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لجنة سنافر البوليتكنك - الاتجاه الإسلامي

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حلول Physics 101

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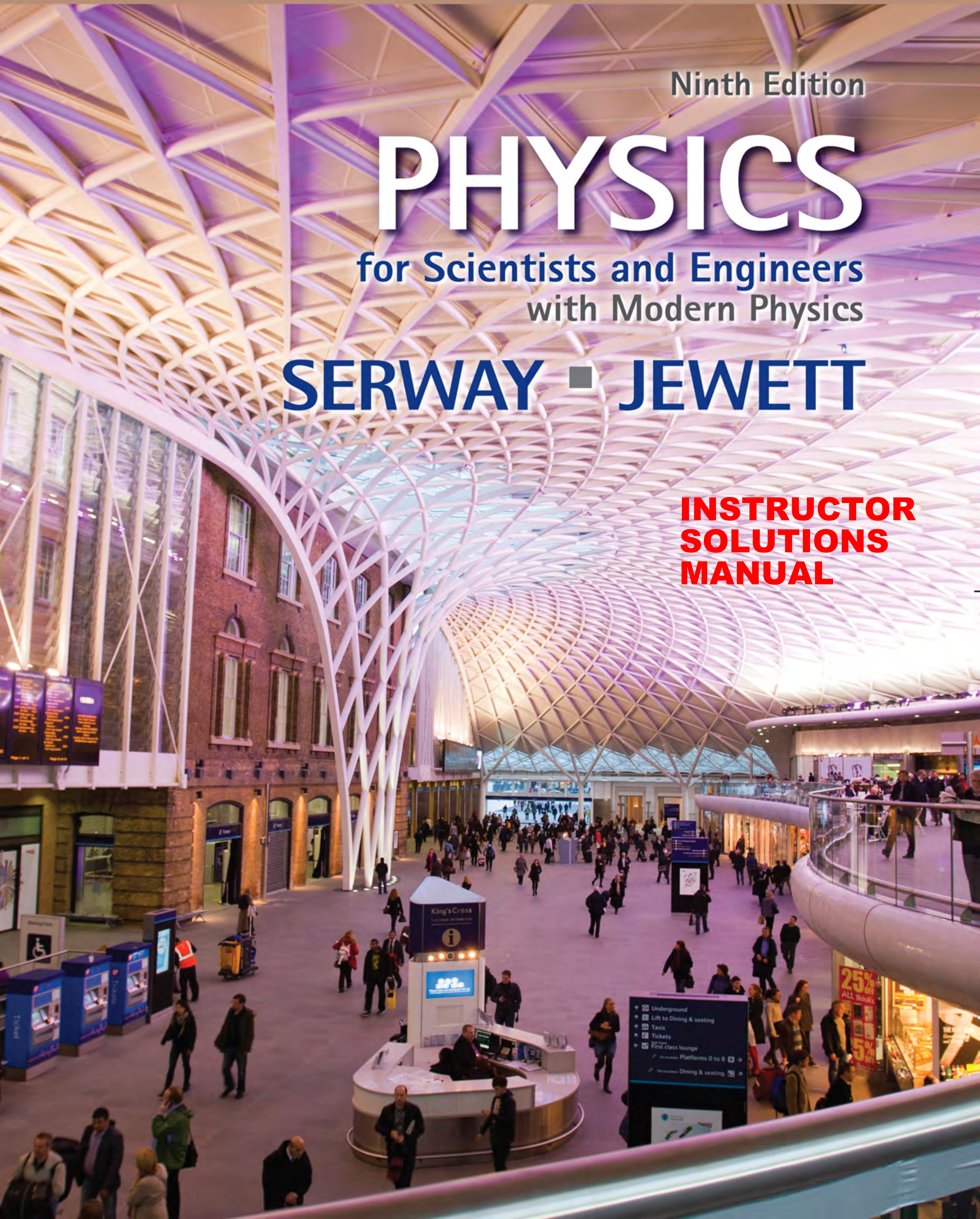
Ninth Edition

PHYSICS

for Scientists and Engineers with Modern Physics

SERWAY ▪ JEWETT

INSTRUCTOR SOLUTIONS MANUAL



1

Physics and Measurement

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ1.1 The meterstick measurement, (a), and (b) can all be 4.31 cm. The meterstick measurement and (c) can both be 4.24 cm. Only (d) does not overlap. Thus (a), (b), and (c) all agree with the meterstick measurement.

OQ1.2 Answer (d). Using the relation

$$1\text{ ft} = 12\text{ in}\left(\frac{2.54\text{ cm}}{1\text{ in}}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right) = 0.304\ 8\text{ m}$$

we find that

$$1\,420\text{ ft}^2\left(\frac{0.304\ 8\text{ m}}{1\text{ ft}}\right)^2 = 132\text{ m}^2$$

OQ1.3 The answer is yes for (a), (c), and (e). You cannot add or subtract a number of apples and a number of jokes. The answer is no for (b) and (d). Consider the gauge of a sausage, 4 kg/2 m, or the volume of a cube, $(2\text{ m})^3$. Thus we have (a) yes; (b) no; (c) yes; (d) no; and (e) yes.

OQ1.4 41 € ≈ 41 € (1 L/1.3 €)(1 qt/1 L)(1 gal/4 qt) ≈ (10/1.3) gal ≈ 8 gallons, answer (c).

OQ1.6 The number of decimal places in a sum of numbers should be the same as the smallest number of decimal places in the numbers summed.

$$\begin{array}{r l} 21.4 & \text{s} \\ 15 & \text{s} \\ 17.17 & \text{s} \\ 4.003 & \text{s} \\ \hline 57.573 & \text{s} = 58 \text{ s, answer (d).} \end{array}$$

OQ1.7 The population is about 6 billion = 6×10^9. Assuming about 100 lb per person = about 50 kg per person (1 kg has the weight of about 2.2 lb), the total mass is about $(6 \times 10^9)(50 \text{ kg}) = 3 \times 10^{11}$ kg, answer (d).

OQ1.8 No: A dimensionally correct equation need not be true. Example: 1 chimpanzee = 2 chimpanzee is dimensionally correct.

Yes: If an equation is not dimensionally correct, it cannot be correct.

OQ1.9 Mass is measured in kg; acceleration is measured in m/s^2. Force = mass × acceleration, so the units of force are answer (a) $\text{kg}\cdot\text{m/s}^2$.

OQ1.10 0.02(1.365) = 0.03. The result is $(1.37 \pm 0.03) \times 10^7$ kg. So (d) 3 digits are significant.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ1.1 Density varies with temperature and pressure. It would be necessary to measure both mass and volume very accurately in order to use the density of water as a standard.

CQ1.2 The metric system is considered superior because units larger and smaller than the basic units are simply related by multiples of 10. Examples: 1 km = 10^3 m, 1 mg = 10^{-3} g = 10^{-6} kg, 1 ns = 10^{-9} s.

CQ1.3 A unit of time should be based on a reproducible standard so it can be used everywhere. The more accuracy required of the standard, the less the standard should change with time. The current, very accurate standard is the period of vibration of light emitted by a cesium atom. Depending on the accuracy required, other standards could be: the period of light emitted by a different atom, the period of the swing of a pendulum at a certain place on Earth, the period of vibration of a sound wave produced by a string of a specific length, density, and tension, and the time interval from full Moon to full Moon.

CQ1.4 (a) 0.3 millimeters; (b) 50 microseconds; (c) 7.2 kilograms

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 1.1 Standards of Length, Mass, and Time

P1.1 (a) Modeling the Earth as a sphere, we find its volume as

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(6.37\times10^6\ \text{m}\right)^3 = 1.08\times10^{21}\ \text{m}^3$$

Its density is then

$$\rho = \frac{m}{V} = \frac{5.98\times10^{24}\ \text{kg}}{1.08\times10^{21}\ \text{m}^3} = \boxed{5.52\times10^3\ \text{kg/m}^3}$$

(b) This value is intermediate between the tabulated densities of aluminum and iron. Typical rocks have densities around 2000 to 3000 kg/m^3. The average density of the Earth is significantly higher, so higher-density material must be down below the surface.

P1.2 With V = (base area)(height), $V = \left(\pi r^2\right)h$ and $\rho = \frac{m}{V}$, we have

$$\rho = \frac{m}{\pi r^2 h} = \frac{1\ \text{kg}}{\pi(19.5\ \text{mm})^2(39.0\ \text{mm})}\left(\frac{10^9\ \text{mm}^3}{1\ \text{m}^3}\right)$$

$$\rho = \boxed{2.15\times10^4\ \text{kg/m}^3}$$

P1.3 Let V represent the volume of the model, the same in $\rho = \frac{m}{V}$, for both.

Then $\rho_{\text{iron}} = 9.35\ \text{kg}/V$ and $\rho_{\text{gold}} = \frac{m_{\text{gold}}}{V}$.

Next, $$\frac{\rho_{\text{gold}}}{\rho_{\text{iron}}} = \frac{m_{\text{gold}}}{9.35\ \text{kg}}$$

and $$m_{\text{gold}} = (9.35\ \text{kg})\left(\frac{19.3\times10^3\ \text{kg/m}^3}{7.87\times10^3\ \text{kg/m}^3}\right) = \boxed{22.9\ \text{kg}}$$

P1.4 (a) $\rho = m/V$ and $V = (4/3)\pi r^3 = (4/3)\pi(d/2)^3 = \pi d^3/6$, where d is the diameter.

Then $$\rho = 6m/\pi d^3 = \frac{6\left(1.67\times10^{-27}\text{kg}\right)}{\pi\left(2.4\times10^{-15}\text{m}\right)^3} = \boxed{2.3\times10^{17}\text{kg/m}^3}$$

(b) $$\frac{2.3\times10^{17}\ \text{kg/m}^3}{22.6\times10^3\ \text{kg/m}^3} = \boxed{1.0\times10^{13}\ \text{times the density of osmium}}$$

P1.5 For either sphere the volume is $V = \frac{4}{3}\pi r^3$ and the mass is $m = \rho V = \rho\frac{4}{3}\pi r^3$. We divide this equation for the larger sphere by the same equation for the smaller:

$$\frac{m_\ell}{m_s} = \frac{\rho(4/3)\pi r_\ell^3}{\rho(4/3)\pi r_s^3} = \frac{r_\ell^3}{r_s^3} = 5$$

Then $r_\ell = r_s\sqrt[3]{5} = (4.50\text{ cm})\sqrt[3]{5} = \boxed{7.69\text{ cm}}$

***P1.6** The volume of a spherical shell can be calculated from

$$V = V_o - V_i = \frac{4}{3}\pi\left(r_2^3 - r_1^3\right)$$

From the definition of density, $\rho = \frac{m}{V}$, so

$$m = \rho V = \rho\left(\frac{4}{3}\pi\right)\left(r_2^3 - r_1^3\right) = \boxed{\frac{4\pi\rho\left(r_2^3 - r_1^3\right)}{3}}$$

Section 1.2 Matter and Model Building

P1.7 From the figure, we may see that the spacing between diagonal planes is half the distance between diagonally adjacent atoms on a flat plane. This diagonal distance may be obtained from the Pythagorean theorem, $L_{\text{diag}} = \sqrt{L^2 + L^2}$. Thus, since the atoms are separated by a distance $L = 0.200$ nm, the diagonal planes are separated by $\frac{1}{2}\sqrt{L^2 + L^2} = \boxed{0.141\text{ nm}}$.

P1.8 (a) Treat this as a conversion of units using $1\text{ Cu-atom} = 1.06 \times 10^{-25}$ kg, and $1\text{ cm} = 10^{-2}$ m:

$$\text{density} = \left(8\ 920\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{10^{-2}\text{ m}}{1\text{ cm}}\right)^3\left(\frac{\text{Cu-atom}}{1.06\times10^{-25}\text{ kg}}\right)$$

$$= \boxed{8.42\times10^{22}\ \frac{\text{Cu-atom}}{\text{cm}^3}}$$

(b) Thinking in terms of units, invert answer (a):

$$(\text{density})^{-1} = \left(\frac{1\ \text{cm}^3}{8.42 \times 10^{22}\ \text{Cu-atoms}}\right)$$

$$= \boxed{1.19 \times 10^{-23}\ \text{cm}^3/\text{Cu-atom}}$$

(c) For a cube of side L,

$$L^3 = 1.19 \times 10^{-23}\ \text{cm}^3 \rightarrow L = \boxed{2.28 \times 10^{-8}\,\text{cm}}$$

Section 1.3 Dimensional Analysis

P1.9 (a) Write out dimensions for each quantity in the equation

$$v_f = v_i + ax$$

The variables v_f and v_i are expressed in units of m/s, so

$$[v_f] = [v_i] = \text{LT}^{-1}$$

The variable a is expressed in units of m/s^2; $[a] = \text{LT}^{-2}$

The variable x is expressed in meters. Therefore, $[ax] = \text{L}^2\text{T}^{-2}$

Consider the right-hand member (RHM) of equation (a):

$$[\text{RHM}] = \text{LT}^{-1} + \text{L}^2\text{T}^{-2}$$

Quantities to be added must have the same dimensions.
Therefore, equation (a) **is not** dimensionally correct.

(b) Write out dimensions for each quantity in the equation

$$y = (2\ \text{m})\cos(kx)$$

For y, $[y] = \text{L}$

for 2 m, $[2\ \text{m}] = \text{L}$

and for (kx), $[kx] = \left[\left(2\ \text{m}^{-1}\right)x\right] = \text{L}^{-1}\text{L}$

Therefore we can think of the quantity kx as an angle in radians, and we can take its cosine. The cosine itself will be a pure number with no dimensions. For the left-hand member (LHM) and the right-hand member (RHM) of the equation we have

$$[\text{LHM}] = [y] = \text{L} \qquad [\text{RHM}] = [2\,\text{m}][\cos(kx)] = \text{L}$$

These are the same, so equation (b) **is** dimensionally correct.

P1.10 Circumference has dimensions L, area has dimensions L^2, and volume has dimensions L^3. Expression (a) has dimensions $L(L^2)^{1/2} = L^2$, expression (b) has dimensions L, and expression (c) has dimensions $L(L^2) = L^3$. $\boxed{\text{The matches are: (a) and (f), (b) and (d), and (c) and (e).}}$

P1.11 (a) Consider dimensions in terms of their mks units. For kinetic energy *K*:

$$[K] = \left[\left(\frac{p^2}{2m}\right)\right] = \frac{[p]^2}{kg} = \frac{kg\cdot m^2}{s^2}$$

Solving for $[p^2]$ and $[p]$ then gives

$$[p]^2 = \frac{kg^2\cdot m^2}{s^2} \quad \rightarrow \quad [p] = \frac{kg\cdot m}{s}$$

$\boxed{\text{The units of momentum are } kg\cdot m/s.}$

(b) Momentum is to be expressed as the product of force (in N) and some other quantity *X*. Considering dimensions in terms of their mks units,

$$[N]\cdot[X] = [p]$$
$$\frac{kg\cdot m}{s^2}\cdot[X] = \frac{kg\cdot m}{s}$$
$$[X] = s$$

Therefore, the units of momentum are $\boxed{N\cdot s}$.

P1.12 We substitute $[kg] = [M]$, $[m] = [L]$, and $[F] = \left[\frac{kg\cdot m}{s^2}\right] = \frac{[M][L]}{[T]^2}$ into Newton's law of universal gravitation to obtain

$$\frac{[M][L]}{[T]^2} = \frac{[G][M]^2}{[L]^2}$$

Solving for $[G]$ then gives

$$[G] = \frac{[L]^3}{[M][T]^2} = \boxed{\frac{m^3}{kg\cdot s^2}}$$

***P1.13** The term *x* has dimensions of L, *a* has dimensions of LT^{-2}, and *t* has dimensions of T. Therefore, the equation $x = ka^m t^n$ has dimensions of

$$L = \left(LT^{-2}\right)^m (T)^n \quad \text{or} \quad L^1T^0 = L^mT^{n-2m}$$

The powers of L and T must be the same on each side of the equation.

Therefore,

$$L^1 = L^m \text{ and } \boxed{m = 1}$$

Likewise, equating terms in T, we see that $n - 2m$ must equal 0. Thus, $\boxed{n = 2}$. The value of k, a dimensionless constant, $\boxed{\text{cannot be obtained by dimensional analysis}}$.

P1.14 Summed terms must have the same dimensions.

(a) $[X] = [At^3] + [Bt]$

$$L = [A]T^3 + [B]T \rightarrow \boxed{[A] = L/T^3, \text{ and } [B] = L/T}.$$

(b) $\left[dx/dt\right] = \left[3At^2\right] + [B] = \boxed{L/T}$.

Section 1.4 Conversion of Units

P1.15 From Table 14.1, the density of lead is 1.13×10^4 kg/m^3, so we should expect our calculated value to be close to this value. The density of water is 1.00×10^3 kg/m^3, so we see that lead is about 11 times denser than water, which agrees with our experience that lead sinks.

Density is defined as $\rho = m/V$. We must convert to SI units in the calculation.

$$\rho = \left(\frac{23.94 \text{ g}}{2.10 \text{ cm}^3}\right)\left(\frac{1 \text{ kg}}{1\,000 \text{ g}}\right)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3$$

$$= \left(\frac{23.94 \text{ g}}{2.10 \text{ cm}^3}\right)\left(\frac{1 \text{ kg}}{1\,000 \text{ g}}\right)\left(\frac{1\,000\,000 \text{ cm}^3}{1 \text{ m}^3}\right)$$

$$= \boxed{1.14 \times 10^4 \text{ kg/m}^3}$$

Observe how we set up the unit conversion fractions to divide out the units of grams and cubic centimeters, and to make the answer come out in kilograms per cubic meter. At one step in the calculation, we note that **one million** cubic centimeters make one cubic meter. Our result is indeed close to the expected value. Since the last reported significant digit is not certain, the difference from the tabulated values is possibly due to measurement uncertainty and does not indicate a discrepancy.

P1.16 The weight flow rate is

$$\left(1\ 200\frac{\text{ton}}{\text{h}}\right)\left(\frac{2000\ \text{lb}}{\text{ton}}\right)\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)=\boxed{667\ \text{lb/s}}$$

P1.17 For a rectangle, Area = Length × Width. We use the conversion 1 m = 3.281 ft. The area of the lot is then

$$A=LW=(75.0\ \text{ft})\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right)(125\ \text{ft})\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right)=\boxed{871\ \text{m}^2}$$

P1.18 Apply the following conversion factors: 1 in = 2.54 cm, 1 d = 86 400 s, 100 cm = 1m, and 10^9 nm = 1 m. Then, the rate of hair growth per second is

$$\text{rate}=\left(\frac{1}{32}\ \text{in/day}\right)\frac{(2.54\ \text{cm/in})(10^{-2}\ \text{m/cm})(10^{9}\ \text{nm/m})}{86\ 400\ \text{s/day}}$$
$$=\boxed{9.19\ \text{nm/s}}$$

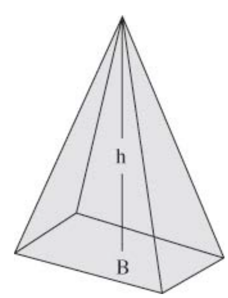
This means the proteins are assembled at a rate of many layers of atoms each second!

P1.19 The area of the four walls is (3.6 + 3.8 + 3.6 + 3.8) m × (2.5 m) = 37 m^2. Each sheet in the book has area (0.21 m)(0.28 m) = 0.059 m^2. The number of sheets required for wallpaper is 37 m^2/0.059 m^2 = 629 sheets = 629 sheets(2 pages/1 sheet) = 1260 pages.

$\boxed{\text{The number of pages in Volume 1 are insufficient.}}$

P1.20 We use the formula for the volume of a pyramid given in the problem and the conversion 43 560 ft^2 = 1 acre. Then,

$$V=Bh$$
$$=\frac{1}{3}\left[(13.0\ \text{acres})(43\ 560\ \text{ft}^2/\text{acre})\right]\times(481\ \text{ft})$$
$$=9.08\times10^7\ \text{ft}^3$$



ANS FIG. P1.20

or

$$V=(9.08\times10^7\ \text{ft}^3)\left(\frac{2.83\times10^{-2}\ \text{m}^3}{1\ \text{ft}^3}\right)$$
$$=\boxed{2.57\times10^6\ \text{m}^3}$$

P1.21 To find the weight of the pyramid, we use the conversion 1 ton = 2 000 lbs:

$$F_g = (2.50\ \text{tons/block})(2.00\times10^6\ \text{blocks})(2\ 000\ \text{lb/ton})$$
$$= \boxed{1.00\times10^{10}\ \text{lbs}}$$

P1.22 (a) $$\text{rate} = \left(\frac{30.0\ \text{gal}}{7.00\ \text{min}}\right)\left(\frac{1\ \text{mi}}{60\ \text{s}}\right) = \boxed{7.14\times10^{-2}\ \frac{\text{gal}}{\text{s}}}$$

(b) $$\text{rate} = 7.14\times10^{-2}\ \frac{\text{gal}}{\text{s}}\left(\frac{231\ \text{in}^3}{1\ \text{gal}}\right)\left(\frac{2.54\ \text{cm}}{1\ \text{in}}\right)^3\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3$$
$$= \boxed{2.70\times10^{-4}\ \frac{\text{m}^3}{\text{s}}}$$

(c) To find the time to fill a 1.00-m^3 tank, find the rate time/volume:

$$2.70\times10^{-4}\ \frac{\text{m}^3}{\text{s}} = \left(\frac{2.70\times10^{-4}\ \text{m}^3}{1\ \text{s}}\right)$$

or $$\left(\frac{2.70\times10^{-4}\ \text{m}^3}{1\ \text{s}}\right)^{-1} = \left(\frac{1\ \text{s}}{2.70\times10^{-4}\ \text{m}^3}\right) = 3.70\times10^3\ \frac{\text{s}}{\text{m}^3}$$

and so: $$3.70\times10^3\ \text{s}\left(\frac{1\ \text{h}}{3\ 600\ \text{s}}\right) = \boxed{1.03\ \text{h}}$$

***P1.23** It is often useful to remember that the 1 600-m race at track and field events is approximately 1 mile in length. To be precise, there are 1 609 meters in a mile. Thus, 1 acre is equal in area to

$$(1\ \text{acre})\left(\frac{1\ \text{mi}^2}{640\ \text{acres}}\right)\left(\frac{1\ 609\ \text{m}}{\text{mi}}\right)^2 = \boxed{4.05\times10^3\ \text{m}^2}$$

***P1.24** The volume of the interior of the house is the product of its length, width, and height. We use the conversion 1 ft = 0.304 8 m and 100 cm = 1 m.

$$V = LWH$$
$$= (50.0\ \text{ft})\left(\frac{0.304\ 8\ \text{m}}{1\ \text{ft}}\right)\times(26\ \text{ft})\left(\frac{0.304\ 8\ \text{m}}{1\ \text{ft}}\right)$$
$$\times(8.0\ \text{ft})\left(\frac{0.304\ 8\ \text{m}}{1\ \text{ft}}\right)$$
$$= 294.5\ \text{m}^3 = \boxed{290\ \text{m}^3}$$
$$= (294.5\ \text{m}^3)\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3 = \boxed{2.9\times10^8\ \text{cm}^3}$$

Both the 26-ft width and 8.0-ft height of the house have two significant figures, which is why our answer was rounded to 290 m^3.

P1.25 The aluminum sphere must be larger in volume to compensate for its lower density. We require equal masses:

$$m_{Al} = m_{Fe} \qquad \text{or} \qquad \rho_{Al}V_{Al} = \rho_{Fe}V_{Fe}$$

then use the volume of a sphere. By substitution,

$$\rho_{Al}\left(\frac{4}{3}\pi r_{Al}{}^3\right) = \rho_{Fe}\left(\frac{4}{3}\pi(2.00\text{ cm})^3\right)$$

Now solving for the unknown,

$$r_{Al}^3 = \left(\frac{\rho_{Fe}}{\rho_{Al}}\right)(2.00\text{ cm})^3 = \left(\frac{7.86\times10^3\text{ kg/m}^3}{2.70\times10^3\text{ kg/m}^3}\right)(2.00\text{ cm})^3$$
$$= 23.3\text{ cm}^3$$

Taking the cube root, $\boxed{r_{Al} = 2.86\text{ cm}}$.

The aluminum sphere is 43% larger than the iron one in radius, diameter, and circumference. Volume is proportional to the cube of the linear dimension, so this excess in linear size gives it the (1.43)(1.43)(1.43) = 2.92 times larger volume it needs for equal mass.

P1.26 The mass of each sphere is $m_{Al} = \rho_{Al}V_{Al} = \dfrac{4\pi\rho_{Al}r_{Al}^3}{3}$

and $m_{Fe} = \rho_{Fe}V_{Fe} = \dfrac{4\pi\rho_{Fe}r_{Fe}^3}{3}$. Setting these masses equal,

$$\frac{4}{3}\pi\rho_{Al}r_{Al}^3 = \frac{4}{3}\pi\rho_{Fe}r_{Fe}^3 \to r_{Al} = r_{Fe}\sqrt[3]{\frac{\rho_{Fe}}{\rho_{Al}}}$$
$$r_{Al} = r_{Fe}\sqrt[3]{\frac{7.86}{2.70}} = \boxed{r_{Fe}(1.43)}$$

The resulting expression shows that the radius of the aluminum sphere is directly proportional to the radius of the balancing iron sphere. The aluminum sphere is 43% larger than the iron one in radius, diameter, and circumference. Volume is proportional to the cube of the linear dimension, so this excess in linear size gives it the $(1.43)^3$ = 2.92 times larger volume it needs for equal mass.

P1.27 We assume the paint keeps the same volume in the can and on the wall, and model the film on the wall as a rectangular solid, with its volume given by its "footprint" area, which is the area of the wall, multiplied by its thickness *t* perpendicular to this area and assumed to be uniform. Then,

$$V = At \quad \text{gives} \quad t = \frac{V}{A} = \frac{3.78 \times 10^{-3}\ \text{m}^3}{25.0\ \text{m}^2} = \boxed{1.51 \times 10^{-4}\ \text{m}}$$

The thickness of 1.5 tenths of a millimeter is comparable to the thickness of a sheet of paper, so this answer is reasonable. The film is many molecules thick.

P1.28 (a) To obtain the volume, we multiply the length, width, and height of the room, and use the conversion 1 m = 3.281 ft.

$$\begin{aligned} V &= (40.0\ \text{m})(20.0\ \text{m})(12.0\ \text{m}) \\ &= \left(9.60 \times 10^3\ \text{m}^3\right)\left(\frac{3.281\ \text{ft}}{1\ \text{m}}\right)^3 \\ &= \boxed{3.39 \times 10^5\ \text{ft}^3} \end{aligned}$$

(b) The mass of the air is

$$m = \rho_{\text{air}} V = \left(1.20\ \text{kg/m}^3\right)\left(9.60 \times 10^3\ \text{m}^3\right) = 1.15 \times 10^4\ \text{kg}$$

The student must look up the definition of weight in the index to find

$$F_g = mg = \left(1.15 \times 10^4\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = 1.13 \times 10^5\ \text{N}$$

where the unit of N of force (weight) is newtons.

Converting newtons to pounds,

$$F_g = (1.13 \times 10^5\ \text{N})\left(\frac{1\ \text{lb}}{4.448\ \text{N}}\right) = \boxed{2.54 \times 10^4\ \text{lb}}$$

P1.29 (a) The time interval required to repay the debt will be calculated by dividing the total debt by the rate at which it is repaid.

$$T = \frac{\$16\ \text{trillion}}{\$1000/\text{s}} = \frac{\$16 \times 10^{12}}{(\$1000/\text{s})\left(3.156 \times 10^7\ \text{s/yr}\right)} = \boxed{507\ \text{yr}}$$

(b) The number of bills is the distance to the Moon divided by the length of a dollar.

$$N = \frac{D}{\ell} = \frac{3.84 \times 10^8\ \text{m}}{0.155\ \text{m}} = \boxed{2.48 \times 10^9\ \text{bills}}$$

Sixteen trillion dollars is larger than this two-and-a-half billion dollars by more than six thousand times. The ribbon of bills

comprising the debt reaches across the cosmic gulf thousands of times. Similar calculations show that the bills could span the distance between the Earth and the Sun sixteen times. The strip could encircle the Earth's equator nearly 62 000 times. With successive turns wound edge to edge without overlapping, the dollars would cover a zone centered on the equator and about 4.2 km wide.

P1.30 (a) To find the scale size of the nucleus, we multiply by the scaling factor

$$d_{\text{nucleus, scale}} = d_{\text{nucleus, real}}\left(\frac{d_{\text{atom, scale}}}{d_{\text{atom, real}}}\right)$$
$$= \left(2.40\times10^{-15}\text{ m}\right)\left(\frac{300\text{ ft}}{1.06\times10^{-10}\text{ m}}\right)$$
$$= 6.79\times10^{-3}\text{ ft}$$

or

$$d_{\text{nucleus, scale}} = \left(6.79\times10^{-3}\text{ ft}\right)\left(\frac{304.8\text{ mm}}{1\text{ ft}}\right) = \boxed{2.07\text{ mm}}$$

(b) The ratio of volumes is simply the ratio of the cubes of the radii:

$$\frac{V_{\text{atom}}}{V_{\text{nucleus}}} = \frac{4\pi r_{\text{atom}}^3/3}{4\pi r_{\text{nucleus}}^3/3} = \left(\frac{r_{\text{atom}}}{r_{\text{nucleus}}}\right)^3 = \left(\frac{d_{\text{atom}}}{d_{\text{nucleus}}}\right)^3$$
$$= \left(\frac{1.06\times10^{-10}\text{ m}}{2.40\times10^{-15}\text{ m}}\right)^3 = \boxed{8.62\times10^{13}\text{ times as large}}$$

Section 1.5 Estimates and Order-of-Magnitude Calculations

P1.31 Since we are only asked to find an estimate, we do not need to be too concerned about how the balls are arranged. Therefore, to find the number of balls we can simply divide the volume of an average-size living room (perhaps 15 ft × 20 ft × 8 ft) by the volume of an individual Ping-Pong ball. Using the approximate conversion 1 ft = 30 cm, we find

$$V_{\text{Room}} = (15\text{ ft})(20\text{ ft})(8\text{ ft})(30\text{ cm/ft})^3 \approx 6\times10^7\text{ cm}^3$$

A Ping-Pong ball has a diameter of about 3 cm, so we can estimate its volume as a cube:

$$V_{\text{ball}} = (3\text{ cm})(3\text{ cm})(3\text{ cm}) \approx 30\text{ cm}^3$$

The number of Ping-Pong balls that can fill the room is

$$N \approx \frac{V_{\text{Room}}}{V_{\text{ball}}} \approx 2 \times 10^6 \text{ balls} \sim \boxed{10^6 \text{ balls}}$$

So a typical room can hold on the order of a million Ping-Pong balls. As an aside, the actual number is smaller than this because there will be a lot of space in the room that cannot be covered by balls. In fact, even in the best arrangement, the so-called "best packing fraction" is $\frac{1}{6}\pi\sqrt{2} = 0.74$, so that at least 26% of the space will be empty.

P1.32 (a) We estimate the mass of the water in the bathtub. Assume the tub measures 1.3 m by 0.5 m by 0.3 m. One-half of its volume is then

$$V = (0.5)(1.3)(0.5)(0.3) = 0.10 \text{ m}^3$$

The mass of this volume of water is

$$m_{\text{water}} = \rho_{\text{water}} V = \left(1\,000 \text{ kg/m}^3\right)\left(0.10 \text{ m}^3\right) = 100 \text{ kg} \boxed{\sim 10^2 \text{ kg}}$$

(b) Pennies are now mostly zinc, but consider copper pennies filling 50% of the volume of the tub. The mass of copper required is

$$m_{\text{copper}} = \rho_{\text{copper}} V = \left(8\,920 \text{ kg/m}^3\right)\left(0.10 \text{ m}^3\right) = 892 \text{ kg} \boxed{\sim 10^3 \text{ kg}}$$

P1.33 Don't reach for the telephone book or do a Google search! Think. Each full-time piano tuner must keep busy enough to earn a living. Assume a total population of 10^7 people. Also, let us estimate that one person in one hundred owns a piano. Assume that in one year a single piano tuner can service about 1 000 pianos (about 4 per day for 250 weekdays), and that each piano is tuned once per year.

Therefore, the number of tuners

$$= \left(\frac{1 \text{ tuner}}{1\,000 \text{ pianos}}\right)\left(\frac{1 \text{ piano}}{100 \text{ people}}\right)\left(10^7 \text{ people}\right) \sim \boxed{100 \text{ tuners}}$$

If you did reach for an Internet directory, you would have to count. Instead, have faith in your estimate. Fermi's own ability in making an order-of-magnitude estimate is exemplified by his measurement of the energy output of the first nuclear bomb (the Trinity test at Alamogordo, New Mexico) by observing the fall of bits of paper as the blast wave swept past his station, 14 km away from ground zero.

P1.34 A reasonable guess for the diameter of a tire might be 2.5 ft, with a circumference of about 8 ft. Thus, the tire would make

$$(50\,000 \text{ mi})(5\,280 \text{ ft/mi})(1 \text{ rev/8 ft}) = 3 \times 10^7 \text{ rev} \sim \boxed{10^7 \text{ rev}}$$

Section 1.6 Significant Figures

P1.35 We will use two different methods to determine the area of the plate and the uncertainty in our answer.

