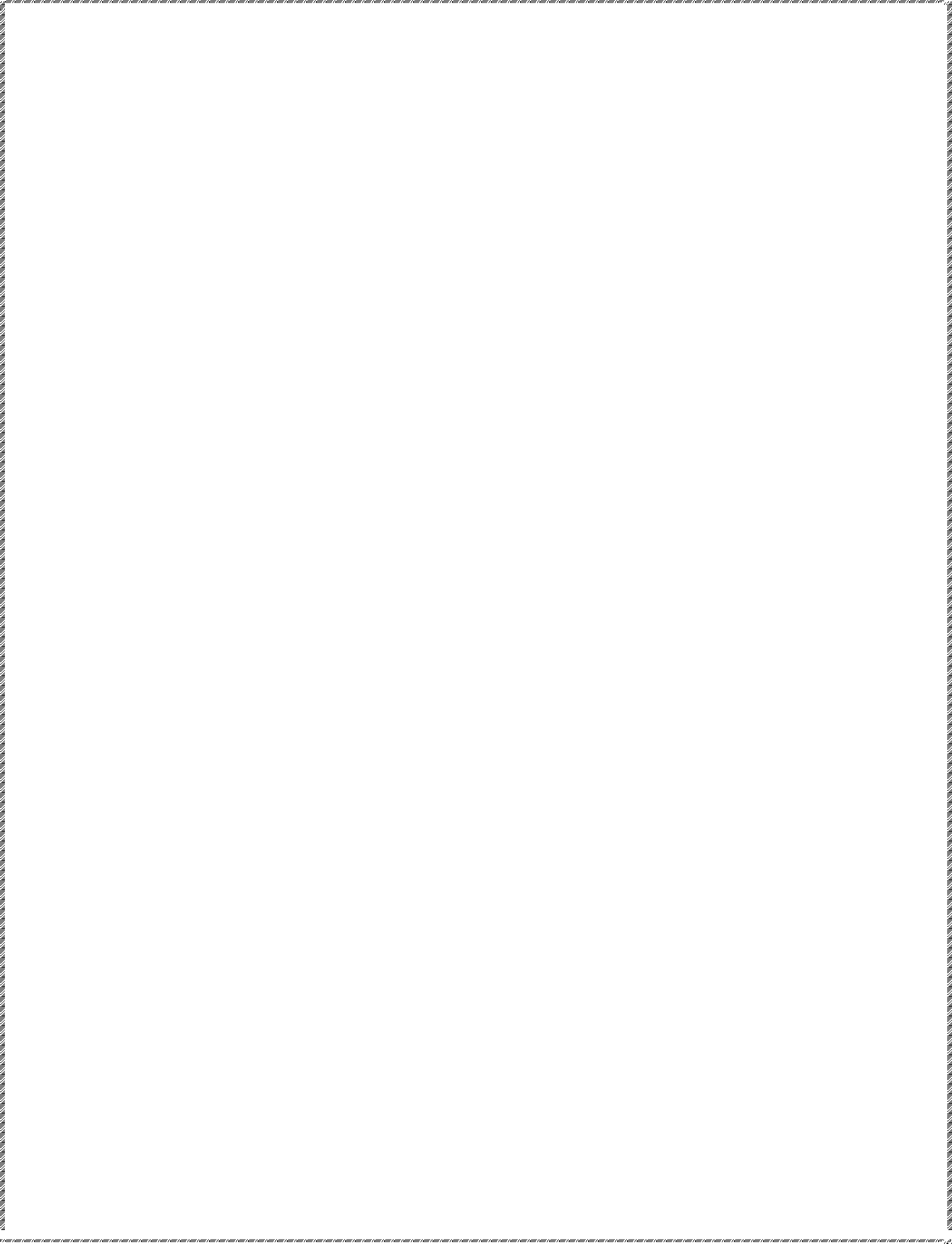
Now we find the Eigen Vectors When 𝜆 = 1

|  |  |  |  |
| --- | --- | --- | --- |
| 2 − 1 | 0 | | 0 |
| = [ 1 | 2 − 1 | | 1 ] |
| −1 | 0 | | 1 − 1 |
| 1 | 0 | 0 | |
| = [ 1 | 1 | 1] | |
| −1 | 0 | 0 | |

Apply row operation

|  |  |  |
| --- | --- | --- |
| 1 | 0 | 0 |
| = [0 | 1 | 1] |
| 0 | 0 | 0 |

𝑅2 − 𝑅1 , 𝑅3 + 𝑅1



To find the null space, solve the matrix equation

1 0 0 𝑥1 0

[0 1 1] [𝑥2] = [0]

0 0 0 𝑥3 0

If we take 𝑥3 = 𝑡, 𝑡ℎ𝑒𝑛 𝑥1 = 0, 𝑥2 = −𝑡 0 0

Thus,𝑋⃗→ = [−𝑡] = [−1] 𝑡

𝑡 1

This is null space

0

Null space {[−1]}

1

When 𝜆 = 2

2 − 2 0 0

= [ 1 2 − 2 1 ]

−1 0 1 − 2

0 0 0

= [ 1 0 1 ]