METHOD ONE: We treat the best value with its uncertainty as a binomial, (21.3 ± 0.2) cm × (9.8 ± 0.1) cm, and obtain the area by expanding:

$$A = \left[21.3(9.8) \pm 21.3(0.1) \pm 0.2(9.8) \pm (0.2)(0.1)\right] \text{cm}^2$$

The first term gives the best value of the area. The cross terms add together to give the uncertainty and the fourth term is negligible.

$$A = \boxed{209\ \text{cm}^2 \pm 4\ \text{cm}^2}$$

METHOD TWO: We add the fractional uncertainties in the data.

$$A = (21.3\ \text{cm})(9.8\ \text{cm}) \pm \left(\frac{0.2}{21.3} + \frac{0.1}{9.8}\right)$$

$$= 209\ \text{cm}^2 \pm 2\% = 209\ \text{cm}^2 \pm 4\ \text{cm}^2$$

P1.36 (a) The ± 0.2 following the 78.9 expresses uncertainty in the last digit. Therefore, there are $\boxed{\textbf{three}}$ significant figures in 78.9 ± 0.2.

(b) Scientific notation is often used to remove the ambiguity of the number of significant figures in a number. Therefore, all the digits in 3.788 are significant, and 3.788×10^9 has $\boxed{\textbf{four}}$ significant figures.

(c) Similarly, 2.46 has three significant figures, therefore 2.46×10^{-6} has $\boxed{\textbf{three}}$ significant figures.

(d) Zeros used to position the decimal point are not significant. Therefore 0.005 3 has $\boxed{\textbf{two}}$ significant figures.

Uncertainty in a measurement can be the result of a number of factors, including the skill of the person doing the measurements, the precision and the quality of the instrument used, and the number of measurements made.

P1.37 We work to nine significant digits:

$$1\ \text{yr} = 1\ \text{yr}\left(\frac{365.\ 242\ 199\ \text{d}}{1\ \text{yr}}\right)\left(\frac{24\ \text{h}}{1\ \text{d}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right)$$

$$= \boxed{315\ 569\ 26.0\ \text{s}}$$

P1.38 (a) $756 + 37.2 + 0.83 + 2 = 796.03 \rightarrow \boxed{796}$, since the number with the fewest decimal places is 2.

(b) $(0.003\ 2)\{2 \text{ s.f.}\} \times (356.3)\{4 \text{ s.f.}\} = 1.140\ 16 = \{2 \text{ s.f.}\}$ $\boxed{1.1}$

(c) $5.620\{4 \text{ s.f.}\} \times \pi\{> 4 \text{ s.f.}\} = 17.656 = \{4 \text{ s.f.}\}$ $\boxed{17.66}$

P1.39 Let o represent the number of ordinary cars and s the number of sport utility vehicles. We know $o = s + 0.947s = 1.947s$, and $o = s + 18$.

We eliminate o by substitution:

$$s + 18 = 1.947s \rightarrow 0.947s = 18 \rightarrow s = 18 / 0.947 = \boxed{19}$$

P1.40 "One and one-third months" = 4/3 months. Treat this problem as a conversion:

$$\left(\frac{1 \text{ bar}}{4/3 \text{ months}}\right)\left(\frac{12 \text{ months}}{1 \text{ year}}\right) = \boxed{9 \text{ bars/year}}$$

P1.41 The tax amount is \$1.36 – \$1.25 = \$0.11. The tax rate is

$$\$0.11/\$1.25 = 0.0880 = \boxed{8.80\%}$$

P1.42 We are given the ratio of the masses and radii of the planets Uranus and Neptune:

$$\frac{M_N}{M_U} = 1.19, \text{ and } \frac{r_N}{r_U} = 0.969$$

The definition of density is $\rho = \dfrac{\text{mass}}{\text{volume}} = \dfrac{M}{V}$, where $V = \dfrac{4}{3}\pi r^3$ for a sphere, and we assume the planets have a spherical shape.

We know $\rho_U = 1.27 \times 10^3 \text{ kg/m}^3$. Compare densities:

$$\frac{\rho_N}{\rho_U} = \frac{M_N / V_N}{M_U / V_U} = \left(\frac{M_N}{M_U}\right)\left(\frac{V_U}{V_N}\right) = \left(\frac{M_N}{M_U}\right)\left(\frac{r_U}{r_N}\right)^3$$

$$= (1.19)\left(\frac{1}{0.969}\right)^3 = 1.307\ 9$$

which gives

$$\rho_N = (1.3079)(1.27 \times 10^3 \text{ kg/m}^3) = \boxed{1.66 \times 10^3 \text{ kg/m}^3}$$

P1.43 Let s represent the number of sparrows and m the number of more interesting birds. We know $s/m = 2.25$ and $s + m = 91$.

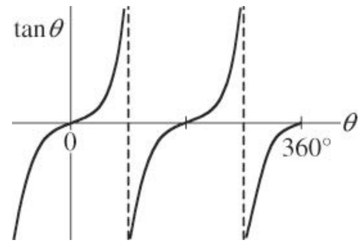
We eliminate m by substitution:

$$m = s/2.25 \rightarrow s + s/2.25 = 91 \rightarrow 1.444s = 91$$

$$\rightarrow s = 91/1.444 = \boxed{63}$$

P1.44 We require

$$\sin\theta = -3\cos\theta, \text{ or } \frac{\sin\theta}{\cos\theta} = \tan\theta = -3$$

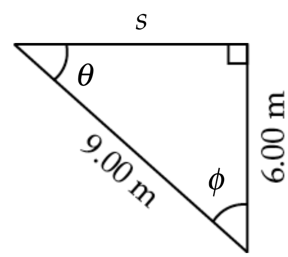


ANS. FIG. P1.44

For $\tan^{-1}(-3) = \arctan(-3)$, your calculator may return –71.6°, but this angle is not between 0° and 360° as the problem requires. The tangent function is negative in the second quadrant (between 90° and 180°) and in the fourth quadrant (from 270° to 360°). The solutions to the equation are then

$$360° - 71.6° = \boxed{288°} \text{ and } 180° - 71.6 = \boxed{108°}$$

***P1.45** (a) ANS. FIG. P1.45 shows that the hypotenuse of the right triangle has a length of 9.00 m and the unknown side is opposite the angle ϕ. Since the two angles in the triangle are not known, we can obtain the length of the unknown side, which we will represent as s, using the Pythagorean Theorem:



ANS. FIG. P1.45

$$s^2 + (6.00\text{ m})^2 = (9.00\text{ m})^2$$
$$s^2 = (9.00\text{ m})^2 - (6.00\text{ m})^2 = 45$$

which gives $s = \boxed{6.71\text{ m}}$. We express all of our answers in three significant figures since the lengths of the two known sides of the triangle are given with three significant figures.

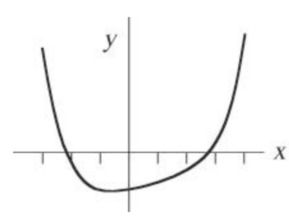
(b) From ANS. FIG. P1.45, the tangent of θ is equal to ratio of the side opposite the angle, 6.00 m in length, and the side adjacent to the angle, s = 6.71 m, and is given by

$$\tan\theta = \frac{6.00\text{ m}}{s} = \frac{6.00\text{ m}}{6.71\text{ m}} = \boxed{0.894}$$

(c) From ANS. FIG. P1.45, the sine of ϕ is equal to ratio of the side opposite the angle, s = 6.71 m, and the hypotenuse of the triangle, 9.00 m in length, and is given by

$$\sin\phi = \frac{s}{9.00\text{ m}} = \frac{6.71\text{ m}}{9.00\text{ m}} = \boxed{0.745}$$

P1.46 For those who are not familiar with solving equations numerically, we provide a detailed solution. It goes beyond proving that the suggested answer works.



ANS. FIG. P1.46

The equation $2x^4 - 3x^3 + 5x - 70 = 0$ is quartic, so

we do not attempt to solve it with algebra. To find how many real solutions the equation has and to estimate them, we graph the expression:

x	–3	–2	–1	0	1	2	3	4
$y = 2x^4 - 3x^3 + 5x - 70$	158	–24	–70	–70	–66	–52	26	270

We see that the equation $y = 0$ has two roots, one around $x = -2.2$ and the other near $x = +2.7$. To home in on the first of these solutions we compute in sequence:

When $x = -2.2$, $y = -2.20$. The root must be between $x = -2.2$ and $x = -3$.
When $x = -2.3$, $y = 11.0$. The root is between $x = -2.2$ and $x = -2.3$.
When $x = -2.23$, $y = 1.58$. The root is between $x = -2.20$ and $x = -2.23$.
When $x = -2.22$, $y = 0.301$. The root is between $x = -2.20$ and -2.22.
When $x = -2.215$, $y = -0.331$. The root is between $x = -2.215$ and -2.22.
We could next try $x = -2.218$, but we already know to three-digit precision that the root is $x = -2.22$.

P1.47 When the length changes by 15.8%, the mass changes by a much larger percentage. We will write each of the sentences in the problem as a mathematical equation.

Mass is proportional to length cubed: $m = k\ell^3$, where k is a constant. This model of growth is reasonable because the lamb gets thicker as it gets longer, growing in three-dimensional space.

At the initial and final points, $m_i = k\ell_i^3$ and $m_f = k\ell_f^3$

Length changes by 15.8%: 15.8% **of** ℓ means 0.158 **times** ℓ.

Thus $\ell_i + 0.158\,\ell_i = \ell_f$ and $\ell_f = 1.158\,\ell_i$

Mass increases by 17.3 kg: $m_i + 17.3\text{ kg} = m_f$

Now we combine the equations using algebra, eliminating the unknowns ℓ_i, ℓ_f, k, and m_i by substitution:

From $\ell_f = 1.158\,\ell_i$, we have $\ell_f^3 = 1.158^3\,\ell_i^3 = 1.553\,\ell_i^3$

Then

$$m_f = k\ell_f^3 = k(1.553)\ell_i^3 = 1.553k\ell_i^3 = 1.553m_i \quad \text{and} \quad m_i = m_f/1.553$$

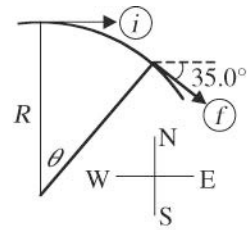
Next,

$$m_i + 17.3\text{ kg} = m_f \quad \text{becomes} \quad m_f/1.553 + 17.3\text{ kg} = m_f$$

Solving, $17.3\text{ kg} = m_f - m_f/1.553 = m_f(1 - 1/1.553) = 0.356\,m_f$

and $$m_f = \frac{17.3\text{ kg}}{0.356} = \boxed{48.6\text{ kg}}.$$

P1.48 We draw the radius to the initial point and the radius to the final point. The angle θ between these two radii has its sides perpendicular, right side to right side and left side to left side, to the 35° angle between the original and final tangential directions of travel. A most useful theorem from geometry then identifies these angles as equal: $\theta = 35°$. The whole circumference of a 360° circle of the same radius is $2\pi R$. By proportion, then



ANS. FIG. P1.48

$$\frac{2\pi R}{360°} = \frac{840\text{ m}}{35°}$$

$$R = \left(\frac{360°}{2\pi}\right)\left(\frac{840\text{ m}}{35°}\right) = \frac{840\text{ m}}{0.611} = \boxed{1.38 \times 10^3\text{ m}}$$

We could equally well say that the measure of the angle in radians is

$$\theta = 35° = 35°\left(\frac{2\pi\text{ radians}}{360°}\right) = 0.611\text{ rad} = \frac{840\text{ m}}{R}$$

Solving yields $R = 1.38$ km.

P1.49 Use substitution to solve simultaneous equations. We substitute $p = 3q$ into each of the other two equations to eliminate p:

$$\begin{cases} 3qr = qs \\ \frac{1}{2}3qr^2 + \frac{1}{2}qs^2 = \frac{1}{2}qt^2 \end{cases}$$

These simplify to $\begin{cases} 3r = s \\ 3r^2 + s^2 = t^2 \end{cases}$, assuming $q \neq 0$.

We substitute the upper relation into the lower equation to eliminate s:

$$3r^2 + (3r)^2 = t^2 \rightarrow 12r^2 = t^2 \rightarrow \frac{t^2}{r^2} = 12$$

We now have the ratio of t to r: $\boxed{\frac{t}{r} = \pm\sqrt{12} = \pm 3.46}$

P1.50 First, solve the given equation for Δt:

$$\Delta t = \frac{4QL}{k\pi d^2(T_h - T_c)} = \left[\frac{4QL}{k\pi(T_h - T_c)}\right]\left[\frac{1}{d^2}\right]$$

(a) Making d three times larger with d^2 in the bottom of the fraction makes Δt $\boxed{\text{nine times smaller}}$.

(b) $\boxed{\Delta t \text{ is inversely proportional to the square of } d.}$

(c) $\boxed{\text{Plot } \Delta t \text{ on the vertical axis and } 1/d^2 \text{ on the horizontal axis.}}$

(d) From the last version of the equation, the slope is $\boxed{4QL / k\pi\left(T_h - T_c\right)}$. Note that this quantity is constant as both Δt and d vary.

P1.51 (a) The fourth experimental point from the top is a circle: this point lies just above the best-fit curve that passes through the point (400 cm^2, 0.20 g). The interval between horizontal grid lines is 1 space = 0.05 g. We estimate from the graph that the circle has a vertical separation of 0.3 spaces = $\boxed{0.015 \text{ g}}$ above the best-fit curve.

(b) The best-fit curve passes through 0.20 g:

$$\left(\frac{0.015 \text{ g}}{0.20 \text{ g}}\right) \times 100 = \boxed{8\%}$$

(c) The best-fit curve passes through the origin and the point (600 cm^3, 3.1 g). Therefore, the slope of the best-fit curve is

$$\text{slope} = \left(\frac{3.1 \text{ g}}{600 \text{ cm}^3}\right) = \boxed{5.2 \times 10^{-3} \frac{\text{g}}{\text{cm}^3}}$$

(d) $\boxed{\text{For shapes cut from this copy paper, the mass of the cutout is proportional to its area. The proportionality constant is } 5.2 \text{ g/m}^2 \pm 8\%\text{, where the uncertainty is estimated.}}$

(e) $\boxed{\text{This result is to be expected if the paper has thickness and density that are uniform within the experimental uncertainty.}}$

(f) $\boxed{\text{The slope is the areal density of the paper, its mass per unit area.}}$

P1.52 $r = (6.50 \pm 0.20) \text{ cm} = (6.50 \pm 0.20) \times 10^{-2} \text{ m}$

$$m = (1.85 + 0.02) \text{kg}$$

$$\rho = \frac{m}{\left(\frac{4}{3}\right)\pi r^3}$$

also, $$\frac{\delta\rho}{\rho} = \frac{\delta m}{m} + \frac{3\delta r}{r}$$

In other words, the percentages of uncertainty are cumulative. Therefore,

$$\frac{\delta\rho}{\rho} = \frac{0.02}{1.85} + \frac{3(0.20)}{6.50} = 0.103,$$

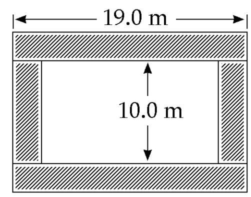
$$\rho = \frac{1.85}{\left(\frac{4}{3}\right)\pi\left(6.5\times10^{-2}\text{ m}\right)^3} = \boxed{1.61\times10^3\text{ kg/m}^3}$$

then $\delta\rho = 0.103\rho = \boxed{0.166\times10^3\text{ kg/m}^3}$

and $\rho \pm \delta\rho = \boxed{(1.61\pm0.17)\times10^3\text{ kg/m}^3} = (1.6\pm0.2)\times10^3\text{ kg/m}^3.$

***P1.53** The volume of concrete needed is the sum of the four sides of sidewalk, or

$$V = 2V_1 + 2V_2 = 2(V_1 + V_2)$$



ANS. FIG. P1.53

The figure on the right gives the dimensions needed to determine the volume of each portion of sidewalk:

$$V_1 = (17.0\text{ m} + 1.0\text{ m} + 1.0\text{ m})(1.0\text{ m})(0.09\text{ m}) = 1.70\text{ m}^3$$

$$V_2 = (10.0\text{ m})(1.0\text{ m})(0.090\text{ m}) = 0.900\text{ m}^3$$

$$V = 2(1.70\text{ m}^3 + 0.900\text{ m}^3) = \boxed{5.2\text{ m}^3}$$

The uncertainty in the volume is the sum of the uncertainties in each dimension:

$$\left.\begin{aligned}\frac{\delta\ell_1}{\ell_1} &= \frac{0.12\text{ m}}{19.0\text{ m}} = 0.0063\\ \frac{\delta w_1}{w_1} &= \frac{0.01\text{ m}}{1.0\text{ m}} = 0.010\\ \frac{\delta t_1}{t_1} &= \frac{0.1\text{ cm}}{9.0\text{ cm}} = 0.011\end{aligned}\right\}\frac{\delta V}{V} = 0.006 + 0.010 + 0.011 = 0.027 = \boxed{3\%}$$

Additional Problems

P1.54 (a) Let d represent the diameter of the coin and h its thickness. The gold plating is a layer of thickness t on the surface of the coin; so, the mass of the gold is

$$m = \rho V = \rho\left[2\pi\frac{d^2}{4} + \pi dh\right]t$$

$$= \left(19.3\frac{\text{g}}{\text{cm}^3}\right)\left[2\pi\frac{(2.41\text{ cm})^2}{4} + \pi(2.41\text{ cm})(0.178\text{ cm})\right]$$

$$\times\left(1.8\times10^{-7}\text{ m}\right)\left(\frac{10^2\text{ cm}}{1\text{ m}}\right)$$

$$= 0.003\ 64\text{ g}$$

and the cost of the gold added to the coin is

$$\text{cost} = (0.003\ 64\text{ g})\left(\frac{\$10}{1\text{ g}}\right) = \$0.036\ 4 = \boxed{3.64\text{ cents}}$$

(b) $\boxed{\text{The cost is negligible compared to \$4.98.}}$

P1.55 It is desired to find the distance x such that

$$\frac{x}{100\text{ m}} = \frac{1\ 000\text{ m}}{x}$$

(i.e., such that x is the same multiple of 100 m as the multiple that 1 000 m is of x). Thus, it is seen that

$$x^2 = (100\text{ m})(1\ 000\text{ m}) = 1.00 \times 10^5\text{ m}^2$$

and therefore

$$x = \sqrt{1.00\times10^5\text{ m}^2} = \boxed{316\text{ m}}$$

P1.56 (a) A Google search yields the following dimensions of the intestinal tract:

small intestines: length $\cong$ 20 ft $\cong$ 6 m, diameter $\cong$ 1.5 in $\cong$ 4 cm

large intestines: length $\cong$ 5 ft $\cong$ 1.5 m, diameter $\cong$ 2.5 in $\cong$ 6 cm

Treat the intestines as two cylinders: the volume of a cylinder of diameter d and length L is $V = \frac{\pi}{4}d^2L$.

The volume of the intestinal tract is

$$V = V_{\text{small}} + V_{\text{large}}$$

$$V = \frac{\pi}{4}(0.04\,\text{m})^2(6\,\text{m}) + \frac{\pi}{4}(0.06\,\text{m})^2(1.5\,\text{m})$$

$$= 0.0117\,\text{m}^3 \cong 10^{-2}\,\text{m}^3$$

Assuming 1% of this volume is occupied by bacteria, the volume of bacteria is

$$V_{\text{bac}} = (10^{-2}\ \text{m}^3)(0.01) = 10^{-4}\ \text{m}^3$$

Treating a bacterium as a cube of side $L = 10^{-6}$ m, the volume of one bacterium is about $L^3 = 10^{-18}$ m^3. The number of bacteria in the intestinal tract is about

$$(10^{-4}\ \text{m}^3)\left(\frac{1\ \text{bacterium}}{10^{-18}\ \text{m}^3}\right) = \boxed{10^{14}\ \text{bacteria!}}$$

(b) The large number of bacteria suggests they must be $\boxed{\text{beneficial}}$, otherwise the body would have developed methods a long time ago to reduce their number. It is well known that certain types of bacteria in the intestinal tract are beneficial: they aid digestion, as well as prevent dangerous bacteria from flourishing in the intestines.

P1.57 We simply multiply the distance between the two galaxies by the scale factor used for the dinner plates. The scale factor used in the "dinner plate" model is

$$S = \left(\frac{0.25\ \text{m}}{1.0\times 10^5\ \text{light-years}}\right) = 2.5\times 10^{-6}\ \text{m/ly}$$

The distance to Andromeda in the scale model will be

$$D_{\text{scale}} = D_{\text{actual}}S = (2.0\times 10^6\ \text{ly})(2.5\times 10^{-6}\ \text{m/ly}) = \boxed{5.0\ \text{m}}$$

P1.58 Assume the winner counts one dollar per second, and the winner tries to maintain the count without stopping. The time interval required for the task would be

$$\$10^6\left(\frac{1\ \text{s}}{\$1}\right)\left(\frac{1\ \text{hour}}{3600\ \text{s}}\right)\left(\frac{1\ \text{work week}}{40\ \text{hours}}\right) = 6.9\ \text{work weeks.}$$

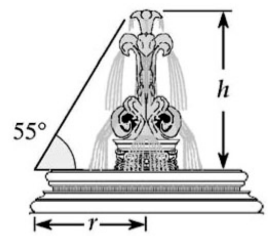
The scenario has the contestants succeeding on the whole. But the calculation shows that is impossible. It just takes too long!

P1.59 We imagine a top view to figure the radius of the pool from its circumference. We imagine a straight-on side view to use trigonometry to find the height.

Define a right triangle whose legs represent the height and radius of the fountain. From the dimensions of the fountain and the triangle, the circumference is $C = 2\pi r$ and the angle satisfies $\tan\phi = h/r$.

Then by substitution

$$h = r\tan\phi = \left(\frac{C}{2\pi}\right)\tan\phi$$



ANS. FIG. P1.59

Evaluating,

$$h = \left(\frac{15.0\text{ m}}{2\pi}\right)\tan 55.0° = \boxed{3.41\text{ m}}$$

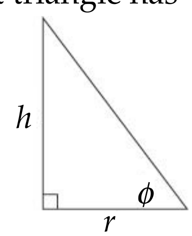
When we look at a three-dimensional system from a particular direction, we may discover a view to which simple mathematics applies.

P1.60 The fountain has height h; the pool has circumference C with radius r. The figure shows the geometry of the problem: a right triangle has base r, height h, and angle ϕ. From the triangle,

$$\tan\phi = h/r$$

We can find the radius of the circle from its circumference, $C = 2\pi r$, and then solve for the height using

$$\boxed{h = r\tan\phi = (\tan\phi)C/2\pi}$$



ANS. FIG. P1.60

P1.61 The density of each material is $\rho = \dfrac{m}{V} = \dfrac{m}{\pi r^2 h} = \dfrac{4m}{\pi D^2 h}$.

$$\text{Al: } \rho = \frac{4(51.5\text{ g})}{\pi(2.52\text{ cm})^2(3.75\text{ cm})} = \boxed{2.75\frac{\text{g}}{\text{cm}^3}\text{; this is 2\% larger}}$$

than the tabulated value, 2.70 g/cm^3.

$$\text{Cu: } \rho = \frac{4(56.3\text{ g})}{\pi(1.23\text{ cm})^2(5.06\text{ cm})} = \boxed{9.36\frac{\text{g}}{\text{cm}^3}\text{; this is 5\% larger}}$$

than the tabulated value, 8.92 g/cm^3.

$$\text{brass: } \rho = \frac{4(94.4\text{ g})}{\pi(1.54\text{ cm})^2(5.69\text{ cm})} = \boxed{8.91\frac{\text{g}}{\text{cm}^3}\text{; this is 5\% larger}}$$

than the tabulated value, 8.47 g/cm³.

$$\text{Sn: } \rho = \frac{4(69.1\text{ g})}{\pi(1.75\text{ cm})^2(3.74\text{ cm})} = \boxed{7.68\frac{\text{g}}{\text{cm}^3}\text{; this is 5\% larger}}$$

than the tabulated value, 7.31 g/cm³.

$$\text{Fe: } \rho = \frac{4(216.1\text{ g})}{\pi(1.89\text{ cm})^2(9.77\text{ cm})} = \boxed{7.88\frac{\text{g}}{\text{cm}^3}\text{; this is 0.3\% larger}}$$

than the tabulated value, 7.86 g/cm³.

P1.62 The volume of the galaxy is

$$\pi r^2 t = \pi\left(10^{21}\text{ m}\right)^2\left(10^{19}\text{ m}\right) \sim 10^{61}\text{ m}^3$$

If the distance between stars is 4×10^{16}, then there is one star in a volume on the order of

$$\left(4\times 10^{16}\text{ m}\right)^3 \sim 10^{50}\text{ m}^3$$

The number of stars is about $\dfrac{10^{61}\text{ m}^3}{10^{50}\text{ m}^3/\text{star}} \sim \boxed{10^{11}\text{ stars}}$.

P1.63 We define an average national fuel consumption rate based upon the total miles driven by all cars combined. In symbols,

$$\text{fuel consumed} = \frac{\text{total miles driven}}{\text{average fuel consumption rate}}$$

or

$$f = \frac{s}{c}$$

For the current rate of 20 mi/gallon we have

$$f = \frac{\left(100 \times 10^6\text{ cars}\right)\left(10^4\text{ (mi/yr)/car}\right)}{20\text{ mi/gal}} = 5 \times 10^{10}\text{ gal/yr}$$

Since we consider the same total number of miles driven in each case, at 25 mi/gal we have

$$f = \frac{\left(100 \times 10^6\text{ cars}\right)\left(10^4\text{ (mi/yr)/car}\right)}{25\text{ mi/gal}} = 4 \times 10^{10}\text{ gal/yr}$$

Thus we estimate a change in fuel consumption of

$$\Delta f = 4\times 10^{10}\text{ gal/yr} - 5\times 10^{10}\text{ gal/yr} = \boxed{-1\times 10^{10}\text{ gal/yr}}$$

The negative sign indicates that the change is a reduction. It is a fuel savings of ten billion gallons each year.

P1.64 (a) The mass is equal to the mass of a sphere of radius 2.6 cm and density 4.7 g/cm^3, minus the mass of a sphere of radius a and density 4.7 g/cm^3, plus the mass of a sphere of radius a and density 1.23 g/cm^3.

$$m = \rho_1\left(\frac{4}{3}\pi r^3\right) - \rho_1\left(\frac{4}{3}\pi a^3\right) + \rho_2\left(\frac{4}{3}\pi a^3\right)$$

$$= \left(\frac{4}{3}\pi\right)\left[\left(4.7\text{ g/cm}^3\right)\left(2.6\text{ cm}\right)^3 - \left(4.7\text{ g/cm}^3\right)a^3 + \left(1.23\text{ g/cm}^3\right)a^3\right]$$

$$m = \boxed{346\text{ g} - \left(14.5\text{ g/cm}^3\right)a^3}$$

(b) The mass is maximum for $\boxed{a = 0}$.

(c) $\boxed{346\text{ g}}$.

(d) $\boxed{\text{Yes}}$. This is the mass of the uniform sphere we considered in the first term of the calculation.

(e) $\boxed{\text{No change, so long as the wall of the shell is unbroken.}}$

P1.65 Answers may vary depending on assumptions:

typical length of bacterium: $L = 10^{-6}$ m

typical volume of bacterium: $L^3 = 10^{-18}$ m^3

surface area of Earth: $A = 4\pi r^2 = 4\pi\left(6.38\times 10^6\text{ m}\right)^2 = 5.12\times 10^{14}\text{m}^2$

(a) If we assume the bacteria are found to a depth $d = 1000$ m below Earth's surface, the volume of Earth containing bacteria is about

$$V = \left(4\pi r^2\right)d = 5.12\times 10^{17}\text{ m}^3$$

If we assume an average of 1000 bacteria in every 1 mm^3 of volume, then the number of bacteria is

$$\left(\frac{1000\text{ bacteria}}{1\text{ mm}^3}\right)\left(\frac{10^3\text{ mm}}{1\text{ m}}\right)^3(5.12\times 10^{17}\text{ m}^3) \approx \boxed{5.12\times 10^{29}\text{ bacteria}}$$

(b) Assuming a bacterium is basically composed of water, the total mass is

$$\left(10^{29}\text{ bacteria}\right)\left(\frac{10^{-18}\text{ m}^3}{1\text{ bacterium}}\right)\left(\frac{10^3\text{ kg}}{1\text{ m}^3}\right) = \boxed{10^{14}\text{ kg}}$$

P1.66 The rate of volume increase is

$$\frac{dV}{dt}=\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)=\frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}=\left(4\pi r^2\right)\frac{dr}{dt}$$

(a) $\frac{dV}{dt}=4\pi(6.5\text{ cm})^2(0.9\text{ cm/s})=\boxed{478\text{ cm}^3/\text{s}}$

(b) The rate of increase of the balloon's radius is

$$\frac{dr}{dt}=\frac{dV/dt}{4\pi r^2}=\frac{478\text{ cm}^3/\text{s}}{4\pi(13\text{ cm})^2}=\boxed{0.225\text{ cm/s}}$$

(c) When the balloon radius is twice as large, its surface area is four times larger. The new volume added in one second in the inflation process is equal to this larger area times an extra radial thickness that is one-fourth as large as it was when the balloon was smaller.

P1.67 (a) We have $B + C(0) = 2.70\text{ g/cm}^3$ and $B + C(14\text{ cm}) = 19.3\text{ g/cm}^3$.

We know $\boxed{B = 2.70\text{ g/cm}^3}$, and we solve for C by subtracting:

$C(14\text{ cm}) = 19.3\text{ g/cm}^3 - B = 16.6\text{ g/cm}^3$, so $\boxed{C = 1.19\text{ g/cm}^4}$

(b) The integral is

$$\begin{aligned} m &= (9.00\text{ cm}^2)\int_0^{14\text{ cm}}(B+Cx)\,dx \\ &= (9.00\text{ cm}^2)\left(Bx+\frac{C}{2}x^2\right)\Bigg|_0^{14\text{ cm}} \end{aligned}$$

$$\begin{aligned} m &= (9.00\text{ cm}^2)\Big\{\left(2.70\text{ g/cm}^3\right)(14\text{ cm}-0) \\ &\qquad +\left(1.19\text{ g/cm}^4/2\right)\left[(14\text{ cm})^2-0\right]\Big\} \\ &= 340\text{ g}+1046\text{ g}=1390\text{ g}=\boxed{1.39\text{ kg}} \end{aligned}$$

P1.68 The table below shows α in degrees, α in radians, tan(α), and sin(α) for angles from 15.0° to 31.1°:

α' (deg)	α (rad)	tan(α)	sin(α)	difference between α and tan α
15.0	0.262	0.268	0.259	2.30%
20.0	0.349	0.364	0.342	4.09%
30.0	0.524	0.577	0.500	9.32%
33.0	0.576	0.649	0.545	11.3%
31.0	0.541	0.601	0.515	9.95%
31.1	0.543	0.603	0.516	10.02%

We see that α in radians, tan(α), and sin(α) start out together from zero and diverge only slightly in value for small angles. Thus $\boxed{31.0°}$ is the largest angle for which $\dfrac{\tan\alpha - \alpha}{\tan\alpha} < 0.1$.

P1.69 We write "millions of cubic feet" as 10^6 ft^3, and use the given units of time and volume to assign units to the equation.

$$\text{V} = (1.50 \times 10^6 \text{ ft}^3/\text{mo})\text{t} + (0.008\,00 \times 10^6 \text{ ft}^3/\text{mo}^2)\text{t}^2$$

To convert the units to seconds, use

$$1 \text{ month} = (30.0 \text{ d})\left(\frac{24 \text{ h}}{1 \text{ d}}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 2.59 \times 10^6 \text{ s}$$

to obtain

$$\begin{aligned}\text{V} &= \left(1.50 \times 10^6 \frac{\text{ft}^3}{\text{mo}}\right)\left(\frac{1 \text{ mo}}{2.59 \times 10^6 \text{ s}}\right)\text{t} \\ &\quad + \left(0.008\,00 \times 10^6 \frac{\text{ft}^3}{\text{mo}^2}\right)\left(\frac{1 \text{ mo}}{2.59 \times 10^6 \text{ s}}\right)^2 \text{t}^2 \\ &= (0.579 \text{ ft}^3/\text{s})\text{t} + (1.19 \times 10^{-9} \text{ ft}^3/\text{s}^2)\text{t}^2\end{aligned}$$

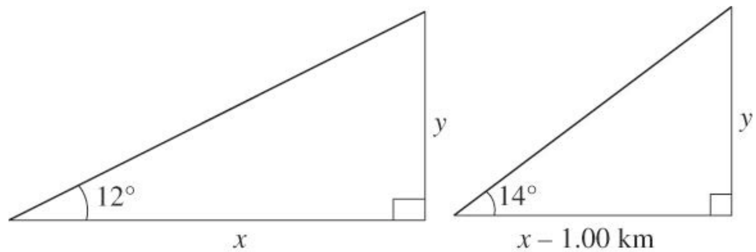
or

$$\text{V} = \boxed{0.579\text{t} + 1.19 \times 10^{-9}\text{t}^2}$$

where *V* is in cubic feet and *t* is in seconds. The coefficient of the first term is the volume rate of flow of gas at the beginning of the month.

The second term's coefficient is related to how much the rate of flow increases every second.

P1.70 (a) and (b), the two triangles are shown.



ANS. FIG. P1.70(a) **ANS. FIG. P1.70(b)**

(c) From the triangles,

$$\tan 12.0° = \frac{y}{x} \rightarrow \boxed{y = x\tan 12.0°}$$

and $$\tan 14.0° = \frac{y}{(x - 1.00\text{ km})} \rightarrow \boxed{y = (x - 1.00\text{ km})\tan 14.0°}.$$

(d) Equating the two expressions for y, we solve to find $\boxed{y = 1.44\text{ km.}}$

P1.71 Observe in Fig. 1.71 that the radius of the horizontal cross section of the bottle is a relative maximum or minimum at the two radii cited in the problem; thus, we recognize that as the liquid level rises, the time rate of change of the diameter of the cross section will be zero at these positions.

The volume of a particular thin cross section of the shampoo of thickness h and area A is $V = Ah$, where $A = \pi r^2 = \pi D^2/4$. Differentiate the volume with respect to time:

$$\frac{dV}{dt} = A\frac{dh}{dt} + h\frac{dA}{dt} = A\frac{dh}{dt} + h\frac{d}{dt}(\pi r^2) = A\frac{dh}{dt} + 2\pi hr\frac{dr}{dt}$$

Because the radii given are a maximum and a minimum value, $dr/dt = 0$, so

$$\frac{dV}{dt} + A\frac{dh}{dt} = Av \rightarrow v = \frac{1}{A}\frac{dV}{dt} = \frac{1}{\pi D^2/4}\frac{dV}{dt} = \frac{4}{\pi D^2}\frac{dV}{dt}$$

where $v = dh/dt$ is the speed with which the level of the fluid rises.

(a) For $D = 6.30$ cm,

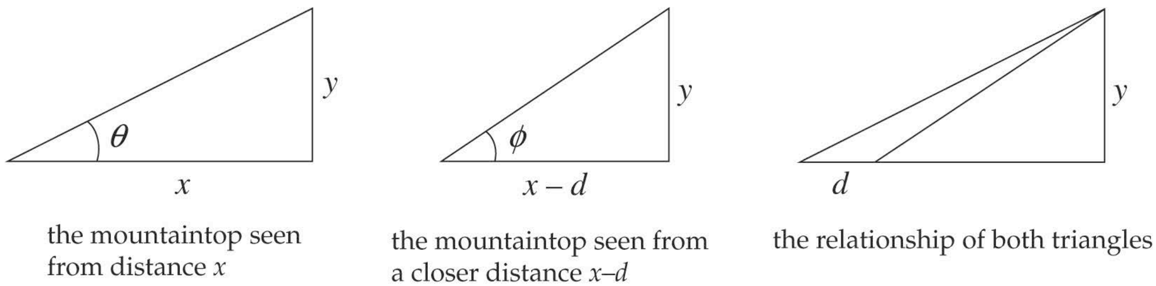
$$v = \frac{4}{\pi(6.30\text{ cm})^2}(16.5\text{ cm}^3/\text{s}) = \boxed{0.529\text{ cm/s}}$$

(b) For D = 1.35 cm,

$$v = \frac{4}{\pi(1.35\text{ cm})^2}(16.5\text{ cm}^3/\text{s}) = \boxed{11.5\text{ cm/s}}$$

Challenge Problems

P1.72 The geometry of the problem is shown below.



ANS. FIG. P1.72

From the triangles in ANS. FIG. P1.72,

$$\tan\theta = \frac{y}{x} \to y = x\tan\theta$$

and

$$\tan\phi = \frac{y}{x-d} \to y = (x-d)\tan\phi$$

Equate these two expressions for y and solve for x:

$$x\tan\theta = (x-d)\tan\phi \to d\tan\phi = x(\tan\phi - \tan\theta)$$

$$\to x = \frac{d\tan\phi}{\tan\phi - \tan\theta}$$

Take the expression for x and substitute it into either expression for y:

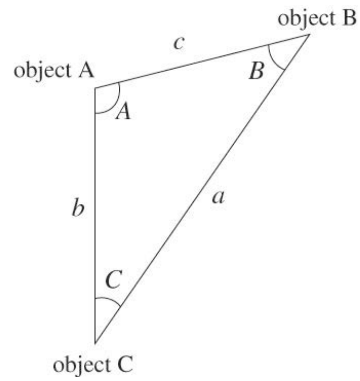
$$y = x\tan\theta = \boxed{\frac{d\tan\phi\tan\theta}{\tan\phi - \tan\theta}}$$

P1.73 The geometry of the problem suggests we use the law of cosines to relate known sides and angles of a triangle to the unknown sides and angles. Recall that the sides a, b, and c with opposite angles A, B, and C have the following relationships:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = c^2 + a^2 - 2ca\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



ANS. FIG. P1.73

For the cows in the meadow, the triangle has sides $a = 25.0$ m and $b = 15.0$ m, and angle $C = 20.0°$, where object A = cow A, object B = cow B, and object C = you.

(a) Find side c:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = (25.0\text{ m})^2 + (15.0\text{ m})^2 - 2(25.0\text{ m})(15.0\text{ m})\cos(20.0°)$$

$$c = \boxed{12.1\text{ m}}$$

(b) Find angle A:

$$a^2 = b^2 + c^2 - 2bc\cos A \rightarrow$$

$$\cos A = \frac{a^2 - b^2 - c^2}{2bc} = \frac{(25.0\text{ m})^2 - (15.0\text{ m})^2 - (12.1\text{ m})^2}{2(15.0\text{ m})(12.1\text{ m})}$$

$$\rightarrow A = 134.8° = \boxed{135°}$$

(c) Find angle B:

$$b^2 = c^2 + a^2 - 2ca\cos B \rightarrow$$

$$\cos B = \frac{b^2 - c^2 - a^2}{2ca} = \frac{(15.0\text{ m})^2 - (25.0\text{ m})^2 - (12.1\text{ m})^2}{2(25.0\text{ m})(12.1\text{ m})}$$

$$\rightarrow B = \boxed{25.2°}$$

(d) For the situation, object A = star A, object B = star B, and object C = our Sun (or Earth); so, the triangle has sides $a = 25.0$ ly, $b = 15.0$ ly, and angle $C = 20.0°$. The numbers are the same, except for units, as in part (b); thus, $\boxed{\text{angle } A = 135°.}$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P1.2 2.15×10^4 kg/m^3

P1.4 (a) 2.3×10^{17} kg/m^3; (b) 1.0×10^{13} times the density of osmium

P1.6 $\dfrac{4\pi\rho\left(r_2^3 - r_1^3\right)}{3}$

P1.8 (a) $8.42 \times 10^{22} \dfrac{\text{Cu-atom}}{\text{cm}^3}$; (b) 1.19×10^{-23} cm^3/Cu-atom; (c) 2.28×10^{-8} cm

P1.10 (a) and (f); (b) and (d); (c) and (e)

P1.12 $\dfrac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$

P1.14 (a) [A] = L/T^3 and [B] = L/T; (b) L/T

P1.16 667 lb/s

P1.18 9.19 nm/s

P1.20 2.57×10^6 m^3

P1.22 (a) $7.14 \times 10^{-2} \dfrac{\text{gal}}{\text{s}}$; (b) $2.70 \times 10^{-4} \dfrac{\text{m}^3}{\text{s}}$; (c) 1.03 h

P1.24 290 m^3, 2.9×10^8 cm^3

P1.26 $r_{\text{Fe}}(1.43)$

P1.28 (a) 3.39×10^5 ft^3; (b) 2.54×10^4 lb

P1.30 (a) 2.07 mm; (b) 8.62×10^{13} times as large

P1.32 (a) ~ 10^2 kg; (b) ~ 10^3 kg

P1.34 10^7 rev

P1.36 (a) 3; (b) 4; (c) 3; (d) 2

P1.38 (a) 796; (b) 1.1; (c) 17.66

P1.40 9 bars / year

P1.42 1.66×10^3 kg/m^3

P1.44 288°; 108°

P1.46 See P1.46 for complete description.

P1.48 1.38×10^3 m

P1.50 (a) nine times smaller; (b) Δt is inversely proportional to the square of d; (c) Plot Δt on the vertical axis and $1/d^2$ on the horizontal axis; (d) $4QL/k\pi\left(T_h - T_c\right)$

P1.52 $1.61\times10^3\ \text{kg/m}^3,\ 0.166\times10^3\ \text{kg/m}^3,\ \left(1.61\pm0.17\right)\times10^3\ \text{kg/m}^3$

P1.54 3.64 cents; the cost is negligible compared to $4.98.

P1.56 (a) 10^{14} bacteria; (b) beneficial

P1.58 The scenario has the contestants succeeding on the whole. But the calculation shows that is impossible. It just takes too long!

P1.60 $h = r\tan\phi = \left(\tan\theta\right)C/2\pi$

P1.62 10^{11} stars

P1.64 (a) $m = 346\text{ g} - (14.5\text{ g/cm}^3)a^3$; (b) a = 0; (c) 346 g; (d) yes; (e) no change

P1.66 (a) 478 cm^3/s; (b) 0.225 cm/s; (c) When the balloon radius is twice as large, its surface area is four times larger. The new volume added in one second in the inflation process is equal to this larger area times an extra radial thickness that is one-fourth as large as it was when the balloon was smaller.

P1.68 31.0°

P1.70 (a-b) see ANS. FIG. P1.70(a) and P1.70(b); (c) $y = x\tan 12.0°$ and $y = (x - 1.00\text{ km})\tan 14.0°$; (d) $y = 1.44$ km

P1.72 $\dfrac{d\tan\phi\tan\theta}{\tan\phi - \tan\theta}$

2

Motion in One Dimension

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ2.1 Count spaces (intervals), not dots. Count 5, not 6. The first drop falls at time zero and the last drop at 5 × 5 s = 25 s. The average speed is 600 m/25 s = 24 m/s, answer (b).

OQ2.2 The initial velocity of the car is $v_0 = 0$ and the velocity at time t is v. The constant acceleration is therefore given by

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0} = \frac{v - 0}{t} = \frac{v}{t}$$

and the average velocity of the car is

$$\bar{v} = \frac{(v + v_0)}{2} = \frac{(v + 0)}{2} = \frac{v}{2}$$

The distance traveled in time t is $\Delta x = \bar{v}t = vt/2$. In the special case where $a = 0$ (and hence $v = v_0 = 0$), we see that statements (a), (b), (c), and (d) are all correct. However, in the general case ($a \neq 0$, and hence

$v \neq 0$) only statements (b) and (c) are true. Statement (e) is not true in either case.

OQ2.3 The bowling pin has a constant downward acceleration while in flight. The velocity of the pin is directed upward on the ascending part of its flight and is directed downward on the descending part of its flight. Thus, only (d) is a true statement.

OQ2.4 The derivation of the equations of kinematics for an object moving in one dimension was based on the assumption that the object had a constant acceleration. Thus, (b) is the correct answer. An object would have constant velocity if its acceleration were zero, so (a) applies to cases of zero acceleration only. The speed (magnitude of the velocity) will increase in time only in cases when the velocity is in the same direction as the constant acceleration, so (c) is not a correct response. An object projected straight upward into the air has a constant downward acceleration, yet its position (altitude) does not always increase in time (it eventually starts to fall back downward) nor is its velocity always directed downward (the direction of the constant acceleration). Thus, neither (d) nor (e) can be correct.

OQ2.5 The maximum height (where $v = 0$) reached by a freely falling object shot upward with an initial velocity $v_0 = +225$ m/s is found from $v_f^2 = v_i^2 + 2a(y_f - y_i) = v_i^2 + 2a\Delta y$, where we replace a with $-g$, the downward acceleration due to gravity. Solving for Δy then gives

$$\Delta y = \frac{\left(v_f^2 - v_i^2\right)}{2a} = \frac{-v_0^2}{2(-g)} = \frac{-(225 \text{ m/s})^2}{2\left(-9.80 \text{ m/s}^2\right)} = 2.58 \times 10^3 \text{ m}$$

Thus, the projectile will be at the $\Delta y = 6.20 \times 10^2$ m level twice, once on the way upward and once coming back down.

The elapsed time when it passes this level coming downward can be found by using $v_f^2 = v_i^2 + 2a\Delta y$ again by substituting $a = -g$ and solving for the velocity of the object at height (displacement from original position) $\Delta y = +6.20 \times 10^2$ m.

$$v_f^2 = v_i^2 + 2a\Delta y$$
$$v^2 = (225 \text{ m/s})^2 + 2\left(-9.80 \text{ m/s}^2\right)\left(6.20 \times 10^2 \text{ m}\right)$$
$$v = \pm 196 \text{ m/s}$$

The velocity coming down is −196 m/s. Using $v_f = v_i + at$, we can solve for the time the velocity takes to change from +225 m/s to −196 m/s:

$$t = \frac{(v_f - v_i)}{a} = \frac{(-196\text{ m/s} - 225\text{ m/s})}{(-9.80\text{ m/s}^2)} = 43.0\text{ s.}$$

The correct choice is (e).

OQ2.6 Once the arrow has left the bow, it has a constant downward acceleration equal to the free-fall acceleration, g. Taking upward as the positive direction, the elapsed time required for the velocity to change from an initial value of 15.0 m/s upward (v_0 = +15.0 m/s) to a value of 8.00 m/s downward (v_f = −8.00 m/s) is given by

$$\Delta t = \frac{\Delta v}{a} = \frac{v_f - v_0}{-g} = \frac{-8.00\text{ m/s } - (+15.0\text{ m/s})}{-9.80\text{ m/s}^2} = 2.35\text{ s}$$

Thus, the correct choice is (d).

OQ2.7 (c) The object has an initial positive (northward) velocity and a negative (southward) acceleration; so, a graph of velocity versus time slopes down steadily from an original positive velocity. Eventually, the graph cuts through zero and goes through increasing-magnitude-negative values.

OQ2.8 (b) Using $v_f^2 = v_i^2 + 2a\Delta y$, with v_i = −12 m/s and Δy = −40 m:

$$v_f^2 = v_i^2 + 2a\Delta y$$
$$v^2 = (-12\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(-40\text{ m})$$
$$v = -30\text{ m/s}$$

OQ2.9 With original velocity zero, displacement is proportional to the square of time in $(1/2)at^2$. Making the time one-third as large makes the displacement one-ninth as large, answer (c).

OQ2.10 We take downward as the positive direction with $y = 0$ and $t = 0$ at the top of the cliff. The freely falling marble then has $v_0 = 0$ and its displacement at t = 1.00 s is Δy = 4.00 m. To find its acceleration, we use

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \rightarrow (y - y_0) = \Delta y = \frac{1}{2}at^2 \rightarrow a = \frac{2\Delta y}{t^2}$$
$$a = \frac{2(4.00\text{ m})}{(1.00\text{ s})^2} = 8.00\text{ m/s}^2$$

The displacement of the marble (from its initial position) at $t = 2.00$ s is found from

$$\Delta y = \frac{1}{2}at^2$$
$$\Delta y = \frac{1}{2}\left(8.00\ \mathrm{m/s^2}\right)\left(2.00\ \mathrm{s}\right)^2 = 16.0\ \mathrm{m}.$$

The distance the marble has fallen in the 1.00 s interval from $t = 1.00$ s to $t = 2.00$ s is then

$$\Delta y = 16.0\ \mathrm{m} - 4.0\ \mathrm{m} = 12.0\ \mathrm{m}.$$

and the answer is (c).

OQ2.11 In a position vs. time graph, the velocity of the object at any point in time is the slope of the line tangent to the graph at that instant in time. The speed of the particle at this point in time is simply the magnitude (or absolute value) of the velocity at this instant in time. The displacement occurring during a time interval is equal to the difference in x coordinates at the final and initial times of the interval, $\Delta x = x_f - x_i$.

The average velocity during a time interval is the slope of the straight line connecting the points on the curve corresponding to the initial and final times of the interval,

$$\bar{v} = \Delta x/\Delta t$$

Thus, we see how the quantities in choices (a), (e), (c), and (d) can all be obtained from the graph. Only the acceleration, choice (b), *cannot be obtained* from the position vs. time graph.

OQ2.12 We take downward as the positive direction with $y = 0$ and $t = 0$ at the top of the cliff. The freely falling pebble then has $v_0 = 0$ and $a = g = +9.8\ \mathrm{m/s^2}$. The displacement of the pebble at $t = 1.0$ s is given: $y_1 = 4.9$ m. The displacement of the pebble at $t = 3.0$ s is found from

$$y_3 = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(9.8\ \mathrm{m/s^2}\right)\left(3.0\ \mathrm{s}\right)^2 = 44\ \mathrm{m}$$

The distance fallen in the 2.0-s interval from $t = 1.0$ s to $t = 3.0$ s is then

$$\Delta y = y_3 - y_1 = 44\ \mathrm{m} - 4.9\ \mathrm{m} = 39\ \mathrm{m}$$

and choice (c) is seen to be the correct answer.

OQ2.13 (c) They are the same. After the first ball reaches its apex and falls back downward past the student, it will have a downward velocity of magnitude v_i. This velocity is the same as the velocity of the second ball, so after they fall through equal heights their impact speeds will

also be the same.

OQ2.14 (b) Above. Your ball has zero initial speed and smaller average speed during the time of flight to the passing point. So your ball must travel a smaller distance to the passing point than the ball your friend throws.

OQ2.15 Take down as the positive direction. Since the pebble is released from rest, $v_f^2 = v_i^2 + 2a\Delta y$ becomes

$$v_f^2 = (4\text{ m/s})^2 = 0^2 + 2gh.$$

Next, when the pebble is thrown with speed 3.0 m/s from the same height *h*, we have

$$v_f^2 = (3\text{ m/s})^2 + 2gh = (3\text{ m/s})^2 + (4\text{ m/s})^2 \rightarrow v_f = 5\text{ m/s}$$

and the answer is (b). Note that we have used the result from the first equation above and replaced $2gh$ with $(4\text{ m/s})^2$ in the second equation.

OQ2.16 Once the ball has left the thrower's hand, it is a freely falling body with a constant, nonzero, acceleration of $a = -g$. Since the acceleration of the ball is not zero at any point on its trajectory, choices (a) through (d) are all false and the correct response is (e).

OQ2.17 (a) Its speed is zero at points B and D where the ball is reversing its direction of motion. Its speed is the same at A, C, and E because these points are at the same height. The assembled answer is A = C = E > B = D.

(b) The acceleration has a very large positive (upward) value at D. At all the other points it is -9.8 m/s^2. The answer is D > A = B = C = E.

OQ2.18 (i) (b) shows equal spacing, meaning constant nonzero velocity and constant zero acceleration. (ii) (c) shows positive acceleration throughout. (iii) (a) shows negative (leftward) acceleration in the first four images.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ2.1 The net displacement must be zero. The object could have moved away from its starting point and back again, but it is at its initial position again at the end of the time interval.

CQ2.2 Tramping hard on the brake at zero speed on a level road, you do not feel pushed around inside the car. The forces of rolling resistance and air resistance have dropped to zero as the car coasted to a stop, so the car's acceleration is zero at this moment and afterward.

Tramping hard on the brake at zero speed on an uphill slope, you feel

thrown backward against your seat. Before, during, and after the zero-speed moment, the car is moving with a downhill acceleration if you do not tramp on the brake.

CQ2.3 Yes. If a car is travelling eastward and slowing down, its acceleration is opposite to the direction of travel: its acceleration is westward.

CQ2.4 Yes. Acceleration is the time rate of change of the velocity of a particle. If the velocity of a particle is zero at a given moment, and if the particle is not accelerating, the velocity will remain zero; if the particle is accelerating, the velocity will change from zero—the particle will begin to move. Velocity and acceleration are independent of each other.

CQ2.5 Yes. Acceleration is the time rate of change of the velocity of a particle. If the velocity of a particle is nonzero at a given moment, and the particle is not accelerating, the velocity will remain the same; if the particle is accelerating, the velocity will change. The velocity of a particle at a given moment and how the velocity is changing at that moment are independent of each other.

CQ2.6 Assuming no air resistance: (a) The ball reverses direction at its maximum altitude. For an object traveling along a straight line, its velocity is zero at the point of reversal. (b) Its acceleration is that of gravity: $-9.80\ \text{m/s}^2$ ($9.80\ \text{m/s}^2$, downward). (c) The velocity is $-5.00\ \text{m/s}^2$. (d) The acceleration of the ball remains $-9.80\ \text{m/s}^2$ as long as it does not touch anything. Its acceleration changes when the ball encounters the ground.

CQ2.7 (a) No. Constant acceleration only: the derivation of the equations assumes that d^2x/dt^2 is constant. (b) Yes. Zero is a constant.

CQ2.8 Yes. If the speed of the object varies at all over the interval, the instantaneous velocity will sometimes be greater than the average velocity and will sometimes be less.

CQ2.9 No: Car A might have greater acceleration than B, but they might both have zero acceleration, or otherwise equal accelerations; or the driver of B might have tramped hard on the gas pedal in the recent past to give car B greater acceleration just then.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 2.1 Position, Velocity, and Speed

P2.1 The average velocity is the slope, not necessarily of the graph line itself, but of a secant line cutting across the graph between specified points. The slope of the graph line itself is the instantaneous velocity, found, for example, in Problem 6 part (b). On this graph, we can tell positions to two significant figures:

(a) $x = 0$ at $t = 0$ and $x = 10$ m at $t = 2$ s:

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{10\text{ m} - 0}{2\text{ s} - 0} = \boxed{5.0\text{ m/s}}$$

(b) $x = 5.0$ m at $t = 4$ s:

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{5.0\text{ m} - 0}{4\text{ s} - 0} = \boxed{1.2\text{ m/s}}$$

(c) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{5.0\text{ m} - 10\text{ m}}{4\text{ s} - 2\text{ s}} = \boxed{-2.5\text{ m/s}}$$

(d) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{-5.0\text{ m} - 5.0\text{ m}}{7\text{ s} - 4\text{ s}} = \boxed{-3.3\text{ m/s}}$$

(e) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{0.0\text{ m} - 0.0\text{ m}}{8\text{ s} - 0\text{ s}} = \boxed{0\text{ m/s}}$$

P2.2 We assume that you are approximately 2 m tall and that the nerve impulse travels at uniform speed. The elapsed time is then

$$\Delta t = \frac{\Delta x}{v} = \frac{2\text{ m}}{100\text{ m/s}} = 2 \times 10^{-2}\text{ s} = \boxed{0.02\text{ s}}$$

P2.3 Speed is positive whenever motion occurs, so the average speed must be positive. For the velocity, we take as positive for motion to the right and negative for motion to the left, so its average value can be positive, negative, or zero.

(a) The average speed during any time interval is equal to the total distance of travel divided by the total time:

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{d_{AB} + d_{BA}}{t_{AB} + t_{BA}}$$

But $d_{AB} = d_{BA}$, $t_{AB} = d/v_{AB}$, and $t_{BA} = d/v_{BA}$

so $$\text{average speed} = \frac{d + d}{(d/v_{AB}) + (d/v_{BA})} = \frac{2(v_{AB})(v_{BA})}{v_{AB} + v_{BA}}$$

and

$$\text{average speed} = 2\left[\frac{(5.00\text{ m/s})(3.00\text{ m/s})}{5.00\text{ m/s} + 3.00\text{ m/s}}\right] = \boxed{3.75\text{ m/s}}$$

(b) The average velocity during any time interval equals total displacement divided by elapsed time.

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t}$$

Since the walker returns to the starting point, $\Delta x = 0$ and $\boxed{v_{x,\text{avg}} = 0}$.

P2.4 We substitute for t in $x = 10t^2$, then use the definition of average velocity:

t (s)	2.00	2.10	3.00
x (m)	40.0	44.1	90.0

(a) $v_{\text{avg}} = \dfrac{\Delta x}{\Delta t} = \dfrac{90.0\text{ m} - 40.0\text{ m}}{1.00\text{ s}} = \dfrac{50.0\text{ m}}{1.00\text{ s}} = \boxed{50.0\text{ m/s}}$

(b) $v_{\text{avg}} = \dfrac{\Delta x}{\Delta t} = \dfrac{44.1\text{ m} - 40.0\text{ m}}{0.100\text{ s}} = \dfrac{4.10\text{ m}}{0.100\text{ s}} = \boxed{41.0\text{ m/s}}$

***P2.5** We read the data from the table provided, assume three significant figures of precision for all the numbers, and use Equation 2.2 for the definition of average velocity.

(a) $v_{x,\text{avg}} = \dfrac{\Delta x}{\Delta t} = \dfrac{2.30\text{ m} - 0\text{ m}}{1.00\text{ s}} = \boxed{2.30\text{ m/s}}$

(b) $v_{x,\text{avg}} = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5\text{ m} - 9.20\text{ m}}{3.00\text{ s}} = \boxed{16.1\text{ m/s}}$

(c) $v_{x,\text{avg}} = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5\text{ m} - 0\text{ m}}{5.00\text{ s}} = \boxed{11.5\text{ m/s}}$

Section 2.2 Instantaneous Velocity and Speed

P2.6 (a) At any time, t, the position is given by $x = (3.00\ \text{m/s}^2)t^2$.

Thus, at $t_i = 3.00$ s: $x_i = (3.00\ \text{m/s}^2)(3.00\ \text{s})^2 = \boxed{27.0\ \text{m}}$.

(b) At $t_f = 3.00\ \text{s} + \Delta t$: : $x_f = (3.00\ \text{m/s}^2)(3.00\ \text{s} + \Delta t)^2$, or

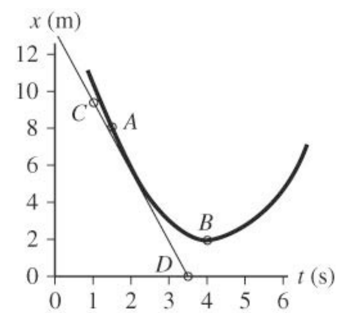
$$x_f = \boxed{27.0\ \text{m} + (18.0\ \text{m/s})\Delta t + (3.00\ \text{m/s}^2)(\Delta t)^2}$$

(c) The instantaneous velocity at $t = 3.00$ s is:

$$\boxed{\begin{aligned}\lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t} &= \lim_{\Delta t\to 0}\frac{(18.0\ \text{m/s})\Delta t + (3.00\ \text{m/s}^2)(\Delta t)^2}{\Delta t} \\ &= \lim_{\Delta t\to 0}(18.0\ \text{m/s}) + (3.00\ \text{m/s}^2)(\Delta t) = \boxed{18.0\ \text{m/s}}\end{aligned}}$$

P2.7 For average velocity, we find the slope of a secant line running across the graph between the 1.5-s and 4-s points. Then for instantaneous velocities we think of slopes of tangent lines, which means the slope of the graph itself at a point.

We place two points on the curve: Point A, at $t = 1.5$ s, and Point B, at $t = 4.0$ s, and read the corresponding values of x.



ANS. FIG. P2.7

(a) At $t_i = 1.5$ s, $x_i = 8.0$ m (Point A)

At $t_f = 4.0$ s, $x_f = 2.0$ m (Point B)

$$v_{avg} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0)\ \text{m}}{(4.0 - 1.5)\ \text{s}}$$

$$= -\frac{6.0\ \text{m}}{2.5\ \text{s}} = \boxed{-2.4\ \text{m/s}}$$

(b) The slope of the tangent line can be found from points C and D. ($t_C = 1.0$ s, $x_C = 9.5$ m) and ($t_D = 3.5$ s, $x_D = 0$),

$$v \approx \boxed{-3.8\ \text{m/s}}$$

The negative sign shows that the **direction** of v_x is along the negative x direction.

(c) The velocity will be zero when the slope of the tangent line is zero. This occurs for the point on the graph where x has its minimum value. This is at $t \approx \boxed{4.0\ \text{s}}$.

P2.8 We use the definition of average velocity.

(a) $$v_{1,x,\text{ave}} = \frac{(\Delta x)_1}{(\Delta t)_1} = \frac{L-0}{t_1} = \boxed{+L/t_1}$$

(b) $$v_{2,x,\text{ave}} = \frac{(\Delta x)_2}{(\Delta t)_2} = \frac{0-L}{t_2} = \boxed{-L/t_2}$$

(c) To find the average velocity for the round trip, we add the displacement and time for each of the two halves of the swim:

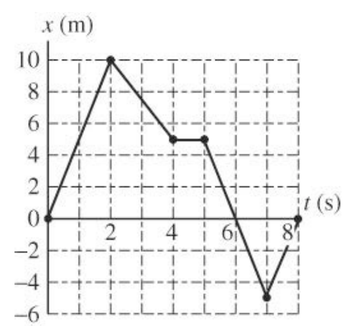
$$v_{x,\text{ave,total}} = \frac{(\Delta x)_{\text{total}}}{(\Delta t)_{\text{total}}} = \frac{(\Delta x)_1 + (\Delta x)_2}{t_1 + t_2} = \frac{+L-L}{t_1+t_2} = \frac{0}{t_1+t_2} = \boxed{0}$$

(d) The average speed of the round trip is the total distance the athlete travels divided by the total time for the trip:

$$v_{\text{ave,trip}} = \frac{\text{total distance traveled}}{(\Delta t)_{\text{total}}} = \frac{|(\Delta x)_1| + |(\Delta x)_2|}{t_1+t_2}$$

$$= \frac{|+L| + |-L|}{t_1+t_2} = \boxed{\frac{2L}{t_1+t_2}}$$

P2.9 The instantaneous velocity is found by evaluating the slope of the $x-t$ curve at the indicated time. To find the slope, we choose two points for each of the times below.



ANS. FIG. P2.9

(a) $$v = \frac{(5-0)\text{ m}}{(1-0)\text{ s}} = \boxed{5\text{ m/s}}$$

(b) $$v = \frac{(5-10)\text{ m}}{(4-2)\text{ s}} = \boxed{-2.5\text{ m/s}}$$

(c) $$v = \frac{(5-5)\text{ m}}{(5\text{ s}-4\text{ s})} = \boxed{0}$$

(d) $$v = \frac{0-(-5\text{ m})}{(8\text{ s}-7\text{ s})} = \boxed{+5\text{ m/s}}$$

Section 2.3 Analysis Model: Particle Under Constant Velocity

P2.10 The plates spread apart distance d of 2.9×10^3 mi in the time interval Δt at the rate of 25 mm/year. Converting units:

$$\left(2.9\times10^3\text{ mi}\right)\left(\frac{1609\text{ m}}{1\text{ mi}}\right)\left(\frac{10^3\text{ mm}}{1\text{ m}}\right)=4.7\times10^9\text{ mm}$$

Use $d = v\Delta t$, and solve for Δt:

$$d = v\Delta t \rightarrow \Delta t = \frac{d}{v}$$

$$\Delta t = \frac{4.7\times10^9\text{ mm}}{25\text{ mm/year}} = \boxed{1.9\times10^8\text{ years}}$$

P2.11 (a) The tortoise crawls through a distance D before the rabbit resumes the race. When the rabbit resumes the race, the rabbit must run through 200 m at 8.00 m/s while the tortoise crawls through the distance (1 000 m – D) at 0.200 m/s. Each takes the same time interval to finish the race:

$$\Delta t = \left(\frac{200\text{ m}}{8.00\text{ m/s}}\right)=\left(\frac{1\ 000\text{ m}-D}{0.200\text{ m/s}}\right)$$

Solving,

$$\rightarrow(0.200\text{ m/s})(200\text{ m})=(8.00\text{ m/s})(1\ 000\text{ m}-D)$$

$$1\ 000\text{ m}-D=\frac{(0.200\text{ m/s})(200\text{ m})}{8.00\text{ m/s}}$$

$$\rightarrow D = 995\text{ m}$$

So, the tortoise is 1 000 m – D = $\boxed{5.00\text{ m}}$ from the finish line when the rabbit resumes running.

(b) Both begin the race at the same time: $t = 0$. The rabbit reaches the 800-m position at time t = 800 m/(8.00 m/s) = 100 s. The tortoise has crawled through 995 m when t = 995 m/(0.200 m/s) = 4 975 s. The rabbit has waited for the time interval Δt = 4 975 s – 100 s = $\boxed{4\ 875\text{ s}}$.

P2.12 The trip has two parts: first the car travels at constant speed v_1 for distance d, then it travels at constant speed v_2 for distance d. The first part takes the time interval $\Delta t_1 = d/v_1$, and the second part takes the time interval $\Delta t_2 = d/v_2$.

(a) By definition, the average velocity for the entire trip is $v_{avg} = \Delta x/\Delta t$, where $\Delta x = \Delta x_1 + \Delta x_2 = 2d$, and

$\Delta t = \Delta t_1 + \Delta t_2 = d/v_1 + d/v_2$. Putting these together, we have

$$v_{avg} = \left(\frac{\Delta d}{\Delta t}\right) = \left(\frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2}\right) = \left(\frac{2d}{d/v_1 + d/v_2}\right) = \left(\frac{2v_1 v_2}{v_1 + v_2}\right)$$

We know $v_{avg} = 30$ mi/h and $v_1 = 60$ mi/h.