−1 0 −1

Apply row operation

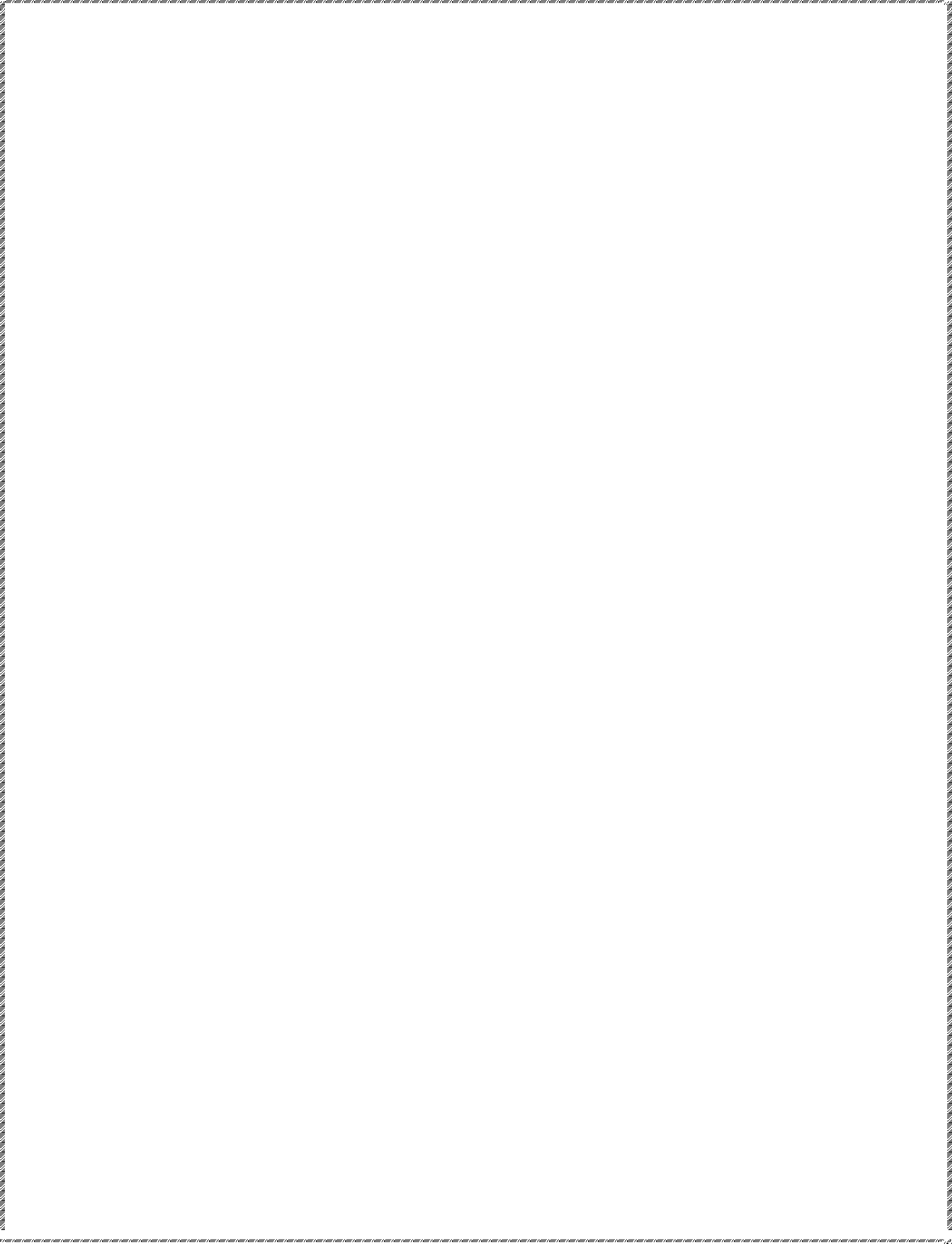
1 0 1

= [0 0 0] 𝑠𝑤𝑎𝑝 𝑟𝑜𝑤 1 𝑎𝑛𝑑 2 , 𝑅3 + 𝑅1

0 0 0

To find the null space, solve the matrix equation

1 0 1 𝑥1 0



[0 0 0] [𝑥2] = [0]

0 0 0 𝑥3 0

If we take 𝑥2 = 𝑡, 𝑡ℎ𝑒𝑛 𝑥3 = 𝑠, 𝑥1 = −𝑠

−𝑠 0 −1

Thus,𝑋⃗→ = [−𝑡] = [1] 𝑡 + [ 0 ] 𝑠

𝑠 0 1

0 −1

The null space {[1] , [ 0 ]}

0 1

So, eigen value 1, 2, 2 Eigen vectors

0 0 −1

{[−1] , [1] , [ 0 ]}

1 0 1

Now, matrix P whose column entries are Eigen vectors

0 0 −1

𝑃 = [−1 1 0 ]

1 0 1

And matrix D whose column entries are Eigen Values

|  |  |  |
| --- | --- | --- |
| 1 | 0 | 0 |
| 𝐷 = [0 | 2 | 0] |
| 0 | 0 | 2 |

The matrices P and D are such that the initial matrix

2 0 0

[ 1 2 1] = 𝑃𝐷𝑃−1

−1 0 1

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