Solving for v_2 gives

$$(v_1 + v_2)v_{avg} = 2v_1 v_2 \rightarrow v_2 = \left(\frac{v_1 v_{avg}}{2v_1 - v_{avg}}\right).$$

$$v_2 = \left[\frac{(30\text{ mi/h})(60\text{ mi/h})}{2(60\text{ mi/h}) - (30\text{ mi/h})}\right] = \boxed{20\text{ mi/h}}$$

(b) The average velocity for this trip is $v_{avg} = \Delta x / \Delta t$, where $\Delta x = \Delta x_1 + \Delta x_2 = d + (-d) = 0$; so, $v_{avg} = \boxed{0}$.

(c) The average speed for this trip is $v_{avg} = d/\Delta t$, where $d = d_1 + d_2 = d + d = 2d$ and $\Delta t = \Delta t_1 + \Delta t_2 = d/v_1 + d/v_2$; so, the average speed is the same as in part (a): $v_{avg} = \boxed{30\text{ mi/h.}}$

***2.13** (a) The total time for the trip is $t_{total} = t_1 + 22.0$ min $= t_1 + 0.367$ h, where t_1 is the time spent traveling at $v_1 = 89.5$ km/h. Thus, the distance traveled is $\Delta x = v_1 t_1 = v_{avg} t_{total}$, which gives

$$\begin{aligned}(89.5\text{ km/h})t_1 &= (77.8\text{ km/h})(t_1 + 0.367\text{ h})\\ &= (77.8\text{ km/h})t_1 + 28.5\text{ km}\end{aligned}$$

or $\quad (89.5\text{ km/h} - 77.8\text{ km/h})t_1 = 28.5\text{ km}$

from which, t_1= 2.44 h, for a total time of

$$t_{total} = t_1 + 0.367\text{ h} = \boxed{2.81\text{ h}}$$

(b) The distance traveled during the trip is $\Delta x = v_1 t_1 = v_{avg} t_{total}$, giving

$$\Delta x = v_{avg} t_{total} = (77.8\text{ km/h})(2.81\text{ h}) = \boxed{219\text{ km}}$$

Section 2.4 Acceleration

P2.14 The ball's motion is entirely in the horizontal direction. We choose the positive direction to be the outward direction, perpendicular to the wall. With outward positive, $v_i = -25.0$ m/s and $v_f = 22.0$ m/s. We use Equation 2.13 for one-dimensional motion with constant acceleration,

$v_f = v_i + at$, and solve for the acceleration to obtain

$$a = \frac{\Delta v}{\Delta t} = \frac{22.0\ \text{m/s} - (-25.0\ \text{m/s})}{3.50 \times 10^{-3}\ \text{s}} = \boxed{1.34 \times 10^4\ \text{m/s}^2}$$

P2.15 (a) Acceleration is the slope of the graph of v versus t.

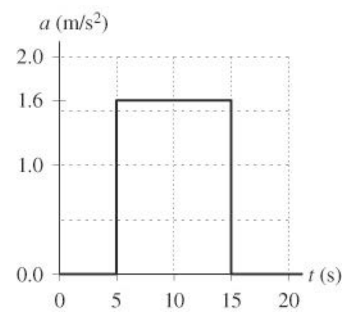
For $0 < t < 5.00$ s, $a = 0$.

For $15.0\ \text{s} < t < 20.0$ s, $a = 0$.

For $5.0\ \text{s} < t < 15.0$ s, $a = \dfrac{v_f - v_i}{t_f - t_i}$.

$$a = \frac{8.00\ \text{m/s} - (-8.00\ \text{m/s})}{15.0\ \text{s} - 5.00\ \text{s}} = \boxed{1.60\ \text{m/s}^2}$$

We can plot $a(t)$ as shown in ANS. FIG. P2.15 below.



ANS. FIG. P2.15

For (b) and (c) we use $a = \dfrac{v_f - v_i}{t_f - t_i}$.

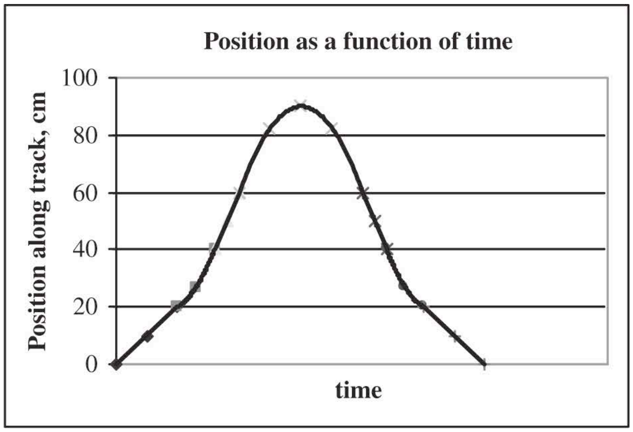
(b) For $5.00\ \text{s} < t < 15.0$ s, $t_i = 5.00$ s, $v_i = -8.00$ m/s, $t_f = 15.0$ s, and $v_f = 8.00$ m/s:

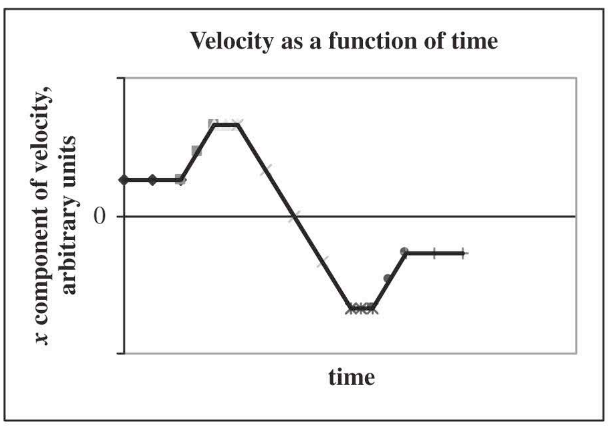
$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00\ \text{m/s} - (-8.00\ \text{m/s})}{15.0\ \text{s} - 5.00\ \text{s}} = \boxed{1.60\ \text{m/s}^2}$$

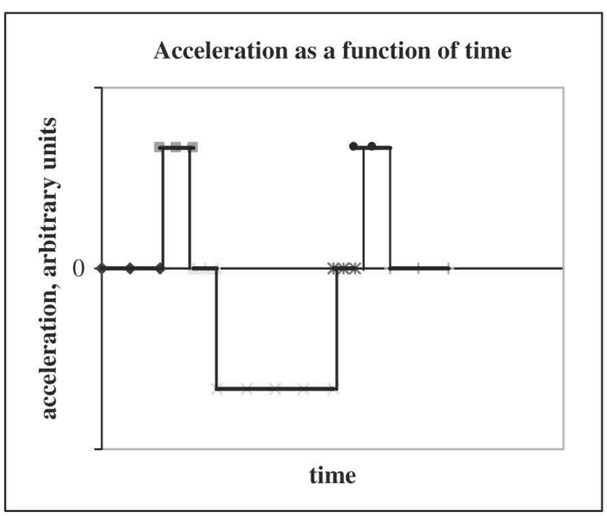
(c) We use $t_i = 0$, $v_i = -8.00$ m/s, $t_f = 20.0$ s, and $v_f = 8.00$ m/s:

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00\ \text{m/s} - (-8.00\ \text{m/s})}{20.0\ \text{s} - 0} = \boxed{0.800\ \text{m/s}^2}$$

P2.16 The acceleration is zero whenever the marble is on a horizontal section. The acceleration has a constant positive value when the marble is rolling on the 20-to-40-cm section and has a constant negative value when it is rolling on the second sloping section. The position graph is a straight sloping line whenever the speed is constant and a section of a parabola when the speed changes.







P2.17 (a) In the interval $t_i = 0$ s and $t_f = 6.00$ s, the motorcyclist's velocity changes from $v_i = 0$ to $v_f = 8.00$ m/s. Then,

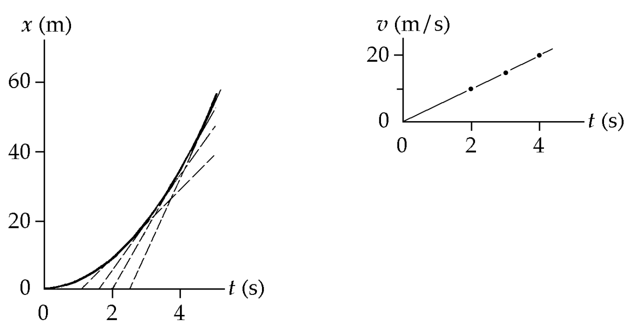
$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{8.0\ \text{m/s} - 0}{6.0\ \text{s} - 0} = \boxed{1.3\ \text{m/s}^2}$$

(b) Maximum positive acceleration occurs when the slope of the velocity-time curve is greatest, at $t = 3$ s, and is equal to the slope of the graph, approximately (6 m/s – 2 m/s)/(4 s – 2 s) = $\boxed{2\ \text{m/s}^2}$.

(c) The acceleration $a = 0$ when the slope of the velocity-time graph is zero, which occurs at $\boxed{t = 6\ \text{s}}$, and also for $\boxed{t > 10\ \text{s}}$.

(d) Maximum negative acceleration occurs when the velocity-time graph has its maximum negative slope, at $t = 8$ s, and is equal to the slope of the graph, approximately $\boxed{-1.5\ \text{m/s}^2}$.

***P2.18** (a) The graph is shown in ANS. FIG. P2.18 below.



ANS. FIG. P2.18

(b) At $t = 5.0$ s, the slope is $v \approx \dfrac{58\ \text{m}}{2.5\ \text{s}} \approx \boxed{23\ \text{m/s}}$.

At $t = 4.0$ s, the slope is $v \approx \dfrac{54\ \text{m}}{3\ \text{s}} \approx \boxed{18\ \text{m/s}}$.

At $t = 3.0$ s, the slope is $v \approx \dfrac{49\ \text{m}}{3.4\ \text{s}} \approx \boxed{14\ \text{m/s}}$.

At $t = 2.0$ s, the slope is $v \approx \dfrac{36\ \text{m}}{4.0\ \text{s}} \approx \boxed{9.0\ \text{m/s}}$.

(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} \approx \dfrac{23\ \text{m/s}}{5.0\ \text{s}} \approx \boxed{4.6\ \text{m/s}^2}$

(d) The initial velocity of the car was $\boxed{\text{zero}}$.

P2.19 (a) The area under a graph of a vs. t is equal to the change in velocity, Δv. We can use Figure P2.19 to find the change in velocity during specific time intervals.

The area under the curve for the time interval 0 to 10 s has the shape of a rectangle. Its area is

$$\Delta v = (2\text{ m/s}^2)(10\text{ s}) = 20\text{ m/s}$$

The particle starts from rest, $v_0 = 0$, so its velocity at the end of the 10-s time interval is

$$v = v_0 + \Delta v = 0 + 20\text{ m/s} = \boxed{20\text{ m/s}}$$

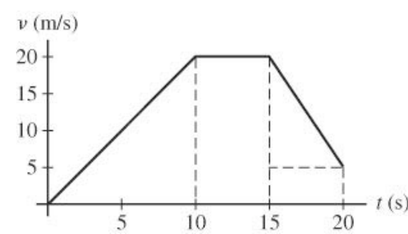
Between $t = 10$ s and $t = 15$ s, the area is zero: $\Delta v = 0$ m/s.

Between $t = 15$ s and $t = 20$ s, the area is a rectangle: $\Delta v = (-3\text{ m/s}^2)(5\text{ s}) = -15\text{ m/s}$.

So, between $t = 0$ s and $t = 20$ s, the total area is $\Delta v = (20\text{ m/s}) + (0\text{ m/s}) + (-15\text{ m/s}) = 5\text{ m/s}$, and the velocity at $t = 20$ s is $\boxed{5\text{ m/s.}}$

(b) We can use the information we derived in part (a) to construct a graph of x vs. t; the area under such a graph is equal to the displacement, Δx, of the particle.

From (a), we have these points (t, v) = (0 s, 0 m/s), (10 s, 20 m/s), (15 s, 20 m/s), and (20 s, 5 m/s). The graph appears below.



The displacements are:

0 to 10 s (area of triangle): $\Delta x = (1/2)(20\text{ m/s})(10\text{ s}) = 100\text{ m}$

10 to 15 s (area of rectangle): $\Delta x = (20\text{ m/s})(5\text{ s}) = 100\text{ m}$

15 to 20 s (area of triangle and rectangle):

$$\Delta x = (1/2)[(20-5)\text{ m/s}](5\text{ s}) + (5\text{ m/s})(5\text{ s})$$
$$= 37.5\text{ m} + 25\text{ m} = 62.5\text{ m}$$

Total displacement over the first 20.0 s:

$$\Delta x = 100\text{ m} + 100\text{ m} + 62.5\text{ m} = 262.5\text{ m} = \boxed{263\text{ m}}$$

P2.20 (a) The average velocity is the change in position divided by the length of the time interval. We plug in to the given equation.

At $t = 2.00$ s, $x = [3.00(2.00)^2 - 2.00(2.00) + 3.00]$ m $= 11.0$ m.

At $t = 3.00$ s, $x = [3.00(3.00)^2 - 2.00(3.00) + 3.00]$ m $= 24.0$ m

so

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{24.0 \text{ m} - 11.0 \text{ m}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{13.0 \text{ m/s}}$$

(b) At all times the instantaneous velocity is

$$v = \frac{d}{dt}\left(3.00t^2 - 2.00t + 3.00\right) = \left(6.00t - 2.00\right) \text{ m/s}$$

At $t = 2.00$ s, $v = [6.00(2.00) - 2.00]$ m/s $= \boxed{10.0 \text{ m/s}}$.

At $t = 3.00$ s, $v = [6.00(3.00) - 2.00]$ m/s $= \boxed{16.0 \text{ m/s}}$.

(c) $$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{16.0 \text{ m/s} - 10.0 \text{ m/s}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{6.00 \text{ m/s}^2}$$

(d) At all times $a = \frac{d}{dt}(6.00t - 2.00) = \boxed{6.00 \text{ m/s}^2}$. This includes both $t = 2.00$ s and $t = 3.00$ s.

(e) From (b), $v = (6.00t - 2.00) = 0 \rightarrow t = (2.00)/(6.00) = \boxed{0.333 \text{ s.}}$

P2.21 To find position we simply evaluate the given expression. To find velocity we differentiate it. To find acceleration we take a second derivative.

With the position given by $x = 2.00 + 3.00t - t^2$, we can use the rules for differentiation to write expressions for the velocity and acceleration as functions of time:

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left(2 + 3t - t^2\right) = 3 - 2t \quad \text{and} \quad a_x = \frac{dv}{dt} = \frac{d}{dt}(3 - 2t) = -2$$

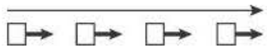
Now we can evaluate x, v, and a at $t = 3.00$ s.

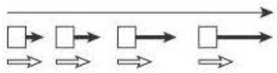
(a) $x = (2.00 + 9.00 - 9.00)$ m $= \boxed{2.00 \text{ m}}$

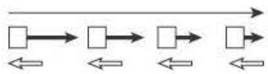
(b) $v = (3.00 - 6.00)$ m/s $= \boxed{-3.00 \text{ m/s}}$

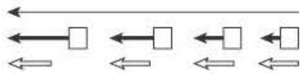
(c) $a = \boxed{-2.00 \text{ m/s}^2}$

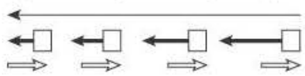
Section 2.5 Motion Diagrams

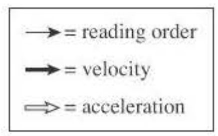
P2.22 (a) 

(b) 

(c) 

(d) 

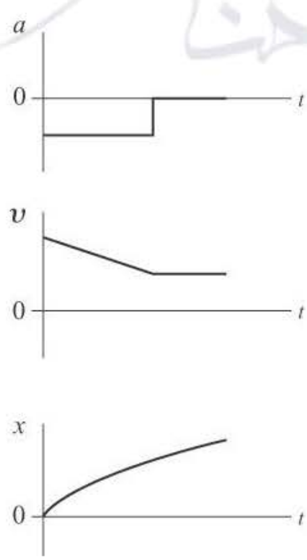
(e) 



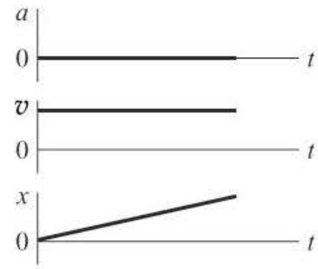
(f) One way of phrasing the answer: The spacing of the successive positions would change with $\boxed{\text{less regularity.}}$

Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the acceleration vectors would vary in magnitude and direction.

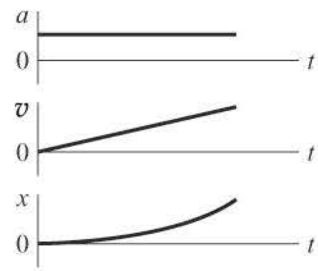
P2.23 (a) The motion is fast at first but slowing until the speed is constant. We assume the acceleration is constant as the object slows.



(b) The motion is constant in speed.



(c) The motion is speeding up, and we suppose the acceleration is constant.



Section 2.6 Analysis Model: Particle Under Constant Acceleration

***P2.24** Method One

Suppose the unknown acceleration is constant as a car moving at $v_{i1} = 35.0 \text{ mi/h}$ comes to a stop, $v_f = 0$ in $x_{f1} - x_i = 40.0$ ft. We find its acceleration from $v_{f1}^2 = v_{i1}^2 + 2a\left(x_{f1} - x_i\right)$:

$$a = \frac{v_{f1}^2 - v_i^2}{2\left(x_{f1} - x_i\right)} = \frac{0 - (35.0 \text{ mi/h})^2}{2(40.0 \text{ ft})}\left(\frac{5\ 280 \text{ ft}}{\text{mi}}\right)^2\left(\frac{1 \text{ h}}{3\ 600 \text{ s}}\right)^2 = -32.9 \text{ ft/s}^2$$

Now consider a car moving at $v_{i2} = 70.0 \text{ mi/h}$ and stopping, $v_f = 0$, with $a = -32.9 \text{ ft/s}^2$. From the same equation, its stopping distance is

$$x_{f2} - x_i = \frac{v_{f2}^2 - v_i^2}{2a} = \frac{0 - (70.0 \text{ mi/h})^2}{2\left(-32.9 \text{ ft/s}^2\right)}\left(\frac{5\ 280 \text{ ft}}{1 \text{ mi}}\right)^2\left(\frac{1 \text{ h}}{3\ 600 \text{ s}}\right)^2$$

$$= \boxed{160 \text{ ft}}$$

Method Two

For the process of stopping from the lower speed v_{i1} we have $v_f^2 = v_{i1}^2 + 2a\left(x_{f1} - x_i\right)$, $0 = v_{i1}^2 + 2ax_{f1}$, and $v_{i1}^2 = -2ax_{f1}$. For stopping

from $v_{i2} = 2v_{i1}$, similarly $0 = v_{i2}^2 + 2ax_{f2}$, and $v_{i2}^2 = -2ax_{f2}$. Dividing gives

$$\frac{v_{i2}^2}{v_{i1}^2} = \frac{x_{f2}}{x_{f1}};\ x_{f2} = 40\ \text{ft} \times 2^2 = \boxed{160\ \text{ft}}$$

***P2.25** We have $v_i = 2.00 \times 10^4\ \text{m/s}$, $v_f = 6.00 \times 10^6\ \text{m/s}$, and $x_f - x_i = 1.50 \times 10^{-2}\ \text{m}$.

(a) $x_f - x_i = \frac{1}{2}(v_i + v_f)t$:

$$t = \frac{2(x_f - x_i)}{v_i + v_f} = \frac{2(1.50 \times 10^{-2}\ \text{m})}{2.00 \times 10^4\ \text{m/s} + 6.00 \times 10^6\ \text{m/s}}$$

$$= \boxed{4.98 \times 10^{-9}\ \text{s}}$$

(b) $v_f^2 = v_i^2 + 2a_x(x_f - x_i)$:

$$a_x = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(6.00 \times 10^6\ \text{m/s})^2 - (2.00 \times 10^4\ \text{m/s})^2}{2(1.50 \times 10^{-2}\ \text{m})}$$

$$= \boxed{1.20 \times 10^{15}\ \text{m/s}^2}$$

***P2.26** (a) Choose the initial point where the pilot reduces the throttle and the final point where the boat passes the buoy: $x_i = 0$, $x_f = 100\ \text{m}$, $v_{xi} = 30\ \text{m/s}$, $v_{xf} = ?$, $a_x = -3.5\ \text{m/s}^2$, and $t = ?$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2:$$

$$100\ \text{m} = 0 + (30\ \text{m/s})t + \frac{1}{2}(-3.5\ \text{m/s}^2)t^2$$

$$(1.75\ \text{m/s}^2)t^2 - (30\ \text{m/s})t + 100\ \text{m} = 0$$

We use the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{30\ \text{m/s} \pm \sqrt{900\ \text{m}^2/\text{s}^2 - 4(1.75\ \text{m/s}^2)(100\ \text{m})}}{2(1.75\ \text{m/s}^2)}$$

$$= \frac{30\ \text{m/s} \pm 14.1\ \text{m/s}}{3.5\ \text{m/s}^2} = 12.6\ \text{s} \quad \text{or} \quad \boxed{4.53\ \text{s}}$$

The smaller value is the physical answer. If the boat kept moving with the same acceleration, it would stop and move backward, then gain speed, and pass the buoy again at 12.6 s.

(b) $v_{xf} = v_{xi} + a_x t = 30\ \text{m/s} - (3.5\ \text{m/s}^2)4.53\ \text{s} = \boxed{14.1\ \text{m/s}}$

P2.27 In parts (a) – (c), we use Equation 2.13 to determine the velocity at the times indicated.

(a) The time given is 1.00 s after 10:05:00 a.m., so

$$v_f = v_i + at = 13.0\ \text{m/s} + (-4.00\ \text{m/s}^2)(1.00\ \text{s}) = \boxed{9.00\ \text{m/s}}$$

(b) The time given is 4.00 s after 10:05:00 a.m., so

$$v_f = v_i + at = 13.0\ \text{m/s} + (-4.00\ \text{m/s}^2)(4.00\ \text{s}) = \boxed{-3.00\ \text{m/s}}$$

(c) The time given is 1.00 s before 10:05:00 a.m., so

$$v_f = v_i + at = 13.0\ \text{m/s} + (-4.00\ \text{m/s}^2)(-1.00\ \text{s}) = \boxed{17.0\ \text{m/s}}$$

(d) The graph of velocity versus time is a slanting straight line, having the value 13.0 m/s at 10:05:00 a.m. on the certain date, and sloping down by 4.00 m/s for every second thereafter.

(e) If we also know the velocity at any one instant, then knowing the value of the constant acceleration tells us the velocity at all other instants

P2.28 (a) We use Equation 2.15:

$x_f - x_i = \frac{1}{2}(v_i + v_f)t$ becomes $40.0\ \text{m} = \frac{1}{2}(v_i + 2.80\ \text{m/s})(8.50\ \text{s})$,

which yields $v_i = \boxed{6.61\ \text{m/s}}$.

(b) From Equation 2.13,

$$a = \frac{v_f - v_i}{t} = \frac{2.80\ \text{m/s} - 6.61\ \text{m/s}}{8.50\ \text{s}} = \boxed{-0.448\ \text{m/s}^2}$$

P2.29 The velocity is always changing; there is always nonzero acceleration and the problem says it is constant. So we can use one of the set of equations describing constant-acceleration motion. Take the initial point to be the moment when $x_i = 3.00$ cm and $v_{xi} = 12.0$ cm/s. Also, at $t = 2.00$ s, $x_f = -5.00$ cm.

Once you have classified the object as a particle moving with constant acceleration and have the standard set of four equations in front of

you, how do you choose which equation to use? Make a list of all of the six symbols in the equations: x_i, x_f, v_{xi}, v_{xf}, a_x, and t. On the list fill in values as above, showing that x_i, x_f, v_{xi}, and t are known. Identify a_x as the unknown. Choose an equation involving only one unknown and the knowns. That is, choose an equation *not* involving v_{xf}. Thus we choose the kinematic equation

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

and solve for a_x:

$$a_x = \frac{2\left[x_f - x_i - v_{xi}t\right]}{t^2}$$

We substitute:

$$a_x = \frac{2[-5.00\text{ cm} - 3.00\text{ cm} - (12.0\text{ cm/s})(2.00\text{ s})]}{(2.00\text{ s})^2}$$

$$= \boxed{-16.0\text{ cm/s}^2}$$

P2.30 We think of the plane moving with maximum-size backward acceleration throughout the landing, so the acceleration is constant, the stopping time a minimum, and the stopping distance as short as it can be. The negative acceleration of the plane as it lands can be called deceleration, but it is simpler to use the single general term *acceleration* for all rates of velocity change.

(a) The plane can be modeled as a particle under constant acceleration, with $a_x = -5.00\text{ m/s}^2$. Given $v_{xi} = 100\text{ m/s}$ and $v_{xf} = 0$, we use the equation $v_{xf} = v_{xi} + a_x t$ and solve for t:

$$t = \frac{v_{xf} - v_{xi}}{a_x} = \frac{0 - 100\text{ m/s}}{-5.00\text{ m/s}^2} = \boxed{20.0\text{ s}}$$

(b) Find the required stopping distance and compare this to the length of the runway. Taking x_i to be zero, we get

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

or $$\Delta x = x_f - x_i = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{0 - \left(100\text{ m/s}\right)^2}{2\left(-5.00\text{ m/s}^2\right)} = \boxed{1\,000\text{ m}}$$

(c) The stopping distance is greater than the length of the runway; $\boxed{\text{the plane cannot land}}$.

P2.31 We assume the acceleration is constant. We choose the initial and final points 1.40 s apart, bracketing the slowing-down process. Then we have a straightforward problem about a particle under constant acceleration. The initial velocity is

$$v_{xi} = 632 \text{ mi/h} = 632 \text{ mi/h}\left(\frac{1\,609 \text{ m}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right) = 282 \text{ m/s}$$

(a) Taking $v_{xf} = v_{xi} + a_x t$ with $v_{xf} = 0$,

$$a_x = \frac{v_{xf} - v_{xi}}{t} = \frac{0 - 282 \text{ m/s}}{1.40 \text{ s}} = \boxed{-202 \text{ m/s}^2}$$

This has a magnitude of approximately $20g$.

(b) From Equation 2.15,

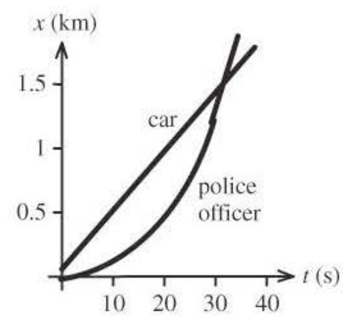
$$x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t = \frac{1}{2}(282 \text{ m/s} + 0)(1.40 \text{ s}) = \boxed{198 \text{ m}}$$

P2.32 As in the algebraic solution to Example 2.8, we let t represent the time the trooper has been moving. We graph

$$x_{\text{car}} = 45 + 45t$$

and $$x_{\text{trooper}} = 1.5t^2$$

They intersect at $t = \boxed{31 \text{ s}}$.



ANS. FIG. P2.32

***P2.33** (a) The time it takes the truck to reach 20.0 m/s is found from $v_f = v_i + at$. Solving for t yields

$$t = \frac{v_f - v_i}{a} = \frac{20.0 \text{ m/s} - 0 \text{ m/s}}{2.00 \text{ m/s}^2} = 10.0 \text{ s}$$

The total time is thus $10.0 \text{ s} + 20.0 \text{ s} + 5.00 \text{ s} = \boxed{35.0 \text{ s}}$.

(b) The average velocity is the total distance traveled divided by the total time taken. The distance traveled during the first 10.0 s is

$$x_1 = \bar{v}t = \left(\frac{0 + 20.0}{2}\right)(10.0) = 100 \text{ m}$$

With $a = 0$ for this interval, the distance traveled during the next 20.0 s is

$$x_2 = v_i t + \frac{1}{2}at^2 = (20.0)(20.0) + 0 = 400 \text{ m}$$

The distance traveled in the last 5.00 s is

$$x_3 = \bar{v}t = \left(\frac{20.0 + 0}{2}\right)(5.00) = 50.0 \text{ m}$$

The total distance $x = x_1 + x_2 + x_3 = 100 + 400 + 50 = 550$ m, and the average velocity is given by $\bar{v} = \frac{x}{t} = \frac{550}{35.0} = \boxed{15.7 \text{ m/s}}$.

P2.34 We ask whether the constant acceleration of the rhinoceros from rest over a period of 10.0 s can result in a final velocity of 8.00 m/s and a displacement of 50.0 m? To check, we solve for the acceleration in two ways.

1) $t_i = 0$, $v_i = 0$; $t = 10.0$ s, $v_f = 8.00$ m/s:

$$v_f = v_i + at \rightarrow a = \frac{v_f}{t}$$

$$a = \frac{8.00 \text{ m/s}}{10.0 \text{ s}} = 0.800 \text{ m/s}^2$$

2) $t_i = 0$, $x_i = 0$, $v_i = 0$; $t = 10.0$ s, $x_f = 50.0$ m:

$$x_f = x_i + v_i t + \frac{1}{2}at^2 \rightarrow x_f = \frac{1}{2}at^2$$

$$a = \frac{2x_f}{t^2} = \frac{2(50.0 \text{ m})}{(10.0 \text{ s})^2} = 1.00 \text{ m/s}^2$$

$\boxed{\text{The accelerations do not match, therefore the situation is impossible.}}$

P2.35 Since we don't know the initial and final velocities of the car, we will need to use two equations simultaneously to find the speed with which the car strikes the tree. From Equation 2.13, we have

$$v_{xf} = v_{xi} + a_x t = v_{xi} + (-5.60 \text{ m/s}^2)(4.20 \text{ s})$$

$$v_{xi} = v_{xf} + (5.60 \text{ m/s}^2)(4.20 \text{ s}) \qquad \textbf{[1]}$$

and from Equation 2.15,

$$x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t$$

$$62.4\text{ m} = \frac{1}{2}(v_{xi} + v_{xf})(4.20\text{ s}) \qquad [2]$$

Substituting for v_{xi} in [2] from [1] gives

$$62.4\text{ m} = \frac{1}{2}\left[v_{xf} + (5.60\text{ m/s}^2)(4.20\text{ s}) + v_{xf}\right](4.20\text{ s})$$

$$14.9\text{ m/s} = v_{xf} + \frac{1}{2}(5.60\text{ m/s}^2)(4.20\text{ s})$$

Thus, $v_{xf} = \boxed{3.10\text{ m/s}}$

P2.36 (a) Take any two of the standard four equations, such as

$$v_{xf} = v_{xi} + a_x t$$

$$x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t$$

Solve one for v_{xi}, and substitute into the other:

$$v_{xi} = v_{xf} - a_x t$$

$$x_f - x_i = \frac{1}{2}(v_{xf} - a_x t + v_{xf})t$$

Thus

$$\boxed{x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2}$$

We note that the equation is dimensionally correct. The units are units of length in each term. Like the standard equation $x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$, this equation represents that displacement is a quadratic function of time.

(b) Our newly derived equation gives us for the situation back in problem 35,

$$62.4\text{ m} = v_{xf}(4.20\text{ s}) - \frac{1}{2}(-5.60\text{ m/s}^2)(4.20\text{ s})^2$$

$$v_{xf} = \frac{62.4\text{ m} - 49.4\text{ m}}{4.20\text{ s}} = \boxed{3.10\text{ m/s}}$$

P2.37 (a) We choose a coordinate system with the x axis positive to the right, in the direction of motion of the speedboat, as shown on the right.

ANS. FIG. P2.37

(b) Since the speedboat is increasing its speed, the particle under constant acceleration model should be used here.

(c) Since the initial and final velocities are given along with the displacement of the speedboat, we use

$$v_{xf}^2 = v_{xi}^2 + 2a\Delta x$$

(d) Solving for the acceleration of the speedboat gives

$$a = \frac{v_{xf}^2 - v_{xi}^2}{2\Delta x}$$

(e) We have $v_i = 20.0$ m/s, $v_f = 30.0$ m/s, and $x_f - x_i = \Delta x = 200$ m:

$$a = \frac{v_{xf}^2 - v_{xi}^2}{2\Delta x} = \frac{(30.0\text{ m/s})^2 - (20.0\text{ m/s})^2}{2(200\text{ m})} = \boxed{1.25\text{ m/s}^2}$$

(f) To find the time interval, we use $v_f = v_i + at$, which gives

$$t = \frac{v_f - v_i}{a} = \frac{30.0\text{ m/s} - 20.0\text{ m/s}}{1.25\text{ m/s}^2} = \boxed{8.00\text{ s}}$$

P2.38 (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to recognize that $x_i = 2.00$ m, $v_i = 3.00$ m/s, and $a = -8.00$ m/s^2. The velocity equation, $v_f = v_i + at$, is then

$$v_f = 3.00\text{ m/s} - (8.00\text{ m/s}^2)t$$

The particle changes direction when $v_f = 0$, which occurs at $t = \frac{3}{8}$ s. The position at this time is

$$x = 2.00\text{ m} + (3.00\text{ m/s})\left(\frac{3}{8}\text{ s}\right) - (4.00\text{ m/s}^2)\left(\frac{3}{8}\text{ s}\right)^2$$

$$= \boxed{2.56\text{ m}}$$

(b) From $x_f = x_i + v_i t + \frac{1}{2}at^2$, observe that when $x_f = x_i$, the time is

given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00\text{ m/s})}{-8.00\text{ m/s}^2} = \frac{3}{4}\text{ s}$$

and the velocity is

$$v_f = 3.00\text{ m/s} - (8.00\text{ m/s}^2)\left(\frac{3}{4}\text{ s}\right) = \boxed{-3.00\text{ m/s}}$$

P2.39 Let the glider enter the photogate with velocity v_i and move with constant acceleration a. For its motion from entry to exit,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$\ell = 0 + v_i\Delta t_d + \frac{1}{2}a\Delta t_d^2 = v_d\Delta t_d$$

$$v_d = v_i + \frac{1}{2}a\Delta t_d$$

(a) The speed halfway through the photogate in space is given by

$$v_{hs}^2 = v_i^2 + 2a\left(\frac{\ell}{2}\right) = v_i^2 + av_d\Delta t_d$$

$$v_{hs} = \sqrt{v_i^2 + av_d\Delta t_d}$$ and this is $\boxed{\text{not equal to } v_d \text{ unless } a = 0}$.

(b) The speed halfway through the photogate in time is given by $v_{ht} = v_i + a\left(\frac{\Delta t_d}{2}\right)$ and this is $\boxed{\text{equal to } v_d}$ as determined above.

P2.40 (a) Let a stopwatch start from $t = 0$ as the front end of the glider passes point A. The average speed of the glider over the interval between $t = 0$ and $t = 0.628$ s is 12.4 cm/(0.628 s) = $\boxed{19.7\text{ cm/s}}$, and this is the instantaneous speed halfway through the time interval, at $t = 0.314$ s.

(b) The average speed of the glider over the time interval between $0.628 + 1.39 = 2.02$ s and $0.628 + 1.39 + 0.431 = 2.45$ s is 12.4 cm/(0.431 s) = 28.8 cm/s and this is the instantaneous speed at the instant $t = (2.02 + 2.45)/2 = 2.23$ s.

Now we know the velocities at two instants, so the acceleration is found from

$$[(28.8 - 19.7)\text{ cm/s}]/[(2.23 - 0.314)\text{ s}] = \boxed{4.70\text{ cm/s}^2}$$

(c) The distance between A and B is not used, but the length of the glider is used to find the average velocity during a known time interval.

P2.41 (a) What we know about the motion of an object is as follows: $a = 4.00\ \text{m/s}^2$, $v_i = 6.00\ \text{m/s}$, and $v_f = 12.0\ \text{m/s}$.

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = v_i^2 + 2a\Delta x$$

$$\Delta x = \frac{\left(v_f^2 - v_i^2\right)}{2a}$$

$$\Delta x = \frac{\left[(12.0\ \text{m/s})^2 - (6.00\ \text{m/s})^2\right]}{2\left(4.00\ \text{m/s}^2\right)} = \boxed{13.5\ \text{m}}$$

(b) From (a), the acceleration and velocity of the object are in the same (positive) direction, so the object speeds up. The distance is $\boxed{13.5\ \text{m}}$ because the object always travels in the same direction.

(c) Given $a = 4.00\ \text{m/s}^2$, $v_i = -6.00\ \text{m/s}$, and $v_f = 12.0\ \text{m/s}$. Following steps similar to those in (a) above, we will find the displacement to be the same: $\boxed{\Delta x = 13.5\ \text{m.}}$ In this case, the object initially is moving in the negative direction but its acceleration is in the positive direction, so the object slows down, reverses direction, and then speeds up as it travels in the positive direction.

(d) We consider the motion in two parts.

(1) Calculate the displacement of the object as it slows down: $a = 4.00\ \text{m/s}^2$, $v_i = -6.00\ \text{m/s}$, and $v_f = 0\ \text{m/s}$.

$$\Delta x = \frac{\left(v_f^2 - v_i^2\right)}{2a}$$

$$\Delta x = \frac{\left[(0\ \text{m/s})^2 - (-6.00\ \text{m/s})^2\right]}{2\left(4.00\ \text{m/s}^2\right)} = -4.50\ \text{m}$$

The object travels 4.50 m in the negative direction.

(2) Calculate the displacement of the object after it has reversed direction: $a = 4.00\ \text{m/s}^2$, $v_i = 0\ \text{m/s}$, $v_f = 12.0\ \text{m/s}$.

$$\Delta x = \frac{\left(v_f^2 - v_i^2\right)}{2a}$$

$$\Delta x = \frac{\left[(12.0 \text{ m/s})^2 - (0 \text{ m/s})^2\right]}{2\left(4.00 \text{ m/s}^2\right)} = 18.0 \text{ m}$$

The object travels 18.0 m in the positive direction.

Total distance traveled: 4.5 m + 18.0 m = $\boxed{22.5 \text{ m.}}$

P2.42 (a) For the first car, the speed as a function of time is

$$v_1 = v_{1i} + a_1 t = -3.50 \text{ cm/s} + \left(2.40 \text{ cm/s}^2\right)t$$

For the second car, the speed is

$$v_2 = v_{2i} + a_2 t = +5.5 \text{ cm/s} + 0$$

Setting the two expressions equal gives

$$-3.50 \text{ cm/s} + \left(2.40 \text{ cm/s}^2\right)t = 5.5 \text{ cm/s}$$

Solving for *t* gives

$$t = \frac{9.00 \text{ cm/s}}{2.40 \text{ cm/s}^2} = \boxed{3.75 \text{ s}}$$

(b) The first car then has speed

$$v_1 = v_{1i} + a_1 t = -3.50 \text{ cm/s} + \left(2.40 \text{ cm/s}^2\right)(3.75 \text{ s}) = \boxed{5.50 \text{ cm/s}}$$

and this is also the constant speed of the second car.

(c) For the first car, the position as a function of time is

$$x_1 = x_{1i} + v_{1i}t + \frac{1}{2}a_1 t^2$$
$$= 15.0 \text{ cm} - (3.50 \text{ cm/s})t + \frac{1}{2}\left(2.40 \text{ cm/s}^2\right)t^2$$

For the second car, the position is

$$x_2 = 10.0 \text{ cm} + (5.50 \text{ cm/s})t$$

At the point where the cars pass one another, their positions are equal:

$$15.0 \text{ cm} - (3.50 \text{ cm/s})t + \frac{1}{2}\left(2.40 \text{ cm/s}^2\right)t^2 = 10.0 \text{ cm} + (5.50 \text{ cm/s})t$$

rearranging gives

$$\left(1.20 \text{ cm/s}^2\right)t^2 - (9.00 \text{ cm/s})t + 5.00 \text{ cm} = 0$$

We solve this with the quadratic formula. Suppressing units,

$$t = \frac{9 \pm \sqrt{(9)^2 - 4(1.2)(5)}}{2(1.2)} = \frac{9 \pm \sqrt{57}}{2.4} = 6.90 \text{ s, or } \boxed{0.604 \text{ s}}$$

(d) At $t = 0.604$ s, the second and also the first car's position is

$$x_{1,2} = 10.0 \text{ cm} + (5.50 \text{ cm/s})(0.604 \text{ s}) = \boxed{13.3 \text{ cm}}$$

At $t = 6.90$ s, both are at position

$$x_{1,2} = 10.0 \text{ cm} + (5.50 \text{ cm/s})(6.90 \text{ s}) = \boxed{47.9 \text{ cm}}$$

(e) The cars are initially moving toward each other, so they soon arrive at the same position x when their speeds are quite different, giving one answer to (c) that is not an answer to (a). The first car slows down in its motion to the left, turns around, and starts to move toward the right, slowly at first and gaining speed steadily. At a particular moment its speed will be equal to the constant rightward speed of the second car, but at this time the accelerating car is far behind the steadily moving car; thus, the answer to (a) is not an answer to (c). Eventually the accelerating car will catch up to the steadily-coasting car, but passing it at higher speed, and giving another answer to (c) that is not an answer to (a).

P2.43 (a) Total displacement = area under the (v, t) curve from $t = 0$ to 50 s. Here, distance is the same as displacement because the motion is in one direction.

$$\Delta x = \frac{1}{2}(50 \text{ m/s})(15 \text{ s}) + (50 \text{ m/s})(40 - 15) \text{ s}$$
$$+ \frac{1}{2}(50 \text{ m/s})(10 \text{ s})$$
$$\Delta x = 1875 \text{ m} = \boxed{1.88 \text{ km}}$$

(b) From $t = 10$ s to $t = 40$ s, displacement is

$$\Delta x = \frac{1}{2}(50 \text{ m/s} + 33 \text{ m/s})(5 \text{ s}) + (50 \text{ m/s})(25 \text{ s}) = \boxed{1.46 \text{ km}}$$

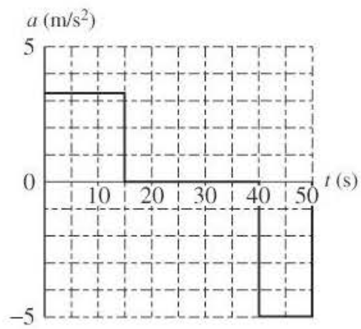
(c) We compute the acceleration for each of the three segments of the car's motion:

$0 \le t \le 15$ s: $\quad a_1 = \dfrac{\Delta v}{\Delta t} = \dfrac{(50-0)\text{ m/s}}{15\text{ s}-0} = \boxed{3.3\text{ m/s}^2}$

$15\text{ s} < t < 40$ s: $\quad \boxed{a_2 = 0}$

$40\text{ s} \le t \le 50$ s: $\quad a_3 = \dfrac{\Delta v}{\Delta t} = \dfrac{(0-50)\text{ m/s}}{50\text{ s}-40\text{ s}} = \boxed{-5.0\text{ m/s}^2}$

ANS. FIG. P2.43 shows the graph of the acceleration during this interval.



ANS FIG. P2.43

(d) For segment *0a*,

$$x_1 = 0 + \frac{1}{2}a_1t^2 = \frac{1}{2}\left(3.3\text{ m/s}^2\right)t^2 \text{ or } \boxed{x_1 = \left(1.67\text{ m/s}^2\right)t^2}$$

For segment *ab*,

$$x_2 = \frac{1}{2}(15\text{ s})[50\text{ m/s}-0] + (50\text{ m/s})(t-15\text{ s})$$

or $\boxed{x_2 = (50\text{ m/s})t - 375\text{ m}}$

For segment *bc*,

$$x_3 = \begin{pmatrix}\text{area under } v \text{ vs. } t \\ \text{from } t = 0 \text{ to } 40\text{ s}\end{pmatrix} + \frac{1}{2}a_3(t-40\text{ s})^2 + (50\text{ m/s})(t-40\text{ s})$$

or

$$x_3 = 375\text{ m} + 1250\text{ m} + \frac{1}{2}(-5.0\text{ m/s}^2)(t-40\text{ s})^2 + (50\text{ m/s})(t-40\text{ s})$$

which reduces to

$$\boxed{x_3 = (250\text{ m/s})t - \left(2.5\text{ m/s}^2\right)t^2 - 4\,375\text{ m}}$$

(e) $\bar{v} = \dfrac{\text{total displacement}}{\text{total elapsed time}} = \dfrac{1\ 875\ \text{m}}{50\ \text{s}} = \boxed{37.5\ \text{m/s}}$

2.44 (a) Take $t = 0$ at the time when the player starts to chase his opponent. At this time, the opponent is a distance $d = (12.0\ \text{m/s})(3.00\ \text{s}) = 36.0\ \text{m}$ in front of the player. At time $t > 0$, the displacements of the players from their initial positions are

$$\Delta x_{\text{player}} = v_{i,\text{player}}t + \frac{1}{2}a_{\text{player}}t^2 = 0 + \frac{1}{2}\left(4.00\ \text{m/s}^2\right)t^2 \qquad \textbf{[1]}$$

and

$$\Delta x_{\text{opponent}} = v_{i,\text{opponent}}t + \frac{1}{2}a_{\text{opponent}}t^2 = \left(12.0\ \text{m/s}\right)t + 0 \qquad \textbf{[2]}$$

When the players are side-by-side, $\Delta x_{\text{player}} = \Delta x_{\text{opponent}} + 36.0\ \text{m}$. **[3]**

Substituting equations [1] and [2] into equation [3] gives

$$\frac{1}{2}\left(4.00\ \text{m/s}^2\right)t^2 = \left(12.0\ \text{m/s}\right)t + 36.0\ \text{m}$$

or $\quad t^2 + \left(-6.00\ \text{s}\right)t + \left(-18.0\ \text{s}^2\right) = 0$

Applying the quadratic formula to this equation gives

$$t = \frac{-(-6.00\ \text{s}) \pm \sqrt{(-6.00\ \text{s})^2 - 4(1)\left(-18.0\ \text{s}^2\right)}}{2(1)}$$

which has solutions of $t = -2.20$ s and $t = +8.20$ s. Since the time must be greater than zero, we must choose $t = \boxed{8.20\ \text{s}}$ as the proper answer.

(b) $\Delta x_{\text{player}} = v_{i,\text{player}}t + \frac{1}{2}a_{\text{player}}t^2 = 0 + \frac{1}{2}\left(4.00\ \text{m/s}^2\right)\left(8.20\ \text{s}\right)^2 = \boxed{134\ \text{m}}$

Section 2.7 Freely Falling Objects

P2.45 This is motion with constant acceleration, in this case the acceleration of gravity. The equation of position as a function of time is

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$

Taking the positve y direction as up, the acceleration is $a = (9.80\ \text{m/s}^2$, downward) $= -g$; we also know that $y_i = 0$ and $v_i = 2.80$ m/s. The above

equation becomes

$$y_f = v_i t - \frac{1}{2} g t^2$$

$$y_f = (2.80 \text{ m/s})t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

(a) At $t = 0.100$ s, $y_f = \boxed{0.231 \text{ m}}$

(b) At $t = 0.200$ s, $y_f = \boxed{0.364 \text{ m}}$

(c) At $t = 0.300$ s, $y_f = \boxed{0.399 \text{ m}}$

(d) At $t = 0.500$ s, $y_f = \boxed{0.175 \text{ m}}$

P2.46 We can solve (a) and (b) at the same time by assuming the rock passes the top of the wall and finding its speed there. If the speed comes out imaginary, the rock will not reach this elevation.

$$\begin{aligned} v_f^2 &= v_i^2 + 2a(y_f - y_i) \\ &= (7.40 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(3.65 \text{ m} - 1.55 \text{ m}) \\ &= 13.6 \text{ m}^2/\text{s}^2 \end{aligned}$$

which gives $v_f = 3.69$ m/s.

$\boxed{\text{So the rock does reach the top of the wall with } v_f = 3.69 \text{ m/s}}$.

(c) The rock travels from $y_i = 3.65$ m to $y_f = 1.55$ m. We find the final speed of the rock thrown down:

$$\begin{aligned} v_f^2 &= v_i^2 + 2a(y_f - y_i) \\ &= (-7.40 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(1.55 \text{ m} - 3.65 \text{ m}) \\ &= 95.9 \text{ m}^2/\text{s}^2 \end{aligned}$$

which gives $v_f = -9.79$ m/s.

The change in speed of the rock thrown down is

$$|9.79 \text{ m/s} - 7.40 \text{ m/s}| = \boxed{2.39 \text{ m/s}}$$

(d) The magnitude of the speed change of the rock thrown up is $|7.40 \text{ m/s} - 3.69 \text{ m/s}| = 3.71$ m/s. This $\boxed{\text{does not agree}}$ with 2.39 m/s.

(e) $\boxed{\text{The upward-moving rock spends more time in flight because its average speed is smaller than the downward-moving rock, so the rock has more time to change its speed.}}$

P2.47 The bill starts from rest, $v_i = 0$, and falls with a downward acceleration of 9.80 m/s^2 (due to gravity). For an average human reaction time of about 0.20 s, we can find the distance the bill will fall:

$$y_f = y_i + v_i t + \frac{1}{2}at^2 \rightarrow \Delta y = v_i t - \frac{1}{2}gt^2$$

$$\Delta y = 0 - \frac{1}{2}\left(9.80 \text{ m/s}^2\right)(0.20 \text{ s})^2 = -0.20 \text{ m}$$

The bill falls about 20 cm—this distance is about twice the distance between the center of the bill and its top edge, about 8 cm. Thus $\boxed{\text{David could not respond fast enough to catch the bill.}}$

P2.48 Since the ball's motion is entirely vertical, we can use the equations for free fall to find the initial velocity and maximum height from the elapsed time. After leaving the bat, the ball is in free fall for $t = 3.00$ s and has constant acceleration $a_y = -g = -9.80$ m/s^2.

(a) The initial speed of the ball can be found from

$$v_f = v_i + at$$

$$0 = v_i - gt \rightarrow v_i = gt$$

$$v_i = \left(9.80 \text{ m/s}^2\right)\left(3.00 \text{ s}\right) = \boxed{29.4 \text{ m/s}}$$

(b) Find the vertical displacement Δy:

$$\Delta y = y_f - y_i = \frac{1}{2}\left(v_i + v_f\right)t$$

$$\Delta y = \frac{1}{2}\left(29.4 \text{ m/s} + 0\right)\left(3.00 \text{ s}\right)$$

$$\Delta y = \boxed{44.1 \text{ m}}$$

***P2.49** (a) Consider the upward flight of the arrow.

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

$$0 = \left(100 \text{ m/s}\right)^2 + 2\left(-9.80 \text{ m/s}^2\right)\Delta y$$

$$\Delta y = \frac{10\,000 \text{ m}^2/\text{s}^2}{19.6 \text{ m/s}^2} = \boxed{510 \text{ m}}$$

(b) Consider the whole flight of the arrow.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = 0 + (100\ \text{m/s})t + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2$$

The root $t = 0$ refers to the starting point. The time of flight is given by

$$t = \frac{100\ \text{m/s}}{4.90\ \text{m/s}^2} = \boxed{20.4\ \text{s}}$$

P2.50 We are given the height of the helicopter: $y = h = 3.00t^3$.

At $t = 2.00$ s, $y = 3.00(2.00\ \text{s})^3 = 24.0$ m and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0\ \text{m/s} \uparrow$$

If the helicopter releases a small mailbag at this time, the mailbag starts its free fall with velocity 36.0 m/s upward. The equation of motion of the mailbag is

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$

$$y_f = (24.0\ \text{m}) + (36.0\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$$

Setting $y_f = 0$, dropping units, and rearranging the equation, we have

$$4.90t^2 - 36.0t - 24.0 = 0$$

We solve for t using the quadratic formula:

$$t = \frac{36.0 \pm \sqrt{(-36.0)^2 - 4(4.90)(-24.0)}}{2(4.90)}$$

Since only positive values of t count, we find $t = \boxed{7.96\ \text{s}}$.

P2.51 The equation for the height of the ball as a function of time is

$$y_f = y_i + v_i t - \frac{1}{2}gt^2$$

$$0 = 30\ \text{m} + (-8.00\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$$

Solving for t,

$$t = \frac{+8.00 \pm \sqrt{(-8.00)^2 - 4(-4.90)(30)}}{2(-4.90)} = \frac{+8.00 \pm \sqrt{64 + 588}}{-9.80}$$

$$t = \boxed{1.79\text{ s}}$$

***P2.52** The falling ball moves a distance of (15 m – h) before they meet, where h is the height above the ground where they meet. We apply

$$y_f = y_i + v_i t - \frac{1}{2}gt^2$$

to the falling ball to obtain

$$-(15.0\text{ m} - h) = -\frac{1}{2}gt^2$$

or $$h = 15.0\text{ m} - \frac{1}{2}gt^2 \qquad \textbf{[1]}$$

Applying $y_f = y_i + v_i t - \frac{1}{2}gt^2$ to the rising ball gives

$$h = (25\text{ m/s})t - \frac{1}{2}gt^2 \qquad \textbf{[2]}$$

Combining equations [1] and [2] gives

$$(25\text{ m/s})t - \frac{1}{2}gt^2 = 15.0\text{ m} - \frac{1}{2}gt^2$$

or $$t = \frac{15\text{ m}}{25\text{ m/s}} = \boxed{0.60\text{ s}}$$

P2.53 We model the keys as a particle under the constant free-fall acceleration. Take the first student's position to be $y_i = 0$ and the second student's position to be $y_f = 4.00$ m. We are given that the time of flight of the keys is $t = 1.50$ s, and $a_y = -9.80\text{ m/s}^2$.

(a) We choose the equation $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$ to connect the data and the unknown.

We solve:

$$v_{yi} = \frac{y_f - y_i - \frac{1}{2}a_y t^2}{t}$$

and substitute:

$$v_{yi} = \frac{4.00\text{ m} - \frac{1}{2}\left(-9.80\text{ m/s}^2\right)(1.50\text{ s})^2}{1.50\text{ s}} = \boxed{10.0\text{ m/s}}$$

(b) The velocity at any time $t > 0$ is given by $v_{yf} = v_{yi} + a_y t$.

Therefore, at $t = 1.50$ s,

$$v_{yf} = 10.0\text{ m/s} - \left(9.80\text{ m/s}^2\right)(1.50\text{ s}) = \boxed{-4.68\text{ m/s}}$$

The negative sign means that the keys are moving **downward** just before they are caught.

P2.54 (a) The keys, moving freely under the influence of gravity ($a = -g$), undergo a vertical displacement of $\Delta y = +h$ in time t. We use $\Delta y = v_i t + \frac{1}{2}at^2$ to find the initial velocity as

$$\Delta y = v_i t + \frac{1}{2}at^2 = h$$

$$\rightarrow h = v_i t - \frac{1}{2}gt^2$$

$$v_i = \frac{h + \frac{1}{2}gt^2}{t} = \boxed{\frac{h}{t} + \frac{gt}{2}}$$

(b) We find the velocity of the keys just before they were caught (at time t) using $v = v_i + at$:

$$v = v_i + at$$

$$v = \left(\frac{h}{t} + \frac{gt}{2}\right) - gt$$

$$v = \boxed{\frac{h}{t} - \frac{gt}{2}}$$

P2.55 Both horse and man have constant accelerations: they are g downward for the man and 0 for the horse. We choose to do part (b) first.

(b) Consider the vertical motion of the man after leaving the limb (with $v_i = 0$ at $y_i = 3.00$ m) until reaching the saddle (at $y_f = 0$).

Modeling the man as a particle under constant acceleration, we find his time of fall from $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$.

When $v_i = 0$,

$$t = \sqrt{\frac{2\left(y_f - y_i\right)}{a_y}} = \sqrt{\frac{2(0 - 3.00\text{ m})}{-9.80\text{ m/s}^2}} = \boxed{0.782\text{ s}}$$

(a) During this time interval, the horse is modeled as a particle under constant velocity in the horizontal direction.

$$v_{xi} = v_{xf} = 10.0 \text{ m/s}$$

$$x_f - x_i = v_{xi}t = (10.0 \text{ m/s})(0.782 \text{ s}) = \boxed{7.82 \text{ m}}$$

and the ranch hand must let go when the horse is 7.82 m from the tree.

P2.56 (a) Let $t = 0$ be the instant the package leaves the helicopter. The package and the helicopter have a common initial velocity of $-v_i$ (choosing upward as positive). The helicopter has zero acceleration, and the package (in free-fall) has constant acceleration $a_y = -g$.

At times $t > 0$, the velocity of the package is

$$v_p = v_{yi} + a_y t \rightarrow v_p = -v_i - gt = -(v_i + gt)$$

so its speed is $|v_p| = \boxed{v_i + gt}$.

(b) Assume the helicopter is at height H when the package is released. Setting our clock to $t = 0$ at the moment the package is released, the position of the helicopter is

$$y_{hel} = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$y_{hel} = H + (-v_i)t$$

and the position of the package is

$$y_p = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$y_p = H + (-v_i)t - \frac{1}{2}gt^2$$

The vertical distance, d, between the helicopter and the package is

$$y_{hel} - y_p = \left[H + (-v_i)t\right] - \left[H + (-v_i)t - \frac{1}{2}gt^2\right]$$

$$d = \boxed{\frac{1}{2}gt^2}$$

The distance is independent of their common initial speed.

(c) Now, the package and the helicopter have a common initial velocity of $+v_i$ (choosing upward as positive). The helicopter has zero acceleration, and the package (in free-fall) has constant

acceleration $a_y = -g$.

At times $t > 0$, the velocity of the package is

$$v_P = v_{yi} + a_y t \rightarrow v_p = +v_i - gt$$

Therefore, the speed of the package at time t is $v_p = \boxed{|v_i - gt|}$.

The position of the helicopter is

$$y_{hel} = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$y_{hel} = H + (+v_i)t$$

and the position of the package is

$$y_p = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$y_p = H + (+v_i)t - \frac{1}{2}gt^2$$

The vertical distance, d, between the helicopter and the package is

$$y_{hel} - y_p = \left[H + (+v_i)t\right] - \left[H + (+v_i)t - \frac{1}{2}gt^2\right]$$
$$d = \boxed{\frac{1}{2}gt^2}$$

As above, the distance is independent of their common initial speed.

Section 2.8 Kinematic Equations Derived from Calculus

P2.57 This is a derivation problem. We start from basic definitions. We are given $J = da_x/dt$ = constant, so we know that $da_x = J dt$.

(a) Integrating from the 'initial' moment when we know the acceleration to any later moment,

$$\int_{a_{ix}}^{a_x} da = \int_0^t J\,dt \quad \rightarrow \quad a_x - a_{ix} = J(t-0)$$

Therefore, $\boxed{a_x = Jt + a_{xi}}$.

From $a_x = dv_x/dt$, $\quad dv_x = a_x\,dt$.

Integration between the same two points tells us the velocity as a function of time:

$$\int_{v_{xi}}^{v_x} dv_x = \int_0^t a_x\,dt = \int_0^t (a_{xi} + Jt)\,dt$$

$$v_x - v_{xi} = a_{xi}t + \frac{1}{2}Jt^2 \quad \text{or} \quad \boxed{v_x = v_{xi} + a_{xi}t + \frac{1}{2}Jt^2}$$

From $v_x = dx/dt$, $dx = v_x dt$. Integrating a third time gives us $x(t)$:

$$\int_{x_i}^{x} dx = \int_0^t v_x\,dt = \int_0^t \left(v_{xi} + a_{xi}t + \frac{1}{2}Jt^2\right) dt$$

$$x - x_i = v_{xi}t + \frac{1}{2}a_{xi}t^2 + \frac{1}{6}Jt^3$$

and $\boxed{x = \frac{1}{6}Jt^3 + \frac{1}{2}a_{xi}t^2 + v_{xi}t + x_i}$.

(b) Squaring the acceleration,

$$a_x^2 = (Jt + a_{xi})^2 = J^2t^2 + a_{xi}^2 + 2Ja_{xi}t$$

Rearranging,

$$a_x^2 = a_{xi}^2 + 2J\left(\frac{1}{2}Jt^2 + a_{xi}t\right)$$

The expression for v_x was

$$v_x = \frac{1}{2}Jt^2 + a_{xi}t + v_{xi}$$

So $\quad (v_x - v_{xi}) = \frac{1}{2}Jt^2 + a_{xi}t$

and by substitution

$$\boxed{a_x^2 = a_{xi}^2 + 2J(v_x - v_{xi})}$$

P2.58 (a) See the x vs. t graph on the top panel of ANS. FIG. P2.58, on the next page. Choose $x = 0$ at $t = 0$.

At t = 3 s, $x = \frac{1}{2}(8\text{ m/s})(3\text{ s}) = 12\text{ m}$.

At t = 5 s, $x = 12\text{ m} + (8\text{ m/s})(2\text{ s}) = 28\text{ m}$.

At t = 7 s, $x = 28\text{ m} + \frac{1}{2}(8\text{ m/s})(2\text{ s})$

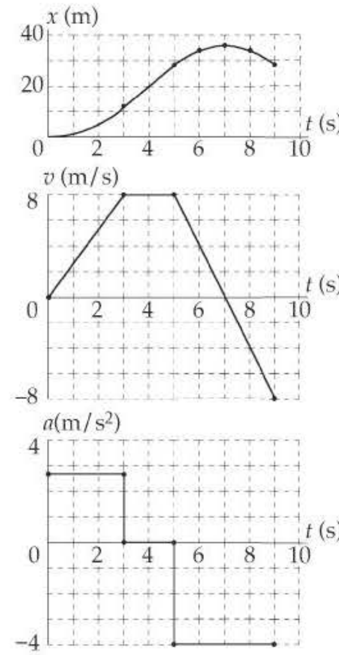
$= 36\text{ m}$

(b) See the a vs. t graph at the bottom right.

For $0 < t < 3$ s, $a = \dfrac{8\text{ m/s}}{3\text{ s}} = 2.67\text{ m/s}^2$.

For $3 < t < 5$ s, $a = 0$.

At the points of inflection, $t = 3$ and 5 s, the slope of the velocity curve changes abruptly, so the acceleration is not defined.



ANS. FIG. P2.58

(c) For $5\text{ s} < t < 9$ s,

$$a = -\frac{16\text{ m/s}}{4\text{ s}} = \boxed{-4\text{ m/s}^2}$$

(d) The average velocity between $t = 5$ and 7 s is

$$v_{\text{avg}} = (8\text{ m/s} + 0)/2 = 4\text{ m/s}$$

At $t = 6$ s, $x = 28\text{ m} + (4\text{ m/s})(1\text{ s}) = \boxed{32\text{ m}}$

(e) The average velocity between $t = 5$ and 9 s is

$$v_{\text{avg}} = [(8\text{ m/s}) + (-8\text{ m/s})]/2 = 0\text{ m/s}$$

At $t = 9$ s, $x = 28\text{ m} + (0\text{ m/s})(1\text{ s}) = \boxed{28\text{ m}}$

P2.59 (a) To find the acceleration, we differentiate the velocity equation with respect to time:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[\left(-5.00\times10^7\right)t^2 + \left(3.00\times10^5\right)t\right]$$

$$\boxed{a = -\left(10.0\times10^7\right)t + 3.00\times10^5}$$

where a is in m/s^2 and t is in seconds.

To find the position, take $x_i = 0$ at $t = 0$. Then, from $v = \dfrac{dx}{dt}$,

$$x - 0 = \int_0^t v\,dt = \int_0^t \left(-5.00\times10^7 t^2 + 3.00\times10^5 t\right)dt$$

$$x = -5.00\times10^7\frac{t^3}{3} + 3.00\times10^5\frac{t^2}{2}$$

which gives

$$\boxed{x = -\left(1.67\times10^7\right)t^3 + \left(1.50\times10^5\right)t^2}$$

where x is in meters and t is in seconds.

(b) The bullet escapes when $a = 0$:

$$a = -\left(10.0\times10^{7}\right)t + 3.00\times10^{5} = 0$$

$$t = \frac{3.00\times10^{5}\text{ s}}{10.0\times10^{7}} = 3.00\times10^{-3}\text{ s} = \boxed{3.00\text{ ms}}$$

(c) Evaluate v when $t = 3.00\times10^{-3}$ s:

$$v = \left(-5.00\times10^{7}\right)\left(3.00\times10^{-3}\right)^{2} + \left(3.00\times10^{5}\right)\left(3.00\times10^{-3}\right)$$

$$v = -450 + 900 = \boxed{450\text{ m/s}}$$

(d) Evaluate x when $t = 3.00\times10^{-3}$ s:

$$x = -\left(1.67\times10^{7}\right)\left(3.00\times10^{-3}\right)^{3} + \left(1.50\times10^{5}\right)\left(3.00\times10^{-3}\right)^{2}$$

$$x = -0.450 + 1.35 = \boxed{0.900\text{ m}}$$

Additional Problems

***P2.60** (a) Assuming a constant acceleration:

$$a = \frac{v_f - v_i}{t} = \frac{42.0\text{ m/s}}{8.00\text{ s}} = \boxed{5.25\text{ m/s}^2}$$

(b) Taking the origin at the original position of the car,

$$x_f = \frac{1}{2}\left(v_i + v_f\right)t = \frac{1}{2}(42.0\text{ m/s})(8.00\text{ s}) = \boxed{168\text{ m}}$$

(c) From $v_f = v_i + at$, the velocity 10.0 s after the car starts from rest is:

$$v_f = 0 + \left(5.25\text{ m/s}^2\right)(10.0\text{ s}) = \boxed{52.5\text{ m/s}}$$

P2.61 (a) From $v^2 = v_i^2 + 2a\Delta y$, the insect's velocity after straightening its legs is

$$v = \sqrt{v_0^2 + 2a(\Delta y)}$$

$$= \sqrt{0 + 2\left(4\ 000\text{ m/s}^2\right)\left(2.00\times10^{-3}\text{ m}\right)} = \boxed{4.00\text{ m/s}}$$

(b) The time to reach this velocity is

$$t = \frac{v - v_0}{a} = \frac{4.00\text{ m/s} - 0}{4\ 000\text{ m/s}^2} = 1.00\times10^{-3}\text{ s} = \boxed{1.00\text{ ms}}$$

(c) The upward displacement of the insect between when its feet leave the ground and its speed is momentarily zero is

$$\Delta y = \frac{v_f^2 - v_i^2}{2a}$$

$$\Delta y = \frac{0-(4.00 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = \boxed{0.816 \text{ m}}$$

P2.62 (a) The velocity is constant between $t_i = 0$ and $t = 4$ s. Its acceleration is $\boxed{0}$.

(b) $a = (v_9 - v_4)/(9 \text{ s} - 4 \text{ s}) = (18 - [-12]) \text{ (m/s)}/5 \text{ s} = \boxed{6.0 \text{ m/s}^2}$

(c) $a = (v_{18} - v_{13})/(18 \text{ s} - 13 \text{ s}) = (0 - 18) \text{ (m/s)}/5 \text{ s} = \boxed{-3.6 \text{ m/s}^2}$

(d) We read from the graph that the speed is zero $\boxed{\text{at } t = 6 \text{ s and at } 18\text{s}}$.

(e) and (f) The object moves away from $x = 0$ into negative coordinates from $t = 0$ to $t = 6$ s, but then comes back again, crosses the origin and moves farther into positive coordinates until $\boxed{t = 18 \text{ s}}$, then attaining its maximum distance, which is the cumulative distance under the graph line:

$$\Delta x = (-12 \text{ m/s})(4 \text{ s}) + \frac{1}{2}(-12 \text{ m/s})(2 \text{ s}) + \frac{1}{2}(18 \text{ m/s})(3 \text{ s})$$

$$+(18 \text{ m/s})(4 \text{ s})\frac{1}{2}(18 \text{ m/s})(5 \text{ s})$$

$$= \boxed{84 \text{ m}}$$

(g) We consider the total distance, rather than the resultant displacement, by counting the contributions computed in part (f) as all positive:

$$d = +60 \text{ m} + 144 \text{ m} = \boxed{204 \text{ m}}$$

P2.63 We set $y_i = 0$ at the top of the cliff, and find the time interval required for the first stone to reach the water using the particle under constant acceleration model:

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

or in quadratic form,

$$-\frac{1}{2}a_y t^2 - v_{yi}t + y_f - y_i = 0$$

(a) If we take the direction downward to be negative,

$$y_f = -50.0 \text{ m}, \quad v_{yi} = -2.00 \text{ m/s}, \quad \text{and} \quad a_y = -9.80 \text{ m/s}^2$$

Substituting these values into the equation, we find

$$(4.90 \text{ m/s}^2)t^2 + (2.00 \text{ m/s})t - 50.0 \text{ m} = 0$$

We now use the quadratic formula. The stone reaches the pool after it is thrown, so time must be positive and only the positive root describes the physical situation:

$$t = \frac{-2.00 \text{ m/s} \pm \sqrt{(2.00 \text{ m/s})^2 - 4(4.90 \text{ m/s}^2)(-50.0 \text{ m})}}{2(4.90 \text{ m/s}^2)}$$

$$= \boxed{3.00 \text{ s}}$$

where we have taken the positive root.

(b) For the second stone, the time of travel is

$$t = 3.00 \text{ s} - 1.00 \text{ s} = 2.00 \text{ s}$$

Since $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$,

$$v_{yi} = \frac{(y_f - y_i) - \frac{1}{2}a_y t^2}{t}$$

$$= \frac{-50.0 \text{ m} - \frac{1}{2}(-9.80 \text{ m/s}^2)(2.00 \text{ s})^2}{2.00 \text{ s}}$$

$$= \boxed{-15.3 \text{ m/s}}$$

The negative value indicates the downward direction of the initial velocity of the second stone.

(c) For the first stone,

$$v_{1f} = v_{1i} + a_1 t_1 = -2.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(3.00 \text{ s})$$

$$v_{1f} = \boxed{-31.4 \text{ m/s}}$$

For the second stone,

$$v_{2f} = v_{2i} + a_2 t_2 = -15.3 \text{ m/s} + (-9.80 \text{ m/s}^2)(2.00 \text{ s})$$

$$v_{2f} = \boxed{-34.8 \text{ m/s}}$$

P2.64 (a) Area A_1 is a rectangle. Thus, $A_1 = hw = v_{xi}t$.

Area A_2 is triangular. Therefore, $A_2 = \frac{1}{2}bh = \frac{1}{2}t(v_x - v_{xi})$.

The total area under the curve is

$$A = A_1 + A_2 = v_{xi}t + \frac{(v_x - v_{xi})t}{2}$$

and since $v_x - v_{xi} = a_x t$,

$$\boxed{A = v_{xi}t + \frac{1}{2}a_x t^2}$$

(b) The displacement given by the equation is: $x = v_{xi}t + \frac{1}{2}a_x t^2$, the same result as above for the total area.

***P2.65** (a) Take initial and final points at top and bottom of the first incline, respectively. If the ball starts from rest, $v_i = 0$, $a = 0.500\ \text{m/s}^2$, and $x_f - x_i = 9.00$ m. Then

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = 0^2 + 2(0.500\ \text{m/s}^2)(9.00\ \text{m})$$

$$v_f = \boxed{3.00\ \text{m/s}}$$

(b) To find the time interval, we use

$$x_f - x_i = v_i t + \frac{1}{2}at^2$$

Plugging in,

$$9.00 = 0 + \frac{1}{2}(0.500\ \text{m/s}^2)t^2$$

$$t = \boxed{6.00\ \text{s}}$$

(c) Take initial and final points at the bottom of the first plane and the top of the second plane, respectively: $v_i = 3.00$ m/s, $v_f = 0$, and $x_f - x_i = 15.0$ m. We use

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

which gives

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{0 - (3.00\ \text{m/s})^2}{2(15.0\ \text{m})} = \boxed{-0.300\ \text{m/s}^2}$$

(d) Take the initial point at the bottom of the first plane and the final point 8.00 m along the second plane:

$v_i = 3.00$ m/s, $x_f - x_i = 8.00$ m, $a = -0.300$ m/s^2

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = (3.00\text{ m/s})^2 + 2(-0.300\text{ m/s}^2)(8.00\text{ m})$$
$$= 4.20\text{ m}^2/\text{s}^2$$
$$v_f = \boxed{2.05\text{ m/s}}$$

***P2.66** Take downward as the positive y direction.

(a) While the woman was in free fall, $\Delta y = 144$ ft, $v_i = 0$, and we take $a = g = 32.0\text{ ft/s}^2$. Thus,

$$\Delta y = v_i t + \frac{1}{2}at^2 \rightarrow 144\text{ ft} = 0 + (16.0\text{ ft/s}^2)t^2$$

giving $t_{\text{fall}} = 3.00$ s. Her velocity just before impact is:

$$v_f = v_i + gt = 0 + (32.0\text{ ft/s}^2)(3.00\text{ s}) = \boxed{96.0\text{ ft/s}}$$

(b) While crushing the box, $v_i = 96.0\text{ ft/s}$, $v_f = 0$, and $\Delta y = 18.0\text{ in.} = 1.50\text{ ft}$. Therefore,

$$a = \frac{v_f^2 - v_i^2}{2(\Delta y)} = \frac{0 - (96.0\text{ ft/s})^2}{2(1.50\text{ ft})} = -3.07 \times 10^3\text{ ft/s}^2$$

or $\boxed{a = 3.07 \times 10^3\text{ ft/s}^2\text{ upward}} = 96.0g.$

(c) Time to crush box:

$$\Delta t = \frac{\Delta y}{\bar{v}} = \frac{\Delta y}{\frac{v_f + v_i}{2}} = \frac{2(1.50\text{ ft})}{0 + 96.0\text{ ft/s}}$$

or $\boxed{\Delta t = 3.13 \times 10^{-2}\text{ s}}$

P2.67 (a) The elevator, moving downward at the constant speed of 5.00 m/s has moved $d = v\Delta t = (5.00\text{ m/s})(5.00\text{ s}) = 25.0$ m below the position from which the bolt drops. Taking the positive direction to be downward, the initial position of the bolt to be $x_B = 0$, and setting $t = 0$ when the bolt drops, the position of the top of the elevator is

$$y_E = y_{Ei} + v_{Ei}t + \frac{1}{2}a_E t^2$$
$$y_E = 25.0\text{ m} + (5.00\text{ m/s})t$$

and the position of the bolt is

$$y_B = y_{Bi} + v_{Bi}t + \frac{1}{2}a_B t^2$$

$$y_B = \frac{1}{2}\left(9.80 \text{ m/s}^2\right)t^2$$

Setting these expressions equal to each other gives

$$y_E = y_B$$

$$25.0 \text{ m} + \left(5.00 \text{ m/s}\right)t = \frac{1}{2}\left(9.80 \text{ m/s}^2\right)t^2$$

$$4.90t^2 - 5.00t - 25.0 = 0$$

The (positive) solution to this is $t = \boxed{2.83 \text{ s}}$.

(b) Both problems have an object traveling at constant velocity being overtaken by an object starting from rest traveling in the same direction at a constant acceleration.

(c) The top of the elevator travels a total distance

$$d = (5.00 \text{ m/s})(5.00 \text{ s} + 2.83 \text{ s}) = 39.1 \text{ m}$$

from where the bolt drops to where the bolt strikes the top of the elevator. Assuming 1 floor $\cong$ 3 m, this distance is about

$$(39.1 \text{ m})(1 \text{ floor}/3 \text{ m}) \cong 13 \text{ floors.}$$

P2.68 For the collision not to occur, the front of the passenger train must not have a position that is equal to or greater than the position of the back of the freight train at any time. We can write expressions of position to see whether the front of the passenger car (P) meets the back of the freight car (F) at some time.

Assume at $t = 0$, the coordinate of the front of the passenger car is $x_{Pi} = 0$; and the coordinate of the back of the freight car is $x_{Fi} = 58.5$ m. At later time t, the coordinate of the front of the passenger car is

$$x_P = x_{Pi} + v_{Pi}t + \frac{1}{2}a_P t^2$$

$$x_P = \left(40.0 \text{ m/s}\right)t + \frac{1}{2}\left(-3.00 \text{ m/s}^2\right)t^2$$

and the coordinate of the back of the freight car is

$$x_F = x_{Fi} + v_{Fi}t + \frac{1}{2}a_F t^2$$

$$x_F = 58.5 \text{ m} + \left(16.0 \text{ m/s}\right)t$$

Setting these expression equal to each other gives

$$x_P = x_F$$

$$(40.0\text{ m/s})t + \frac{1}{2}\left(-3.00\text{ m/s}^2\right)t^2 = 58.5\text{ m} + (16.0\text{ m/s})t$$

or $$(1.50)t^2 + (-24.0)t + 58.5 = 0$$

after simplifying and suppressing units.

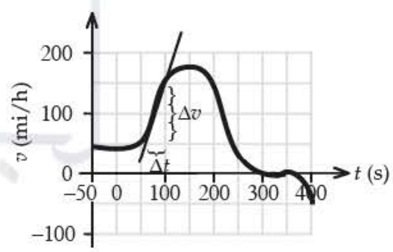
We do not have to solve this equation, we just want to check if a solution exists; if a solution does exist, then the trains collide. A solution does exist:

$$t = \frac{-(-24.0) \pm \sqrt{(-24.0)^2 - 4(1.50)(58.5)}}{2(1.50)}$$

$$t = \frac{24.0 \pm \sqrt{576 - 351}}{3.00} \to t = \frac{24.0 \pm \sqrt{225}}{3.00} = \frac{24.0 \pm 15}{3.00}$$

$\boxed{\text{The situation is impossible since there is a finite time for which the front of the passenger train and the back of the freight train are at the same location.}}$

P2.69 (a) As we see from the graph, from about –50 s to 50 s Acela is cruising at a constant positive velocity in the $+x$ direction. From 50 s to 200 s, Acela accelerates in the $+x$ direction reaching a top speed of about 170 mi/h. Around 200 s, the engineer applies the brakes, and the train, still traveling in the $+x$ direction, slows down and then stops at 350 s. Just after 350 s, Acela reverses direction (v becomes negative) and steadily gains speed in the $-x$ direction.



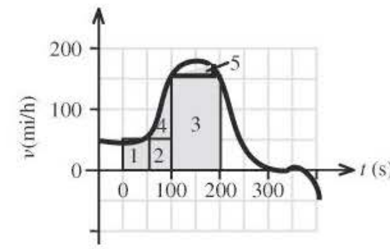
ANS. FIG. P2.69(a)

(b) The peak acceleration between 45 and 170 mi/h is given by the slope of the steepest tangent to the v versus t curve in this interval. From the tangent line shown, we find

$$a = \text{slope} = \frac{\Delta v}{\Delta t} = \frac{(155 - 45)\text{ mi/h}}{(100 - 50)\text{ s}}$$

$$= \boxed{2.2\text{ (mi/h)/s}} = 0.98\text{ m/s}^2$$

(c) Let us use the fact that the area under the v versus t curve equals the displacement. The train's displacement between 0 and 200 s is equal to the area of the gray shaded region, which we have approximated with a series of triangles and rectangles.



ANS. FIG. P2.69(c)

$$\begin{aligned}\Delta x_{0\to 200\text{ s}} &= \text{area}_1 + \text{area}_2 + \text{area}_3 + \text{area}_4 + \text{area}_5\\ &\approx (50\text{ mi/h})(50\text{ s}) + (50\text{ mi/h})(50\text{ s})\\ &\quad + (160\text{ mi/h})(100\text{ s})\\ &\quad + \frac{1}{2}(50\text{ s})(100\text{ mi/h})\\ &\quad + \frac{1}{2}(100\text{ s})(170\text{ mi/h} - 160\text{ mi/h})\\ &= 24\ 000(\text{mi/h})(\text{s})\end{aligned}$$

Now, at the end of our calculation, we can find the displacement in miles by converting hours to seconds. As 1 h = 3 600 s,

$$\Delta x_{0\to 200\text{ s}} = \left(\frac{24\ 000\text{ mi}}{3\ 600\text{ s}}\right)(\text{s}) = \boxed{6.7\text{ mi}}$$

P2.70 We use the relation $v_f^2 = v_i^2 + 2a(x_f - x_i)$, where $v_i = -8.00$ m/s and $v_f = 16.0$ m/s.

(a) The displacement of the first object is $\Delta x = +20.0$ m. Solving the above equation for the acceleration a, we obtain

$$a = \frac{v_f^2 - v_i^2}{2\Delta x}$$

$$a = \frac{(16.0\text{ m/s})^2 - (-8.00\text{ m/s})^2}{2(20.0\text{ m})}$$

$$a = \boxed{+4.80\text{ m/s}^2}$$

(b) Here, the total distance $d = 22.0$ m. The initial negative velocity and final positive velocity indicate that first the object travels through a negative displacement, slowing down until it reverses direction (where $v = 0$), then it returns to, and passes, its starting point, continuing to speed up until it reaches a speed of 16.0 m/s. We must consider the motion as comprising three displacements; the total distance d is the sum of the lengths of these displacements.

We split the motion into three displacements in which the acceleration remains constant throughout. We can find each displacement using

$$\Delta x = \frac{v_f^2 - v_i^2}{2a}$$

Displacement $\Delta x_1 = -d_1$ for velocity change $-8.00 \rightarrow 0$ m/s:

$$\Delta x_1 = \frac{v_f^2 - v_i^2}{2a} = \frac{0-(-8.00 \text{ m/s})^2}{2a} = \frac{(-8)^2}{2a} \quad \rightarrow \quad d_1 = \frac{8^2}{2a}$$

Displacement $\Delta x_2 = +d_1$ for velocity change $0 \rightarrow +8.00$ m/s:

$$\Delta x_2 = \frac{v_f^2 - v_i^2}{2a} = \frac{(8.00 \text{ m/s})^2 - 0}{2a} = \frac{8^2}{2a} \quad \rightarrow \quad d_2 = \frac{8^2}{2a}$$

Displacement $\Delta x_3 = +d_2$ for velocity change $+8.00 \rightarrow +16.0$ m/s:

$$\Delta x_3 = \frac{v_f^2 - v_i^2}{2a} = \frac{(16.0 \text{ m/s})^2 - (8.00 \text{ m/s})^2}{2a} = \frac{16^2 - 8^2}{2a}$$

$$\rightarrow \quad d_3 = \frac{16^2 - 8^2}{2a}$$

Suppressing units, the total distance is $d = d_1 + d_2 + d_3$, or

$$d = d_1 + d_2 + d_3 = 2\left(\frac{8^2}{2a}\right) + \frac{16^2 - 8^2}{2a} = \frac{16^2 + 8^2}{2a}$$

Solving for the acceleration gives

$$a = \frac{v_f^2 - v_i^2}{2d} = \frac{(16 \text{ m/s})^2 + (8 \text{ m/s})^2}{2d} = \frac{(16 \text{ m/s})^2 + (8 \text{ m/s})^2}{2(22.0 \text{ m})}$$

$$a = \boxed{7.27 \text{ m/s}^2}$$

P2.71 (a) In order for the trailing athlete to be able to catch the leader, his speed (v_1) must be greater than that of the leading athlete (v_2), and the distance between the leading athlete and the finish line must be great enough to give the trailing athlete sufficient time to make up the deficient distance, d.

(b) During a time interval t the leading athlete will travel a distance $d_2 = v_2 t$ and the trailing athlete will travel a distance $d_1 = v_1 t$. Only when $d_1 = d_2 + d$ (where d is the initial distance the trailing athlete was behind the leader) will the trailing athlete have caught the leader. Requiring that this condition be satisfied gives the elapsed time required for the second athlete to overtake the first:

$$d_1 = d_2 + d \quad \text{or} \quad v_1 t = v_2 t + d$$

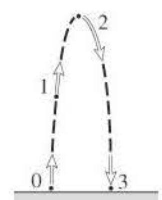
giving

$$v_1t - v_2t = d \quad \text{or} \quad t = \boxed{\frac{d}{(v_1 - v_2)}}$$

(c) In order for the trailing athlete to be able to at least tie for first place, the initial distance D between the leader and the finish line must be greater than or equal to the distance the leader can travel in the time t calculated above (i.e., the time required to overtake the leader). That is, we must require that

$$D \geq d_2 = v_2t = v_2\left[\frac{d}{(v_1 - v_2)}\right] \quad \text{or} \quad \boxed{d_2 = \frac{v_2 d}{v_1 - v_2}}$$

P2.72 Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact. The data in the table below are found for each phase of the rocket's motion.



ANS. FIG. P2.72

(0 to 1): $v_f^2 - (80.0\text{ m/s})^2 = 2(4.00\text{ m/s}^2)(1\,000\text{ m})$

so $v_f = 120$ m/s. Then, $120\text{ m/s} = 80.0\text{ m/s} + (4.00\text{ m/s}^2)t$

giving $t = 10.0$ s.

(1 to 2) $0 - (120\text{ m/s})^2 = 2(-9.80\text{ m/s}^2)(y_f - y_i)$

giving $y_f - y_i = 735$ m,

$0 - 120\text{ m/s} = (-9.80\text{ m/s}^2)t$

giving $t = 12.2$ s.

This is the time of maximum height of the rocket.

(2 to 3) $v_f^2 - 0 = 2(-9.80\text{ m/s}^2)(-1\,735\text{ m})$ or $v_f = -184$ m/s

Then $v_f = -184\text{ m/s} = (-9.80\text{ m/s}^2)t$

giving $t = 18.8$ s.

(a) $t_{total} = 10\text{ s} + 12.2\text{ s} + 18.8\text{ s} = \boxed{41.0\text{ s}}$

(b) $(y_f - y_i)_{total} = \boxed{1.73\text{ km}}$

(c) $v_{final} = \boxed{-184\text{ m/s}}$

		t	x	v	a
0	Launch	0.0	0	80	+4.00
#1	End Thrust	10.0	1 000	120	+4.00
#2	Rise Upwards	22.2	1 735	0	–9.80
#3	Fall to Earth	41.0	0	–184	–9.80

P2.73 We have constant-acceleration equations to apply to the two cars separately.

(a) Let the times of travel for Kathy and Stan be t_K and t_S, where

$$t_S = t_K + 1.00 \text{ s}$$

Both start from rest ($v_{xi,K} = v_{xi,S} = 0$), so the expressions for the distances traveled are

$$x_K = \frac{1}{2}a_{x,K}t_K^2 = \frac{1}{2}(4.90 \text{ m/s}^2)t_K^2$$

and $$x_S = \frac{1}{2}a_{x,S}t_S^2 = \frac{1}{2}(3.50 \text{ m/s}^2)(t_K + 1.00 \text{ s})^2$$

When Kathy overtakes Stan, the two distances will be equal. Setting $x_K = x_S$ gives

$$\frac{1}{2}(4.90 \text{ m/s}^2)t_K^2 = \frac{1}{2}(3.50 \text{ m/s}^2)(t_K + 1.00 \text{ s})^2$$

This we simplify and write in the standard form of a quadratic as

$$t_K^2 - (5.00\ t_K)\text{s} - 2.50 \text{ s}^2 = 0$$

We solve using the quadratic formula $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, suppressing units, to find

$$t_K = \frac{5 \pm \sqrt{5^2 - 4(1)(-2.5)}}{2(1)} = \frac{5 + \sqrt{35}}{2} = \boxed{5.46 \text{ s}}$$

Only the positive root makes sense physically, because the overtake point must be after the starting point in time.

(b) Use the equation from part (a) for distance of travel,

$$x_K = \frac{1}{2}a_{x,K}t_K^2 = \frac{1}{2}(4.90 \text{ m/s}^2)(5.46 \text{ s})^2 = \boxed{73.0 \text{ m}}$$

(c) Remembering that $v_{xi,K} = v_{xi,S} = 0$, the final velocities will be:

$$v_{xf,K} = a_{x,K}t_K = (4.90\ \text{m/s}^2)(5.46\ \text{s}) = \boxed{26.7\ \text{m/s}}$$

$$v_{xf,S} = a_{x,S}t_S = (3.50\ \text{m/s}^2)(6.46\ \text{s}) = \boxed{22.6\ \text{m/s}}$$

P2.74 (a) While in the air, both balls have acceleration $a_1 = a_2 = -g$ (where upward is taken as positive). Ball 1 (thrown downward) has initial velocity $v_{01} = -v_0$, while ball 2 (thrown upward) has initial velocity $v_{02} = v_0$. Taking $y = 0$ at ground level, the initial y coordinate of each ball is $y_{01} = y_{02} = +h$. Applying $\Delta y = y - y_i = v_i t + \frac{1}{2}at^2$ to each ball gives their y coordinates at time t as

Ball 1: $y_1 - h = -v_0 t + \frac{1}{2}(-g)t^2$ or $\boxed{y_1 = h - v_0 t - \frac{1}{2}gt^2}$

Ball 2: $y_2 - h = +v_0 t + \frac{1}{2}(-g)t^2$ or $\boxed{y_2 = h + v_0 t - \frac{1}{2}gt^2}$

At ground level, $y = 0$. Thus, we equate each of the equations found above to zero and use the quadratic formula to solve for the times when each ball reaches the ground. This gives the following:

Ball 1: $0 = h - v_0 t_1 - \frac{1}{2}gt_1^2 \rightarrow gt_1^2 + (2v_0)t_1 + (-2h) = 0$

so $t_1 = \dfrac{-2v_0 \pm \sqrt{(2v_0)^2 - 4(g)(-2h)}}{2g} = -\dfrac{v_0}{g} \pm \sqrt{\left(\dfrac{v_0}{g}\right)^2 + \dfrac{2h}{g}}$

Using only the *positive* solution gives

$$t_1 = -\frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Ball 2: $0 = h + v_0 t_2 - \frac{1}{2}gt_2^2 \rightarrow gt_2^2 + (-2v_0)t_2 + (-2h) = 0$

and $t_2 = \dfrac{-(-2v_0) \pm \sqrt{(-2v_0)^2 - 4(g)(-2h)}}{2g} = +\dfrac{v_0}{g} \pm \sqrt{\left(\dfrac{v_0}{g}\right)^2 + \dfrac{2h}{g}}$

Again, using only the *positive* solution,

$$t_2 = \frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Thus, the difference in the times of flight of the two balls is

$$\Delta t = t_2 - t_1$$

$$= \frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}} - \left(-\frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}\right) = \boxed{\frac{2v_0}{g}}$$

(b) Realizing that the balls are going *downward* ($v < 0$) as they near the ground, we use $v_f^2 = v_i^2 + 2a(\Delta y)$ with $\Delta y = -h$ to find the velocity of each ball just before it strikes the ground:

Ball 1:

$$v_{1f} = -\sqrt{v_{1i}^2 + 2a_1(-h)} = -\sqrt{(-v_0)^2 + 2(-g)(-h)} = \boxed{-\sqrt{v_0^2 + 2gh}}$$

Ball 2:

$$v_{2f} = -\sqrt{v_{2i}^2 + 2a_2(-h)} = -\sqrt{(+v_0)^2 + 2(-g)(-h)} = \boxed{-\sqrt{v_0^2 + 2gh}}$$

(c) While both balls are still in the air, the distance separating them is

$$d = y_2 - y_1 = \left(h + v_0 t - \frac{1}{2}gt^2\right) - \left(h - v_0 t - \frac{1}{2}gt^2\right) = \boxed{2v_0 t}$$

P2.75 We translate from a pictorial representation through a geometric model to a mathematical representation by observing that the distances x and y are always related by $x^2 + y^2 = L^2$.

(a) Differentiating this equation with respect to time, we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Now the unknown velocity of B is $\frac{dy}{dt} = v_B$ and $\frac{dx}{dt} = -v$,

so the differentiated equation becomes

$$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\left(\frac{x}{y}\right)(-v) = v_B$$

But $\frac{y}{x} = \tan\theta$, so $\qquad v_B = \boxed{\left(\frac{1}{\tan\theta}\right)v}$

(b) We assume that θ starts from zero. At this instant $1/\tan\theta$ is infinite, and the velocity of B is infinitely larger than that of A. As θ increases, the velocity of object B decreases, becoming equal to v when $\theta = 45°$. After that instant, B continues to slow down with non-constant acceleration, coming to rest as θ goes to $90°$.

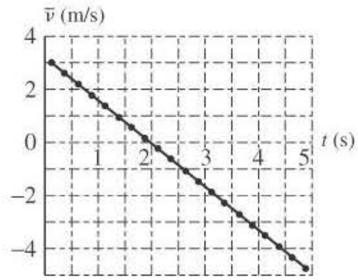
P2.76

Time t (s)	Height h (m)	Δh (m)	Δt (s)	$\bar{v}$ (m/s)	midpoint time t (s)
0.00	5.00	0.75	0.25	3.00	0.13
0.25	5.75	0.65	0.25	2.60	0.38
0.50	6.40	0.54	0.25	2.16	0.63
0.75	6.94	0.44	0.25	1.76	0.88
1.00	7.38	0.34	0.25	1.36	1.13
1.25	7.72	0.24	0.25	0.96	1.38
1.50	7.96	0.14	0.25	0.56	1.63
1.75	8.10	0.03	0.25	0.12	1.88
2.00	8.13	–0.06	0.25	–0.24	2.13
2.25	8.07	–0.17	0.25	–0.68	2.38
2.50	7.90	–0.28	0.25	–1.12	2.63
2.75	7.62	–0.37	0.25	–1.48	2.88
3.00	7.25	–0.48	0.25	–1.92	3.13
3.25	6.77	–0.57	0.25	–2.28	3.38
3.50	6.20	–0.68	0.25	–2.72	3.63
3.75	5.52	–0.79	0.25	–3.16	3.88
4.00	4.73	–0.88	0.25	–3.52	4.13
4.25	3.85	–0.99	0.25	–3.96	4.38
4.50	2.86	–1.09	0.25	–4.36	4.63
4.75	1.77	–1.19	0.25	–4.76	4.88
5.00	0.58				

TABLE P2.76

The very convincing fit of a single straight line to the points in the graph of velocity versus time indicates that the rock does fall with constant acceleration. The acceleration is the slope of line:

$$a_{avg} = -1.63\ \text{m/s}^2 = \boxed{1.63\ \text{m/s}^2\ \text{downward}}$$



***P2.77** Distance traveled by motorist = (15.0 m/s)t

Distance traveled by policeman $= \dfrac{1}{2}(2.00\ \text{m/s}^2)t^2$

(a) Intercept occurs when $15.0t = t^2$, or $t = \boxed{15.0\ \text{s}}$.

(b) $v(\text{officer}) = (2.00\ \text{m/s}^2)t = \boxed{30.0\ \text{m/s}}$

(c) $x(\text{officer}) = \dfrac{1}{2}(2.00\ \text{m/s}^2)t^2 = \boxed{225\ \text{m}}$

***P2.78** The train accelerates with $a_1 = 0.100\ \text{m/s}^2$ then decelerates with $a_2 = -0.500\ \text{m/s}^2$. We can write the 1.00-km displacement of the train as

$$x = 1\,000\ \text{m} = \frac{1}{2}a_1\Delta t_1^2 + v_{1f}\Delta t_2 + \frac{1}{2}a_2\Delta t_2^2$$

with $t = t_1 + t_2$. Now, $v_{1f} = a_1\Delta t_1 = -a_2\Delta t_2$; therefore

$$1\,000\ \text{m} = \frac{1}{2}a_1\Delta t_1^2 + a_1\Delta t_1\left(-\frac{a_1\Delta t_1}{a_2}\right) + \frac{1}{2}a_2\left(\frac{a_1\Delta t_1}{a_2}\right)^2$$

$$1\,000\ \text{m} = \frac{1}{2}a_1\left(1 - \frac{a_1}{a_2}\right)\Delta t_1^2$$

$$1\,000\ \text{m} = \frac{1}{2}(0.100\ \text{m/s}^2)\left(1 - \frac{0.100\ \text{m/s}^2}{-0.500\ \text{m/s}^2}\right)\Delta t_1^2$$

$$\Delta t_1 = \sqrt{\frac{20\,000}{1.20}}\ \text{s} = 129\ \text{s}$$

$$\Delta t_2 = \frac{a_1\Delta t_1}{-a_2} = \frac{12.9}{0.500}\ \text{s} \approx 26\ \text{s}$$

$$\text{Total time} = \Delta t = \Delta t_1 + \Delta t_2 = 129\text{ s} + 26\text{ s} = \boxed{155\text{ s}}$$

***P2.79** The average speed of every point on the train as the first car passes Liz is given by:

$$\frac{\Delta x}{\Delta t} = \frac{8.60\text{ m}}{1.50\text{ s}} = 5.73\text{ m/s}$$

The train has this as its instantaneous speed halfway through the 1.50-s time. Similarly, halfway through the next 1.10 s, the speed of the train is $\frac{8.60\text{ m}}{1.10\text{ s}} = 7.82\text{ m/s}$. The time required for the speed to change from 5.73 m/s to 7.82 m/s is

$$\frac{1}{2}(1.50\text{ s}) + \frac{1}{2}(1.10\text{ s}) = 1.30\text{ s}$$

so the acceleration is: $a_x = \frac{\Delta v_x}{\Delta t} = \frac{7.82\text{ m/s} - 5.73\text{ m/s}}{1.30\text{ s}} = \boxed{1.60\text{ m/s}^2}$

P2.80 Let the ball fall freely for 1.50 m after starting from rest. It strikes at speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a(x_f - x_i)$$

$$v_{xf}^2 = 0 + 2(-9.80\text{ m/s}^2)(-1.50\text{ m})$$

$$v_{xf} = -5.42\text{ m/s}$$

If its acceleration were constant, its stopping would be described by

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$0 = (-5.42\text{ m/s})^2 + 2a_x(-10^{-2}\text{ m})$$

$$a_x = \frac{-29.4\text{ m}^2/\text{s}^2}{-2.00\times10^{-2}\text{ m}} = +1.47\times10^3\text{ m/s}^2$$

Upward acceleration of this same order of magnitude will continue for some additional time after the dent is at its maximum depth, to give the ball the speed with which it rebounds from the pavement. The ball's maximum acceleration will be larger than the average acceleration we estimate by imagining constant acceleration, but will still be of order of magnitude $\boxed{\sim10^3\text{ m/s}^2}$.

Challenge Problems

P2.81 (a) From the information in the problem, we model the blue car as a particle under constant acceleration. The important "particle" for this part of the problem is the nose of the car. We use the position equation from the particle under constant acceleration model to find the velocity v_0 of the particle as it enters the intersection

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$\rightarrow 28.0 \text{ m} = 0 + v_0(3.10 \text{ s}) + \frac{1}{2}(-2.10 \text{ m/s}^2)(3.10 \text{ s})^2$$

$$\rightarrow v_0 = 12.3 \text{ m/s}$$

Now we use the velocity-position equation in the particle under constant acceleration model to find the displacement of the particle from the first edge of the intersection when the blue car stops:

$$v^2 = v_0^2 + 2a(x - x_0)$$

or $$x - x_0 = \Delta x = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12.3 \text{ m/s})^2}{2(-2.10 \text{ m/s}^2)} = \boxed{35.9 \text{ m}}$$

(b) The time interval during which any part of the blue car is in the intersection is that time interval between the instant at which the nose enters the intersection and the instant when the tail leaves the intersection. Thus, the change in position of the nose of the blue car is 4.52 m + 28.0 m = 32.52 m. We find the time at which the car is at position $x = 32.52$ m if it is at $x = 0$ and moving at 12.3 m/s at $t = 0$:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$\rightarrow 32.52 \text{ m} = 0 + (12.3 \text{ m/s})t + \frac{1}{2}(-2.10 \text{ m/s}^2)t^2$$

$$\rightarrow -1.05t^2 + 12.3t - 32.52 = 0$$

The solutions to this quadratic equation are $t = 4.04$ s and 7.66 s. Our desired solution is the lower of two, so $t = \boxed{4.04 \text{ s}}$, (The later time corresponds to the blue car stopping and reversing, which it must do if the acceleration truly remains constant, and arriving again at the position $x = 32.52$ m.)

(c) We again define $t = 0$ as the time at which the nose of the blue car enters the intersection. Then at time $t = 4.04$ s, the tail of the blue

car leaves the intersection. Therefore, to find the minimum distance from the intersection for the silver car, its nose must enter the intersection at $t = 4.04$ s. We calculate this distance from the position equation:

$$x - x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(5.60 \text{ m/s}^2)(4.04 \text{ s})^2 = \boxed{45.8 \text{ m}}$$

(d) We use the velocity equation:

$$v = v_0 + at = 0 + (5.60 \text{ m/s}^2)(4.04 \text{ s}) = \boxed{22.6 \text{ m/s}}$$

P2.82 (a) Starting from rest and accelerating at $a_b = 13.0$ mi/h · s, the bicycle reaches its maximum speed of $v_{b,\max} = 20.0$ mi/h in a time

$$t_{b,1} = \frac{v_{b,\max} - 0}{a_b} = \frac{20.0 \text{ mi/h}}{13.0 \text{ mi/h} \cdot \text{s}} = 1.54 \text{ s}$$

Since the acceleration a_c of the car is less than that of the bicycle, the car cannot catch the bicycle until some time $t > t_{b,1}$ (that is, until the bicycle is at its maximum speed and coasting). The total displacement of the bicycle at time t is

$$\begin{aligned}\Delta x_b &= \frac{1}{2}a_b t_{b,1}^2 + v_{b,\max}(t - t_{b,1}) \\ &= \left(\frac{1.47 \text{ ft/s}}{1 \text{ mi/h}}\right) \times \\ &\quad \left[\frac{1}{2}\left(13.0\frac{\text{mi/h}}{\text{s}}\right)(1.54 \text{ s})^2 + (20.0 \text{ mi/h})(t - 1.54 \text{ s})\right] \\ &= (29.4 \text{ ft/s})t - 22.6 \text{ ft}\end{aligned}$$

The total displacement of the car at this time is

$$\Delta x_c = \frac{1}{2}a_c t^2 = \left(\frac{1.47 \text{ ft/s}}{1 \text{ mi/h}}\right)\left[\frac{1}{2}\left(9.00\frac{\text{mi/h}}{\text{s}}\right)t^2\right] = (6.62 \text{ ft/s}^2)t^2$$

At the time the car catches the bicycle, $\Delta x_c = \Delta x_b$. This gives

$$(6.62 \text{ ft/s}^2)t^2 = (29.4 \text{ ft/s})t - 22.6 \text{ ft}$$

or $\quad t^2 - (4.44 \text{ s})t + 3.42 \text{ s}^2 = 0$

that has only one physically meaningful solution $t > t_{b,1}$. This solution gives the total time the bicycle leads the car and is $t = \boxed{3.45 \text{ s}}$.

(b) The lead the bicycle has over the car continues to increase as long as the bicycle is moving faster than the car. This means until the

car attains a speed of $v_c = v_{b,\max} = 20.0$ mi/h. Thus, the elapsed time when the bicycle's lead ceases to increase is

$$t = \frac{v_{b,\max}}{a_c} = \frac{20.0 \text{ mi/h}}{9.00 \text{ mi/h} \cdot \text{s}} = 2.22 \text{ s}$$

At this time, the lead is

$$\begin{aligned}(\Delta x_b - \Delta x_c)_{\max} &= (\Delta x_b - \Delta x_c)\big|_{t=2.22 \text{ s}} \\ &= \left[(29.4 \text{ ft/s})(2.22 \text{ s}) - 22.6 \text{ ft}\right] \\ &\quad - \left[(6.62 \text{ ft/s}^2)(2.22 \text{ s})^2\right]\end{aligned}$$

or $\quad (\Delta x_b - \Delta x_c)_{\max} = \boxed{10.0 \text{ ft}}$

P2.83 Consider the runners in general. Each completes the race in a total time interval T. Each runs at constant acceleration a for a time interval Δt, so each covers a distance (displacement) $\Delta x_a = \frac{1}{2}a\Delta t^2$ where they eventually reach a final speed (velocity) $v = a\Delta t$, after which they run at this constant speed for the remaining time $(T - \Delta t)$ until the end of the race, covering distance $\Delta x_v = v(T - \Delta t) = a\Delta t(T - \Delta t)$. The total distance (displacement) each covers is the same:

$$\begin{aligned}\Delta x &= \Delta x_a + \Delta x_v \\ &= \frac{1}{2}a\Delta t^2 + a\Delta t(T - \Delta t) \\ &= a\left[\frac{1}{2}\Delta t^2 + \Delta t(T - \Delta t)\right]\end{aligned}$$

so $\quad a = \dfrac{\Delta x}{\frac{1}{2}\Delta t^2 + \Delta t(T - \Delta t)}$

where $\Delta x = 100$ m and $T = 10.4$ s.

(a) For Laura (runner 1), $\Delta t_1 = 2.00$ s:

$$a_1 = (100 \text{ m})/(18.8 \text{ s}^2) = \boxed{5.32 \text{ m/s}^2}$$

For Healan (runner 2), $\Delta t_2 = 3.00$ s:

$$a_2 = (100 \text{ m})/(26.7 \text{ s}^2) = \boxed{3.75 \text{ m/s}^2}$$

(b) Laura (runner 1): $v_1 = a_1\Delta t_1 = \boxed{10.6 \text{ m/s}}$

Healan (runner 2): $v_2 = a_2\Delta t_2 = \boxed{11.2 \text{ m/s}}$

(c) The 6.00-s mark occurs after either time interval Δt. From the reasoning above, each has covered the distance

$$\Delta x = a\left[\frac{1}{2}\Delta t^2 + \Delta t(t - \Delta t)\right]$$

where $t = 6.00$ s.

Laura (runner 1): $\Delta x_1 = 53.19$ m

Healan (runner 2): $\Delta x_2 = 50.56$ m

$\boxed{\text{So, Laura is ahead by } (53.19 \text{ m} - 50.56 \text{ m}) = 2.63 \text{ m.}}$

(d) Laura accelerates at the greater rate, so she will be ahead of Healen at, and immediately after, the 2.00-s mark. After the 3.00-s mark, Healan is travelling faster than Laura, so the distance between them will shrink. In the time interval

$\boxed{\text{from the 2.00-s mark to the 3.00-s mark, the distance between them will be the greatest.}}$

During that time interval, the distance between them (the position of Laura relative to Healan) is

$$D = \Delta x_1 - \Delta x_2 = a_1\left[\frac{1}{2}\Delta t_1^2 + \Delta t_1(t - \Delta t_1)\right] - \frac{1}{2}a_2 t^2$$

because Laura has ceased to accelerate but Healan is still accelerating. Differentiating with respect to time, (and doing some simplification), we can solve for the time t when D is an maximum:

$$\frac{dD}{dt} = a_1\Delta t_1 - a_2 t = 0$$

which gives

$$t = \Delta t_1\left(\frac{a_1}{a_2}\right) = (2.00 \text{ s})\left(\frac{5.32 \text{ m/s}^2}{3.75 \text{ m/s}^2}\right) = 2.84 \text{ s}$$

Substituting this time back into the expression for D, we find that $D = 4.47$ m, that is, Laura ahead of Healan by $\boxed{4.47 \text{ m.}}$

P2.84 (a) The factors to consider are as follows. The red bead falls through a greater distance with a downward acceleration of g. The blue bead travels a shorter distance, but with acceleration of $g\sin\theta$. A first guess would be that the blue bead "wins," but not by much. We do note, however, that points Ⓐ, Ⓑ, and Ⓒ are the vertices of a right triangle with ⒶⒸ as the hypotenuse.

(b) The red bead is a particle under constant acceleration. Taking downward as the positive direction, we can write

$$\Delta y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

as $$D = \frac{1}{2}gt_R^2$$

which gives $\boxed{t_R = \sqrt{\dfrac{2D}{g}}}$.

(c) The blue bead is a particle under constant acceleration, with $a = g\sin\theta$. Taking the direction along L as the positive direction, we can write

$$\Delta y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

as $$L = \frac{1}{2}(g\sin\theta)t_B^2$$

which gives $\boxed{t_B = \sqrt{\dfrac{2L}{g\sin\theta}}}$.

(d) For the two beads to reach point Ⓒ simultaneously, $t_R = t_B$. Then,

$$\sqrt{\frac{2D}{g}} = \sqrt{\frac{2L}{g\sin\theta}}$$

Squaring both sides and cross-multiplying gives

$$2gD\sin\theta = 2gL$$

or $$\sin\theta = \frac{L}{D}.$$

We note that the angle between chords ⒶⒸ and ⒷⒸ is $90° - \theta$, so that the angle between chords ⒶⒸ and ⒶⒷ is

θ. Then, $\sin\theta = \frac{L}{D}$, and the beads arrive at point Ⓒ simultaneously.

(e) Once we recognize that the two rods form one side and the hypotenuse of a right triangle with θ as its smallest angle, then the result becomes obvious.

P2.85 The rock falls a distance *d* for a time interval Δt_1 and the sound of the splash travels upward through the same distance *d* for a time interval Δt_2 before the man hears it. The total time interval $\Delta t = \Delta t_1 + \Delta t_2 = 2.40$ s.

(a) Relationship between distance the rock falls and time interval Δt_1:

$$d = \frac{1}{2} g \Delta t_1^2$$

Relationship between distance the sound travels and time interval Δt_2: $d = v_s \Delta t_2$, where $v_s = 336$ m/s.

$$d = v_s \Delta t_2 = \frac{1}{2} g \Delta t_1^2$$

Substituting $\Delta t_1 = \Delta t - \Delta t_2$ gives

$$2\frac{v_s \Delta t_2}{g} = (\Delta t - \Delta t_2)^2$$

$$(\Delta t_2)^2 - 2\left(\Delta t + \frac{v_s}{g}\right)\Delta t_2 + \Delta t^2 = 0$$

$$(\Delta t_2)^2 - 2\left(2.40 \text{ s} + \frac{336 \text{ m/s}}{9.80 \text{ m/s}^2}\right)\Delta t_2 + (2.40 \text{ s})^2 = 0$$

$$(\Delta t_2)^2 - (73.37)\Delta t_2 + 5.76 = 0$$

Solving the quadratic equation gives

$$\Delta t_2 = 0.078\ 6 \text{ s} \quad \rightarrow \quad d = v_s \Delta t_2 = \boxed{26.4 \text{ m}}$$

(b) Ignoring the sound travel time,

$d = \frac{1}{2}(9.80 \text{ m/s}^2)(2.40 \text{ s})^2 = 28.2$ m, an error of $\boxed{6.82\%}$.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P2.2 0.02 s

P2.4 (a) 50.0 m/s; (b) 41.0 m/s

P2.6 (a) 27.0 m; (b) 27.0 m + (18.0 m/s) Δt + (3.00 m/s^2)(Δt^2); (c) 18.0 m/s

P2.8 (a) $+L/t_1$; (b) $-L/t_2$; (c) 0; (d) $2L/t_1 + t_2$

P2.10 1.9×10^8 years

P2.12 (a) 20 mi/h; (b) 0; (c) 30 mi/h

P2.14 1.34×10^4 m/s^2

P2.16 See graphs in P2.16.

P2.18 (a) See ANS. FIG. P2.18; (b) 23 m/s, 18 m/s, 14 m/s, and 9.0 m/s; (c) 4.6 m/s^2; (d) zero

P2.20 (a) 13.0 m/s; (b) 10.0 m/s, 16.0 m/s; (c) 6.00 m/s^2; (d) 6.00 m/s^2; (e) 0.333 s

P2.22 (a–e) See graphs in P2.22; (f) with less regularity

P2.24 160 ft.

P2.26 4.53 s

P2.28 (a) 6.61 m/s; (b) –0.448 m/s^2

P2.30 (a) 20.0 s; (b) No; (c) The plane would overshoot the runway.

P2.32 31 s

P2.34 The accelerations do not match.

P2.36 (a) $x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2$; (b) 3.10 m/s

P2.38 (a) 2.56 m; (b) –3.00 m/s

P2.40 19.7 cm/s; (b) 4.70 cm/s^2; (c) The length of the glider is used to find the average velocity during a known time interval.

P2.42 (a) 3.75 s; (b) 5.50 cm/s; (c) 0.604 s; (d) 13.3 cm, 47.9 cm; (e) See P2.42 part (e) for full explanation.

P2.44 (a) 8.20 s; (b) 134 m

P2.46 (a and b) The rock does not reach the top of the wall with v_f = 3.69 m/s; (c) 2.39 m/s; (d) does not agree; (e) The average speed of the upward-moving rock is smaller than the downward moving rock.

P2.48 (a) 29.4 m/s; (b) 44.1 m

P2.50 7.96 s

P2.52 0.60 s

P2.54 (a) $\frac{h}{t}+\frac{gt}{2}$; (b) $\frac{h}{t}-\frac{gt}{2}$

P2.56 (a) $(v_i + gt)$; (b) $\frac{1}{2}gt^2$; (c) $|v_i - gt|$; (d) $\frac{1}{2}gt^2$

P2.58 (a) See graphs in P2.58; (b) See graph in P2.58; (c) $-4\ \mathrm{m/s^2}$; (d) 32 m; (e) 28 m

P2.60 (a) $5.25\ \mathrm{m/s^2}$; (b) 168 m; (c) 52.5 m/s

P2.62 (a) 0; (b) $6.0\ \mathrm{m/s^2}$; (c) $-3.6\ \mathrm{m/s^2}$; (d) at $t = 6$ s and at 18 s; (e and f) $t = 18$ s; (g) 204 m

P2.64 (a) $A = v_{xi}t + \frac{1}{2}a_x t^2$; (b) The displacement is the same result for the total area.

P2.66 (a) 96.0 ft/s; (b) $3.07\times10^3\ \mathrm{ft/s^2}$ upward; (c) 3.13×10^{-2} s

P2.68 The trains do collide.

P2.70 (a) $+4.8\ \mathrm{m/s^2}$; (b) $7.27\ \mathrm{m/s^2}$

P2.72 (a) 41.0 s; (b) 1.73 km; (c) -184 m/s

P2.74 (a) Ball 1: $y_1 = h - v_0t - \frac{1}{2}gt^2$, Ball 2: $y_2 = h + v_0t - \frac{1}{2}gt^2$, $\frac{2v_0}{g}$; (b) Ball 1: $-\sqrt{v_0^2 + 2gh}$, Ball 2: $-\sqrt{v_0^2 + 2gh}$; (c) $2v_0t$

P2.76 (a and b) See TABLE P2.76; (c) $1.63\ \mathrm{m/s^2}$ downward and see graph in P2.76

P2.78 155 s

P2.80 $\sim10^3\ \mathrm{m/s^2}$

P2.82 (a) 3.45 s; (b) 10.0 ft.

P2.84 (a) The red bead falls through a greater distance with a downward acceleration of g. The blue bead travels a shorter distance, but with acceleration of $g\sin\theta$. A first guess would be that the blue bead "wins," but not by much. (b) $\sqrt{\frac{2D}{g}}$; (c) $\sqrt{\frac{2L}{g\sin\theta}}$; (d) the beads arrive at point Ⓒ simultaneously; (e) Once we recognize that the two rods form one side and the hypotenuse of a right triangle with θ as its smallest angle, then the result becomes obvious.

3

Vectors

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ3.1 Answer (e). The magnitude is $\sqrt{10^2+10^2}$ m/s.

OQ3.2 Answer (e). If the quantities x and y are positive, a vector with components $(-x, y)$ or $(x, -y)$ would lie in the second or fourth quadrant, respectively.

***OQ3.3** Answer (a). The vector $-2\vec{\mathbf{D}}_1$ will be twice as long as $\vec{\mathbf{D}}_1$ and in the opposite direction, namely northeast. Adding $\vec{\mathbf{D}}_2$, which is about equally long and southwest, we get a sum that is still longer and due east.

OQ3.4 The ranking is c = e > a > d > b. The magnitudes of the vectors being added are constant, and we are considering the magnitude only—not the direction—of the resultant. So we need look only at the angle between the vectors being added in each case. The smaller this angle, the larger the resultant magnitude.

OQ3.5 Answers (a), (b), and (c). The magnitude can range from the sum of the individual magnitudes, 8 + 6 = 14, to the difference of the individual magnitudes, 8 – 6 = 2. Because magnitude is the "length" of a vector, it is always positive.

OQ3.6 Answer (d). If we write vector $\vec{\mathbf{A}}$ as

$$\left(A_x, A_y\right) = \left(-|A_x|, |A_y|\right)$$

and vector $\vec{\mathbf{B}}$ as

$$\left(B_x, B_y\right) = \left(|B_x|, -|B_y|\right)$$

then

$$\vec{\mathbf{B}} - \vec{\mathbf{A}} = \left(|B_x| - \left(-|A_x|\right), -|B_y| - |A_y|\right) = \left(|B_x| + |A_x|, -|B_y| - |A_y|\right)$$

which would be in the fourth quadrant.

OQ3.7 The answers are (a) yes (b) no (c) no (d) no (e) no (f) yes (g) no. Only force and velocity are vectors. None of the other quantities requires a direction to be described.

OQ3.8 Answer (c). The vector has no y component given. It is therefore 0.

OQ3.9 Answer (d). Take the difference of the x coordinates of the ends of the vector, head minus tail: $-4 - 2 = -6$ cm.

OQ3.10 Answer (a). Take the difference of the y coordinates of the ends of the vector, head minus tail: $1 - (-2) = 3$ cm.

OQ3.11 Answer (c). The signs of the components of a vector are the same as the signs of the points in the quadrant into which it points. If a vector arrow is drawn to scale, the coordinates of the point of the arrow equal the components of the vector. All x and y values in the third quadrant are negative.

OQ3.12 Answer (c). The vertical component is opposite the 30° angle, so $\sin 30° = (\textit{vertical component})/50$ m.

OQ3.13 Answer (c). A vector in the second quadrant has a negative x component and a positive y component.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ3.1 Addition of a vector to a scalar is not defined. Try adding the speed and velocity, 8.0 m/s + $(15.0 \text{ m/s } \hat{\mathbf{i}})$: Should you consider the sum to be a vector or a scalar? What meaning would it have?

CQ3.2 No, the magnitude of a vector is always positive. A minus sign in a vector only indicates direction, not magnitude.

CQ3.3 (a) The book's displacement is zero, as it ends up at the point from which it started. (b) The distance traveled is 6.0 meters.

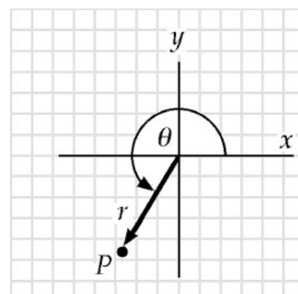
CQ3.4 Vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other.

CQ3.5 The inverse tangent function gives the correct angle, relative to the $+x$ axis, for vectors in the first or fourth quadrant, and it gives an incorrect answer for vectors in the second or third quadrant. If the x and y components are both positive, their ratio y/x is positive and the vector lies in the first quadrant; if the x component is positive and the y component negative, their ratio y/x is negative and the vector lies in the fourth quadrant. If the x and y components are both negative, their ratio y/x is positive but the vector lies in the third quadrant; if the x component is negative and the y component positive, their ratio y/x is negative but the vector lies in the second quadrant.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 3.1 Coordinate Systems

P3.1 ANS. FIG. P3.1 helps to visualize the x and y coordinates, and trigonometric functions will tell us the coordinates directly. When the polar coordinates (r, θ) of a point P are known, the Cartesian coordinates are found as



$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

Then,

$$x = r\cos\theta = (5.50\text{ m})\cos 240°$$
$$= (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$$
$$y = r\sin\theta = (5.50\text{ m})\sin 240°$$
$$= (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$$

P3.2 (a) We use $x = r\cos\theta$. Substituting, we have $2.00 = r\cos 30.0°$, so

$$r = \frac{2.00}{\cos 30.0°} = \boxed{2.31}$$

(b) From $y = r\sin\theta$, we have $y = r\sin 30.0° = 2.31\sin 30.0° = \boxed{1.15}$.

***P3.3** (a) The distance between the points is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2.00 - [-3.00])^2 + (-4.00 - 3.00)^2}$$

$$d = \sqrt{25.0 + 49.0} = \boxed{8.60 \text{ m}}$$

(b) To find the polar coordinates of each point, we measure the radial distance to that point and the angle it makes with the $+x$ axis:

$$r_1 = \sqrt{(2.00)^2 + (-4.00)^2} = \sqrt{20.0} = \boxed{4.47 \text{ m}}$$

$$\theta_1 = \tan^{-1}\left(-\frac{4.00}{2.00}\right) = \boxed{-63.4°}$$

$$r_2 = \sqrt{(-3.00)^2 + (3.00)^2} = \sqrt{18.0} = \boxed{4.24 \text{ m}}$$

$\theta_2 = \boxed{135°}$ measured from the $+x$ axis.

P3.4 (a) $x = r\cos\theta$ and $y = r\sin\theta$, therefore,

$x_1 = (2.50 \text{ m}) \cos 30.0°$, $y_1 = (2.50 \text{ m}) \sin 30.0°$, and

$(x_1, y_1) = \boxed{(2.17, 1.25) \text{ m}}$

$x_2 = (3.80 \text{ m}) \cos 120°$, $y_2 = (3.80 \text{ m}) \sin 120°$, and

$(x_2, y_2) = \boxed{(-1.90, 3.29) \text{ m}}$

(b) $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{4.07^2 + 2.04^2} \text{ m} = \boxed{4.55 \text{ m}}$

P3.5 For polar coordinates (r, θ), the Cartesian coordinates are $(x = r\cos\theta, y = r\sin\theta)$, if the angle is measured relative to the $+x$ axis.

(a) $\boxed{(-3.56 \text{ cm}, -2.40 \text{ cm})}$

(b) $(+3.56 \text{ cm}, -2.40 \text{ cm}) \to \boxed{(4.30 \text{ cm}, -34.0°)}$

(c) $(7.12 \text{ cm}, 4.80 \text{ cm}) \to \boxed{(8.60 \text{ cm}, 34.0°)}$

(d) $(-10.7 \text{ cm}, 7.21 \text{ cm}) \to \boxed{(12.9 \text{ cm}, 146°)}$

P3.6 We have $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

(a) The radius for this new point is

$$\sqrt{(-x)^2 + y^2} = \sqrt{x^2 + y^2} = \boxed{r}$$

and its angle is

$$\tan^{-1}\left(\frac{y}{-x}\right) = \boxed{180° - \theta}$$

(b) $\sqrt{(-2x)^2+(-2y)^2} = \boxed{2r}$. This point is in the third quadrant if (x, y) is in the first quadrant or in the fourth quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{180° + \theta}$.

(c) $\sqrt{(3x)^2+(-3y)^2} = \boxed{3r}$. This point is in the fourth quadrant if (x, y) is in the first quadrant or in the third quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{-\theta \text{ or } 360-\theta}$.

Section 3.2 Vector and Scalar Quantities

Section 3.3 Some Properties of Vectors

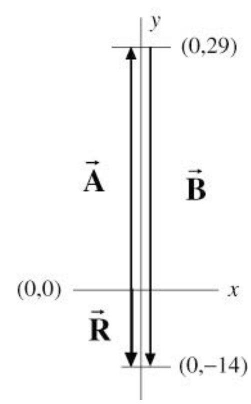
P3.7 Figure P3.7 suggests a right triangle where, relative to angle θ, its adjacent side has length d and its opposite side is equal to width of the river, y; thus,

$$\tan\theta = \frac{y}{d} \rightarrow y = d\tan\theta$$

$$y = (100\text{ m})\tan(35.0°) = 70.0\text{ m}$$

The width of the river is $\boxed{70.0\text{ m}}$.

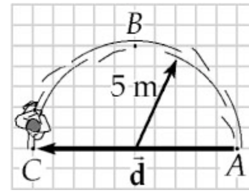
P3.8 We are given $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. When two vectors are added graphically, the second vector is positioned with its tail at the tip of the first vector. The resultant then runs from the tail of the first vector to the tip of the second vector. In this case, vector $\vec{\mathbf{A}}$ will be positioned with its tail at the origin and its tip at the point (0, 29). The resultant is then drawn, starting at the origin (tail of first vector) and going 14 units in the negative y direction to the point (0, −14). The second vector, $\vec{\mathbf{B}}$, must then start from the tip of $\vec{\mathbf{A}}$ at point (0, 29) and end on the tip of $\vec{\mathbf{R}}$ at point (0, −14) as shown in the sketch at the right. From this, it is seen that



ANS. FIG. P3.8

$\boxed{\vec{\mathbf{B}} \text{ is 43 units in the negative } y \text{ direction}}$

P3.9 In solving this problem we must contrast displacement with distance traveled. We draw a diagram of the skater's path in ANS. FIG. P3.9, which is the view from a hovering helicopter so that we can see the circular path as circular in shape. To start with a concrete example, we have chosen to draw motion *ABC* around one half of a circle of radius 5 m.



ANS. FIG. P3.9

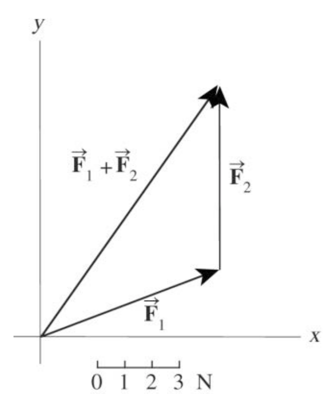
The displacement, shown as $\vec{\mathbf{d}}$ in the diagram, is the straight-line change in position from starting point *A* to finish *C*. In the specific case we have chosen to draw, it lies along a diameter of the circle. Its magnitude is $\left|\vec{\mathbf{d}}\right| = \left|-10.0\hat{\mathbf{i}}\right| = 10.0$ m.

The distance skated is greater than the straight-line displacement. The distance follows the curved path of the semicircle (*ABC*). Its length is half of the circumference: $s = \frac{1}{2}(2\pi r) = 5.00\pi \text{ m} = 15.7 \text{ m}$.

A straight line is the shortest distance between two points. For any nonzero displacement, less or more than across a semicircle, the distance along the path will be greater than the displacement magnitude. Therefore:

$\boxed{\text{The situation can never be true because the distance is an arc of a circle between two points, whereas the magnitude of the displacement vector is a straight-line cord of the circle between the same points.}}$

P3.10 We find the resultant $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ graphically by placing the tail of $\vec{\mathbf{F}}_2$ at the head of $\vec{\mathbf{F}}_1$. The resultant force vector $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ is of magnitude $\boxed{9.5 \text{ N}}$ and at an angle of $\boxed{57° \text{ above the } x \text{ axis}}$.



ANS. FIG. P3.10

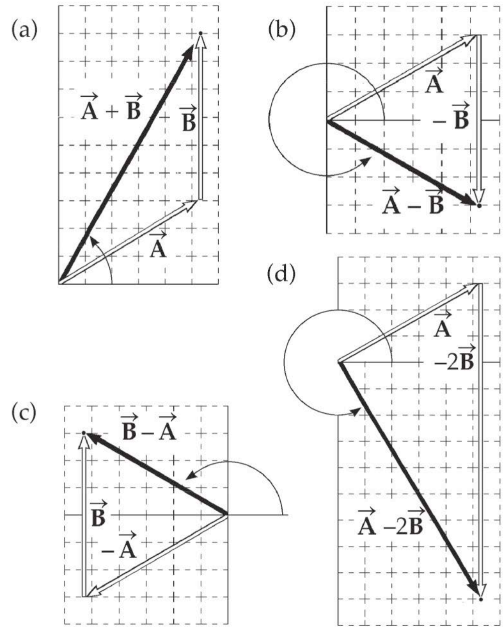
P3.11 To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor. (Scale: 1 unit = 0.5 m)

(a) $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \boxed{5.2 \text{ m at } 60^\circ}$

(b) $\vec{\mathbf{A}} - \vec{\mathbf{B}} = \boxed{3.0 \text{ m at } 330^\circ}$

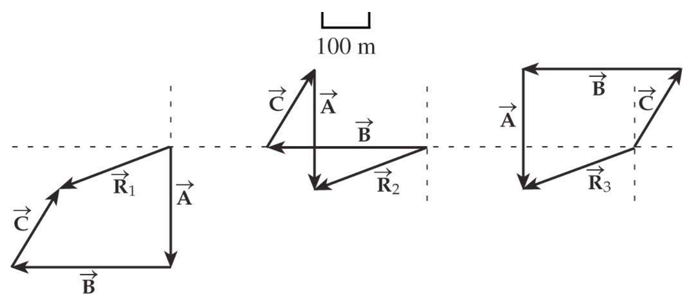
(c) $\vec{\mathbf{B}} - \vec{\mathbf{A}} = \boxed{3.0 \text{ m at } 150^\circ}$

(d) $\vec{\mathbf{A}} - 2\mathbf{B} = \boxed{5.2 \text{ m at } 300^\circ}$



ANS. FIG. P3.11

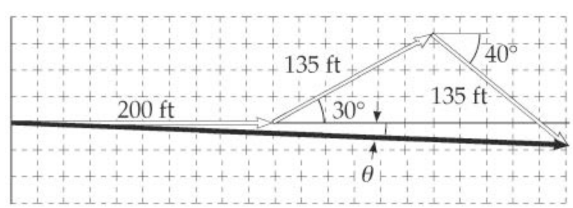
P3.12 (a) The three diagrams are shown in ANS. FIG. P3.12a below.



ANS. FIG. P3.12a

(b) The diagrams in ANS. FIG. P3.12a represent the graphical solutions for the three vector sums: $\vec{\mathbf{R}}_1 = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$, $\vec{\mathbf{R}}_2 = \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{A}}$, and $\vec{\mathbf{R}}_3 = \vec{\mathbf{C}} + \vec{\mathbf{B}} + \vec{\mathbf{A}}$.

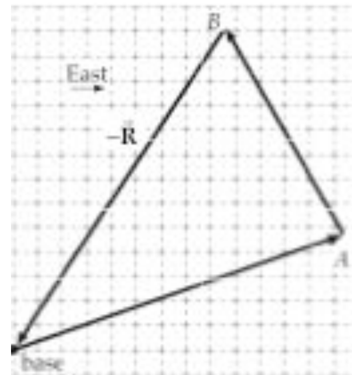
P3.13 The scale drawing for the graphical solution should be similar to the figure to the right. The magnitude and direction of the final displacement from the starting point are obtained by measuring d and θ on the drawing and applying the scale factor used in making the drawing. The results should be $\boxed{d = 420 \text{ ft and } \theta = -3^\circ}$.



(Scale: 1unit = 20 ft)

ANS. FIG. P3.13

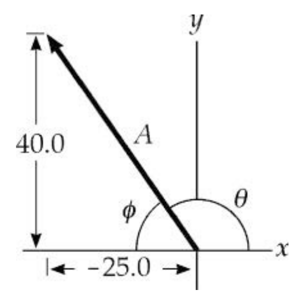
***P3.14** ANS. FIG. P3.14 shows the graphical addition of the vector from the base camp to lake A to the vector connecting lakes A and B, with a scale of 1 unit = 20 km. The distance from lake B to base camp is then the negative of this resultant vector, or $-\vec{\mathbf{R}} = \boxed{310 \text{ km at } 57° \text{ S of W}}$.



ANS. FIG. P3.14

Section 3.4 Components of a Vector and Unit Vectors

P3.15 First we should visualize the vector either in our mind or with a sketch, as shown in ANS. FIG. P3.15. The magnitude of the vector can be found by the Pythagorean theorem:



ANS. FIG. P3.15

$$A_x = -25.0$$

$$A_y = 40.0$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2}$$

$$= \boxed{47.2 \text{ units}}$$

We observe that

$$\tan\phi = \frac{|A_y|}{|A_x|}$$

so

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan^{-1}\left(\frac{40.0}{25.0}\right) = \tan^{-1}(1.60) = 58.0°$$

The diagram shows that the angle from the $+x$ axis can be found by subtracting from 180°: $\theta = 180° - 58° = \boxed{122°}$

P3.16 We can calculate the components of the vector A using $(A_x, A_y) = (A\cos\theta, A\sin\theta)$ if the angle θ is measured from the $+x$ axis, which is true here. For $A = 35.0$ units and $\theta = 325°$,

$$\boxed{A_x = 28.7 \text{ units}, \; A_y = -20.1 \text{ units}}$$

P3.17 (a) $\boxed{\text{Yes}}$.

(b) Let v represent the speed of the camper. The northward component of its velocity is $v \cos 8.50°$. To avoid crowding the minivan we require $v \cos 8.50° \geq 28$ m/s.

$$\boxed{\begin{array}{l}\text{We can satisfy this requirement simply by taking}\\ v \geq (28.0 \text{ m/s})/\cos 8.50° = 28.3 \text{ m/s.}\end{array}}$$

P3.18 The person would have to walk

$$(3.10 \text{ km})\sin 25.0° = \boxed{1.31 \text{ km north}}$$

and $$(3.10 \text{ km})\cos 25.0° = \boxed{2.81 \text{ km east}}$$

P3.19 Do not think of $\sin\theta$ = opposite/hypotenuse, but jump right to $y = R\sin\theta$. The angle does not need to fit inside a triangle. We find the x and y components of each vector using $x = r\cos\theta$ and $y = r\sin\theta$. In unit vector notation, $\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$.

(a) $x = 12.8 \cos 150°$, $y = 12.8 \sin 150°$, and $\boxed{(x, y) = (-11.1\hat{\mathbf{i}} + 6.40\hat{\mathbf{j}}) \text{ m}}$

(b) $x = 3.30 \cos 60.0°$, $y = 3.30 \sin 60.0°$, and $\boxed{(x, y) = (1.65\hat{\mathbf{i}} + 2.86\hat{\mathbf{j}}) \text{ cm}}$

(c) $x = 22.0 \cos 215°$, $y = 22.0 \sin 215°$, and $\boxed{(x, y) = (-18.0\hat{\mathbf{i}} - 12.6\hat{\mathbf{j}}) \text{ in}}$

P3.20 (a) Her net x (east-west) displacement is $-3.00 + 0 + 6.00 = +3.00$ blocks, while her net y (north-south) displacement is $0 + 4.00 + 0 = +4.00$ blocks. The magnitude of the resultant displacement is

$$R = \sqrt{(x_{\text{net}})^2 + (y_{\text{net}})^2} = \sqrt{(3.00)^2 + (4.00)^2} = 5.00 \text{ blocks}$$

and the angle the resultant makes with the x axis (eastward direction) is

$$\theta = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}(1.33) = 53.1°.$$

The resultant displacement is then $\boxed{5.00 \text{ blocks at } 53.1° \text{ N of E}}$.

(b) The total distance traveled is $3.00 + 4.00 + 6.00 = \boxed{13.00 \text{ blocks}}$.

P3.21 Let $+x$ be East and $+y$ be North. We can sum the total x and y displacements of the spelunker as

$$\sum x = 250 \text{ m} + (125 \text{ m})\cos 30° = 358 \text{ m}$$
$$\sum y = 75 \text{ m} + (125 \text{ m})\sin 30° - 150 \text{ m} = -12.5 \text{ m}$$

the total displacement is then

$$d = \sqrt{\left(\sum x\right)^2 + \left(\sum y\right)^2} = \sqrt{(358\text{ m})^2 + (-12.5\text{ m})^2} = 358\text{ m}$$

at an angle of

$$\theta = \tan^{-1}\left(\frac{\sum y}{\sum x}\right) = \tan^{-1}\left(-\frac{12.5\text{ m}}{358\text{ m}}\right) = -2.00°$$

or $\boxed{\vec{\mathbf{d}} = 358\text{ m at }2.00°\text{ S of E}}$

P3.22 We use the numbers given in Problem 3.11:

$$\vec{\mathbf{A}} = 3.00\text{ m},\ \theta_A = 30.0°$$

$$A_x = A\cos\theta_A = 3.00\cos 30.0° = 2.60\text{ m},$$

$$A_y = A\sin\theta_A = 3.00\sin 30.0° = 1.50\text{ m}$$

So $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = (2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}})\text{ m}$

$$\vec{\mathbf{B}} = 3.00\text{ m},\ \theta_B = 90.0°$$

$$B_x = 0,\ B_y = 3.00\text{ m} \rightarrow \vec{\mathbf{B}} = 3.00\hat{\mathbf{j}}\text{ m}$$

then $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right) + 3.00\hat{\mathbf{j}} = \boxed{\left(2.60\hat{\mathbf{i}} + 4.50\hat{\mathbf{j}}\right)\text{m}}$

P3.23 We can get answers in unit-vector form just by doing calculations with each term labeled with an $\hat{\mathbf{i}}$ or a $\hat{\mathbf{j}}$. There are, in a sense, only two vectors to calculate, since parts (c), (d), and (e) just ask about the magnitudes and directions of the answers to (a) and (b). Note that the whole numbers appearing in the problem statement are assumed to have three significant figures.

We use the property of vector addition that states that the components of $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ are computed as $R_x = A_x + B_x$ and $R_y = A_y + B_y$.

(a) $\left(\vec{\mathbf{A}} + \vec{\mathbf{B}}\right) = \left(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\right) + \left(-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\right) = \boxed{2\hat{\mathbf{i}} - 6\hat{\mathbf{j}}}$

(b) $\left(\vec{\mathbf{A}} - \vec{\mathbf{B}}\right) = \left(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\right) - \left(-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\right) = \boxed{4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}}$

(c) $\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \sqrt{2^2 + 6^2} = \boxed{6.32}$

(d) $\left|\vec{\mathbf{A}} - \vec{\mathbf{B}}\right| = \sqrt{4^2 + 2^2} = \boxed{4.47}$

(e) $\theta_{|\mathbf{A}+\mathbf{B}|} = \tan^{-1}\left(-\dfrac{6}{2}\right) = -71.6° = \boxed{288°}$

$\theta_{|\mathbf{A}-\mathbf{B}|} = \tan^{-1}\left(\dfrac{2}{4}\right) = \boxed{26.6°}$

P3.24 The east and north components of the displacement from Dallas (D) to Chicago (C) are the sums of the east and north components of the displacements from Dallas to Atlanta (A) and from Atlanta to Chicago. In equation form:

$$\begin{aligned} d_{\text{DC east}} &= d_{\text{DA east}} + d_{\text{AC east}} \\ &= (730 \text{ mi})\cos 5.00° - (560 \text{ mi})\sin 21.0° = 527 \text{ miles} \end{aligned}$$

$$\begin{aligned} d_{\text{DC north}} &= d_{\text{DA north}} + d_{\text{AC north}} \\ &= (730 \text{ mi})\sin 5.00° + (560 \text{ mi})\cos 21.0° = 586 \text{ miles} \end{aligned}$$

By the Pythagorean theorem,

$$d = \sqrt{(d_{\text{DC east}})^2 + (d_{\text{DC north}})^2} = 788 \text{ mi}$$

Then, $\theta = \tan^{-1}\left(\dfrac{d_{\text{DC north}}}{d_{\text{DC east}}}\right) = 48.0°$

Thus, Chicago is $\boxed{\text{788 miles at } 48.0° \text{ northeast of Dallas}}$.

P3.25 We use the unit-vector addition method. It is just as easy to add three displacements as to add two. We take the direction east to be along $+\hat{\mathbf{i}}$. The three displacements can be written as:

$$\begin{aligned} \vec{\mathbf{d}}_1 &= (-3.50 \text{ m})\hat{\mathbf{j}} \\ \vec{\mathbf{d}}_2 &= (8.20 \text{ m})\cos 45.0°\hat{\mathbf{i}} + (8.20 \text{ m})\sin 45.0°\hat{\mathbf{j}} \\ &= (5.80 \text{ m})\hat{\mathbf{i}} + (5.80 \text{ m})\hat{\mathbf{j}} \\ \vec{\mathbf{d}}_3 &= (-15.0 \text{ m})\hat{\mathbf{i}} \end{aligned}$$

The resultant is

$$\begin{aligned} \vec{\mathbf{R}} &= \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 = (-15.0 \text{ m} + 5.80 \text{ m})\hat{\mathbf{i}} + (5.80 \text{ m} - 3.50 \text{ m})\hat{\mathbf{j}} \\ &= (-9.20 \text{ m})\hat{\mathbf{i}} + (2.30 \text{ m})\hat{\mathbf{j}} \end{aligned}$$

(or 9.20 m west and 2.30 m north).

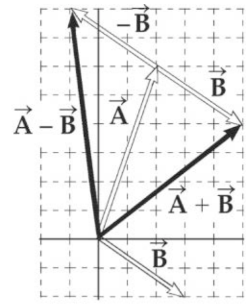
The magnitude of the resultant displacement is

$$|\vec{\mathbf{R}}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(-9.20 \text{ m})^2 + (2.30 \text{ m})^2} = \boxed{9.48 \text{ m}}$$

The direction of the resultant vector is given by

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{2.30\text{ m}}{-9.20\text{ m}}\right) = \boxed{166°}$$

P3.26 (a) See figure to the right.



ANS. FIG. P3.26

(b) $$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$$
$$= \boxed{5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}}$$
$$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}}$$
$$= \boxed{-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}}$$

(c) $$\vec{\mathbf{C}} = \sqrt{25.0 + 16.0} \text{ at } \tan^{-1}\left(\frac{4}{5}\right) = \boxed{6.40 \text{ at } 38.7°}$$
$$\vec{\mathbf{D}} = \sqrt{(-1.00)^2 + (8.00)^2} \text{ at } \tan^{-1}\left(\frac{8.00}{-1.00}\right)$$
$$\vec{\mathbf{D}} = 8.06 \text{ at } (180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$$

P3.27 We first tabulate the three strokes of the novice golfer, with the x direction corresponding to East and the y direction corresponding to North. The sum of the displacement in each of the directions is shown as the last row of the table.

East	North
x (m)	y (m)
0	4.00
1.41	1.41
–0.500	–0.866
+0.914	4.55

The "hole-in-one" single displacement is then

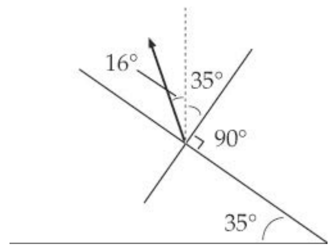
$$|\vec{\mathbf{R}}| = \sqrt{|x|^2 + |y|^2} = \sqrt{(0.914\text{ m})^2 + (4.55\text{ m})^2} = 4.64\text{ m}$$

The angle of the displacement with the horizontal is

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{4.55\text{ m}}{0.914\text{ m}}\right) = 78.6°$$

The expert golfer would accomplish the hole in one with the displacement $\boxed{4.64 \text{ m at } 78.6° \text{ N of E}}$.

P3.28 We take the x axis along the slope downhill. (Students, get used to this choice!) The y axis is perpendicular to the slope, at 35.0° to the vertical. Then the displacement of the snow makes an angle of 90.0° + 35.0° + 16.0° = 141° with the x axis.



ANS. FIG. P3.28

(a) $\boxed{\begin{array}{l}\text{Its component parallel to the surface is} \\ (1.50 \text{ m}) \cos 141° = -1.17 \text{ m}, \\ \text{or 1.17 m toward the top of the hill.}\end{array}}$

(b) $\boxed{\begin{array}{l}\text{Its component perpendicular to the surface is} \\ (1.50 \text{ m}) \sin 141° = 0.944 \text{ m, or 0.944 m away} \\ \text{from the snow.}\end{array}}$

P3.29 (a) The single force is obtained by summing the two forces:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$$

$$\vec{\mathbf{F}} = 120 \cos(60.0°)\hat{\mathbf{i}} + 120 \sin(60.0°)\hat{\mathbf{j}}$$
$$- 80.0 \cos(75.0°)\hat{\mathbf{i}} + 80.0 \sin(75.0°)\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}} = 60.0\hat{\mathbf{i}} + 104\hat{\mathbf{j}} - 20.7\hat{\mathbf{i}} + 77.3\hat{\mathbf{j}} = \left(39.3\hat{\mathbf{i}} + 181\hat{\mathbf{j}}\right) \text{ N}$$

We can also express this force in terms of its magnitude and direction:

$$\left|\vec{\mathbf{F}}\right| = \sqrt{39.3^2 + 181^2} \text{ N} = \boxed{185 \text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{181}{39.3}\right) = \boxed{77.8°}$$

(b) A force equal and opposite the resultant force from part (a) is required for the total force to equal zero:

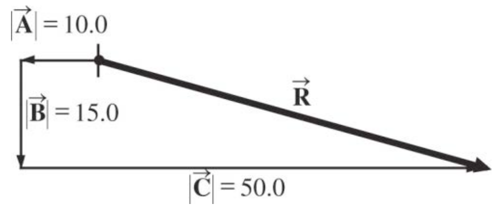
$$\vec{\mathbf{F}}_3 = -\vec{\mathbf{F}} = \boxed{\left(-39.3\hat{\mathbf{i}} - 181\hat{\mathbf{j}}\right) \text{ N}}$$

P3.30 ANS. FIG. P3.30 is a graphical depiction of the three displacements the football undergoes, with $\vec{\mathbf{A}}$ corresponding to the 10.0-yard backward run, $\vec{\mathbf{B}}$ corresponding to the 15.0-yard sideways run, and $\vec{\mathbf{C}}$ corresponding to the 50.0-yard downfield pass. The resultant vector is then

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = -10.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}} + 50.0\hat{\mathbf{i}}$$

$$= 40.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}$$

$$\left|\vec{\mathbf{R}}\right| = \left[(40.0)^2 + (-15.0)^2\right]^{1/2} = \boxed{42.7 \text{ yards}}$$



ANS. FIG. P3.30

P3.31 (a) We add the components of the three vectors:

$$\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = 6\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$$

$$\left|\vec{\mathbf{D}}\right| = \sqrt{6^2 + 2^2} = \boxed{6.32 \text{ m at } \theta = 342°}$$

(b) Again, using the components of the three vectors,

$$\vec{\mathbf{E}} = -\vec{\mathbf{A}} - \vec{\mathbf{B}} + \vec{\mathbf{C}} = -2\hat{\mathbf{i}} + 12\hat{\mathbf{j}}$$

$$\left|\vec{\mathbf{E}}\right| = \sqrt{2^2 + 12^2} = \boxed{12.2 \text{ m at } \theta = 99.5°}$$

P3.32 We are given $\vec{\mathbf{A}} = -8.70\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}$, and $\vec{\mathbf{B}} = 13.2\hat{\mathbf{i}} - 6.60\hat{\mathbf{j}}$, and $\vec{\mathbf{A}} - \vec{\mathbf{B}} + 3\vec{\mathbf{C}} = 0$. Solving for $\vec{\mathbf{C}}$ gives

$$3\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}} = 21.9\hat{\mathbf{i}} - 21.6\hat{\mathbf{j}}$$

$$\vec{\mathbf{C}} = 7.30\hat{\mathbf{i}} - 7.20\hat{\mathbf{j}} \text{ or } \quad C_x = \boxed{7.30 \text{ cm}}; C_y = \boxed{-7.20 \text{ cm}}$$

P3.33 Hold your fingertip at the center of the front edge of your study desk, defined as point *O*. Move your finger 8 cm to the right, then 12 cm vertically up, and then 4 cm horizontally away from you. Its location relative to the starting point represents position vector $\vec{\mathbf{A}}$. Move three-fourths of the way straight back toward *O*. Now your fingertip is at the location of $\vec{\mathbf{B}}$. Now move your finger 50 cm straight through *O*, through your left thigh, and down toward the floor. Its position vector now is $\vec{\mathbf{C}}$.

We use unit-vector notation throughout. There is no adding to do here, but just multiplication of a vector by two different scalars.

(a) $\vec{\mathbf{A}} = \boxed{8.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}}$

(b) $\vec{\mathbf{B}} = \dfrac{\vec{\mathbf{A}}}{4} = \boxed{2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{k}}}$

(c) $\vec{\mathbf{C}} = -3\vec{\mathbf{A}} = \boxed{-24.0\hat{\mathbf{i}} - 36.0\hat{\mathbf{j}} + 12.0\hat{\mathbf{k}}}$

P3.34 We are given $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} = 4.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}}$. The magnitude of the vector is therefore

$$\left|\vec{\mathbf{B}}\right| = \sqrt{4.00^2 + 6.00^2 + 3.00^2} = \boxed{7.81}$$

And the angle of the vector with the three coordinate axes is

$$\alpha = \cos^{-1}\left(\frac{4.00}{7.81}\right) = \boxed{59.2°} \text{ is the angle with the x axis}$$

$$\beta = \cos^{-1}\left(\frac{6.00}{7.81}\right) = \boxed{39.8°} \text{ is the angle with the y axis}$$

$$\gamma = \cos^{-1}\left(\frac{3.00}{7.81}\right) = \boxed{67.4°} \text{ is the angle with the z axis}$$

P3.35 The component description of $\vec{\mathbf{A}}$ is just restated to constitute the answer to part (a): $A_x = -3.00$, $A_y = 2.00$.

(a) $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = \boxed{-3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}}}$

(b) $\left|\vec{\mathbf{A}}\right| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3.00)^2 + (2.00)^2} = \boxed{3.61}$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{2.00}{-3.00}\right) = -33.7°$$

θ is in the second quadrant, so $\theta = 180° + (-33.7°) = \boxed{146°}$.

(c) $R_x = 0$, $R_y = -4.00$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, thus $\vec{\mathbf{B}} = \vec{\mathbf{R}} - \vec{\mathbf{A}}$ and

$B_x = R_x - A_x = 0 - (-3.00) = 3.00$, $B_y = R_y - A_y = -4.00 - 2.00 = -6.00$.

Therefore, $\vec{\mathbf{B}} = \boxed{3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}}$.

P3.36 We carry out the prescribed mathematical operations using unit vectors.

(a) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \boxed{\left(5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right)\text{ m}}$

$$\left|\vec{\mathbf{C}}\right| = \sqrt{(5.00\text{ m})^2 + (1.00\text{ m})^2 + (3.00\text{ m})^2} = \boxed{5.92\text{ m}}$$

(b) $\vec{\mathbf{D}} = 2\vec{\mathbf{A}} - \vec{\mathbf{B}} = \boxed{\left(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}}\right)\text{m}}$

$$\left|\vec{\mathbf{D}}\right| = \sqrt{(4.00\text{ m})^2 + (11.0\text{ m})^2 + (15.0\text{ m})^2} = \boxed{19.0\text{ m}}$$

P3.37 (a) Taking components along $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get two equations:

$$6.00a - 8.00b + 26.0 = 0$$

and

$$-8.00a + 3.00b + 19.0 = 0$$

Substituting $a = 1.33b - 4.33$ into the second equation, we find

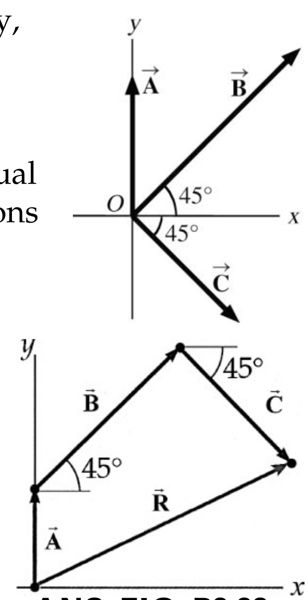
$$-8(1.33b - 4.33) + 3b + 19 = 0 \rightarrow 7.67b = 53.67 \rightarrow b = 7.00$$

and so $a = 1.33(7.00) - 4.33 = 5.00$.

Thus $\boxed{a = 5.00,\ b = 7.00}$. Therefore, $5.00\vec{\mathbf{A}} + 7.00\vec{\mathbf{B}} + \vec{\mathbf{C}} = 0$.

(b) In order for vectors to be equal, all of their components must be equal. A vector equation contains more information than a scalar equation, as each component gives us one equation.

P3.38 The given diagram shows the vectors individually, but not their addition. The second diagram represents a map view of the motion of the ball. According to the definition of a displacement, we ignore any departure from straightness of the actual path of the ball. We model each of the three motions as straight. The simplified problem is solved by straightforward application of the component method of vector addition. It works for adding two, three, or any number of vectors.



ANS. FIG. P3.38

(a) We find the two components of each of the three vectors

$$A_x = (20.0\text{ units})\cos 90° = 0$$

and $A_y = (20.0\text{ units})\sin 90° = 20.0\text{ units}$

$$B_x = (40.0\text{ units})\cos 45° = 28.3 \text{ units}$$

and $B_y = (40.0\text{ units})\sin 45° = 28.3 \text{ units}$

$$C_x = (30.0\text{ units})\cos 315° = 21.2 \text{ units}$$

and $C_y = (30.0\text{ units})\sin 315° = -21.2 \text{ units}$

Now adding,

$$R_x = A_x + B_x + C_x = (0 + 28.3 + 21.2) \text{ units} = 49.5 \text{ units}$$

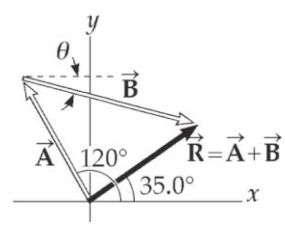
and $R_y = A_y + B_y + C_y = (20 + 28.3 - 21.2) \text{ units} = 27.1 \text{ units}$

so $\vec{\mathbf{R}} = \boxed{49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}}$

(b) $\left|\vec{\mathbf{R}}\right| = \sqrt{(49.5)^2 + (27.1)^2} = \boxed{56.4}$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{27.1}{49.5}\right) = \boxed{28.7°}$$

P3.39 We will use the component method for a precise answer. We already know the total displacement, so the algebra of solving a vector equation will guide us to do a subtraction.



ANS. FIG. P3.39

We have $\vec{\mathbf{B}} = \vec{\mathbf{R}} - \vec{\mathbf{A}}$:

$$A_x = 150\cos 120° = -75.0 \text{ cm}$$

$$A_y = 150\sin 120° = 130 \text{ cm}$$

$$R_x = 140\cos 35.0° = 115 \text{ cm}$$

$$R_y = 140\sin 35.0° = 80.3 \text{ cm}$$

Therefore,

$$\vec{\mathbf{B}} = [115 - (-75)\hat{\mathbf{i}} + [80.3 - 130]\hat{\mathbf{j}} = \left(190\hat{\mathbf{i}} - 49.7\hat{\mathbf{j}}\right) \text{ cm}$$

$$\left|\vec{\mathbf{B}}\right| = \sqrt{190^2 + 49.7^2} = \boxed{196 \text{ cm}}$$

$$\theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7°}$$

P3.40 First, we sum the components of the two vectors for the male:

$$d_{3mx} = d_{1mx} + d_{2mx} = 0 + (100 \text{ cm})\cos 23.0° = 92.1 \text{ cm}$$
$$d_{3my} = d_{1my} + d_{2my} = 104 \text{ cm} + (100 \text{ cm})\sin 23.0° = 143.1 \text{ cm}$$

$$\boxed{\begin{array}{l}\text{magnitude: } d_{3m} = \sqrt{(92.1 \text{ cm})^2 + (143.1 \text{ cm})^2} = 170.1 \text{ cm} \\ \text{direction: } \tan^{-1}(143.1/92.1) = 57.2° \text{ above } +x \text{ axis (first quadrant)}\end{array}}$$

followed by the components of the two vectors for the female:

$$d_{3fx} = d_{1fx} + d_{2fx} = 0 + (86.0 \text{ cm})\cos 28.0° = 75.9 \text{ cm}$$
$$d_{3fy} = d_{1fy} + d_{2fy} = 84.0 \text{ cm} + (86.0 \text{ cm})\sin 28.0° = 124.4 \text{ cm}$$

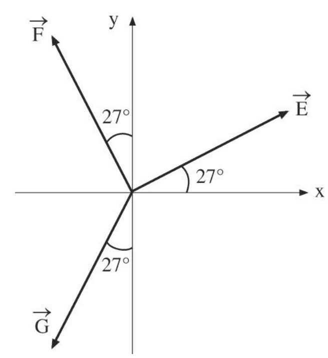
$$\boxed{\begin{array}{l}\text{magnitude: } d_{3f} = \sqrt{(75.9 \text{ cm})^2 + (124.4 \text{ cm})^2} = 145.7 \text{ cm} \\ \text{direction: } \tan^{-1}(124.4/75.9) = 58.6° \text{ above } +x \text{ axis (first quadrant)}\end{array}}$$

P3.41 (a) $\vec{\mathbf{E}} = (17.0 \text{ cm})\cos(27.0°)\hat{\mathbf{i}} + (17.0 \text{ cm})\sin(27.0°)\hat{\mathbf{j}}$

$$\vec{\mathbf{E}} = \boxed{(15.1\hat{\mathbf{i}} + 7.72\hat{\mathbf{j}}) \text{ cm}}$$

(b) $\vec{\mathbf{F}} = (17.0 \text{ cm})\cos(117.0°)\hat{\mathbf{i}} + (17.0 \text{ cm})\sin(117.0°)\hat{\mathbf{j}}$

$$\vec{\mathbf{F}} = \boxed{\left(-7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right) \text{ cm}}$$



ANS. FIG. P3.41

Note that we did not need to explicitly identify the angle with the positive x axis, but by doing so, we don't have to keep track of minus signs for the components.

(c) $\vec{\mathbf{G}} = \left[(-17.0 \text{ cm})\cos(243.0°)\right]\hat{\mathbf{i}} + \left[(-17.0 \text{ cm})\sin(243.0°)\right]\hat{\mathbf{j}}$

$$\boxed{\vec{\mathbf{G}} = \left(-7.72\hat{\mathbf{i}} - 15.1\hat{\mathbf{j}}\right) \text{ cm}}$$

P3.42 The position vector from radar station to ship is

$$\vec{\mathbf{S}} = \left(17.3\sin 136°\hat{\mathbf{i}} + 17.3\cos 136°\hat{\mathbf{j}}\right) \text{ km} = \left(12.0\hat{\mathbf{i}} - 12.4\hat{\mathbf{j}}\right) \text{ km}$$

From station to plane, the position vector is

$$\vec{\mathbf{P}} = \left(19.6\sin 153°\hat{\mathbf{i}} + 19.6\cos 153°\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right) \text{ km}$$

or

$$\vec{\mathbf{P}} = \left(8.90\hat{\mathbf{i}} - 17.5\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right) \text{ km}$$

(a) To fly to the ship, the plane must undergo displacement

$$\vec{\mathbf{D}} = \vec{\mathbf{S}} - \vec{\mathbf{P}} = \boxed{\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right)\text{ km}}$$

(b) The distance the plane must travel is

$$D = \left|\vec{\mathbf{D}}\right| = \sqrt{(3.12)^2 + (5.02)^2 + (2.20)^2}\text{ km} = \boxed{6.31\text{ km}}$$

P3.43 The hurricane's first displacement is

$$(41.0\text{ km/h})(3.00\text{ h})\text{ at }60.0°\text{ N of W}$$

and its second displacement is

$$(25.0\text{ km/h})(1.50\text{ h})\text{ due North}$$

With $\hat{\mathbf{i}}$ representing east and $\hat{\mathbf{j}}$ representing north, its total displacement is:

$$\begin{aligned}&\left[(41.0\text{ km/h})\cos 60.0°\right](3.00\text{ h})\left(-\hat{\mathbf{i}}\right)\\ &\qquad + \left[(41.0\text{ km/h})\sin 60.0°\right](3.00\text{ h})\hat{\mathbf{j}}\\ &\qquad + (25.0\text{ km/h})(1.50\text{ h})\hat{\mathbf{j}}\\ &= 61.5\text{ km}\left(-\hat{\mathbf{i}}\right) + 144\text{ km }\hat{\mathbf{j}}\end{aligned}$$

with magnitude $\sqrt{(61.5\text{ km})^2 + (144\text{ km})^2} = \boxed{157\text{ km}}$.

P3.44 Note that each shopper must make a choice whether to turn 90° to the left or right, each time he or she makes a turn. One set of such choices, following the rules in the problem, results in the shopper heading in the positive y direction and then again in the positive x direction.

Find the magnitude of the sum of the displacements:

$$\vec{\mathbf{d}} = (8.00\text{ m})\hat{\mathbf{i}} + (3.00\text{ m})\hat{\mathbf{j}} + (4.00\text{ m})\hat{\mathbf{i}} = (12.00\text{ m})\hat{\mathbf{i}} + (3.00\text{ m})\hat{\mathbf{j}}$$

$$\text{magnitude: } d = \sqrt{(12.00\text{ m})^2 + (3.00\text{ m})^2} = 12.4\text{ m}$$

$$\boxed{\text{Impossible because 12.4 m is greater than 5.00 m.}}$$

P3.45 The y coordinate of the airplane is constant and equal to 7.60×10^3 m whereas the x coordinate is given by $x = v_i t$, where v_i is the constant speed in the horizontal direction.

At $t = 30.0$ s we have $x = 8.04 \times 10^3$, so $v_i = 8\,040\text{ m}/30\text{ s} = 268\text{ m/s}$. The position vector as a function of time is

$$\vec{\mathbf{P}} = (268\text{ m/s})t\hat{\mathbf{i}} + (7.60 \times 10^3\text{ m})\hat{\mathbf{j}}$$

At $t = 45.0\text{ s}$, $\vec{\mathbf{P}} = \left[1.21 \times 10^4\,\hat{\mathbf{i}} + 7.60 \times 10^3\,\hat{\mathbf{j}}\right]$ m. The magnitude is

$$\vec{\mathbf{P}} = \sqrt{\left(1.21 \times 10^4\right)^2 + \left(7.60 \times 10^3\right)^2}\ \text{m} = \boxed{1.43 \times 10^4\ \text{m}}$$

and the direction is

$$\theta = \tan^{-1}\left(\frac{7.60 \times 10^3}{1.21 \times 10^4}\right) = \boxed{32.2°\ \text{above the horizontal}}$$

P3.46 The displacement from the start to the finish is

$$16\hat{\mathbf{i}} + 12\hat{\mathbf{j}} - (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) = (11\hat{\mathbf{i}} + 9\hat{\mathbf{j}})$$

The displacement from the starting point to *A* is $f\left(11\hat{\mathbf{i}} + 9\hat{\mathbf{j}}\right)$ meters.

(a) The position vector of point *A* is

$$5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + f\left(11\hat{\mathbf{i}} + 9\hat{\mathbf{j}}\right) = \boxed{\left[(5 + 11f)\hat{\mathbf{i}} + (3 + 9f)\hat{\mathbf{j}}\right]\ \text{m}}$$

(b) For $f = 0$ we have the position vector $\boxed{(5 + 0)\hat{\mathbf{i}} + (3 + 0)\hat{\mathbf{j}}\ \text{meters.}}$

(c) This is reasonable because it is the location of the starting point, $5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ meters.

(d) For $f = 1 = 100\%$, we have position vector
$(5 + 11)\hat{\mathbf{i}} + (3 + 9)\hat{\mathbf{j}}$ meters $= \boxed{16\hat{\mathbf{i}} + 12\hat{\mathbf{j}}\ \text{meters}}$.

(e) This is reasonable because we have completed the trip, and this is the position vector of the endpoint.

P3.47 Let the positive *x* direction be eastward, the positive *y* direction be vertically upward, and the positive *z* direction be southward. The total displacement is then

$$\vec{\mathbf{d}} = \left(4.80\hat{\mathbf{i}} + 4.80\hat{\mathbf{j}}\right)\text{ cm} + \left(3.70\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\text{ cm}$$
$$= \left(4.80\hat{\mathbf{i}} + 8.50\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\text{ cm}$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2}\ \text{cm} = \boxed{10.4\ \text{cm}}$.

(b) Its angle with the *y* axis follows from
$\cos\theta = \dfrac{8.50}{10.4}$, giving $\boxed{\theta = 35.5°}$.

Additional Problems

P3.48 The Pythagorean theorem and the definition of the tangent will be the starting points for our calculation.

(a) Take the wall as the xy plane so that the coordinates are $x = 2.00$ m and $y = 1.00$ m; and the fly is located at point P. The distance between two points in the xy plane is

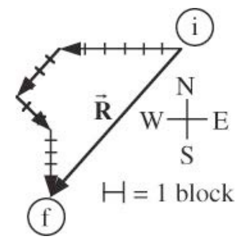
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

so here $d = \sqrt{(2.00\ \text{m} - 0)^2 + (1.00\ \text{m} - 0)^2} = \boxed{2.24\ \text{m}}$

(b) $\theta = \tan^{-1}\left(\dfrac{y}{x}\right) = \tan^{-1}\left(\dfrac{1.00\ \text{m}}{2.00\ \text{m}}\right) = 26.6°$, so $\vec{\mathbf{r}} = \boxed{2.24\ \text{m},\ 26.6°}$

P3.49 We note that $-\hat{\mathbf{i}}$ = west and $-\hat{\mathbf{j}}$ = south. The given mathematical representation of the trip can be written as 6.30 b west + 4.00 b at 40° south of west +3.00 b at 50° south of east +5.00 b south.

(a) The figure on the right shows a map of the successive displacements that the bus undergoes.



ANS. FIG. P3.49

(b) The total odometer distance is the sum of the magnitudes of the four displacements:

$$6.30\ \text{b} + 4.00\ \text{b} + 3.00\ \text{b} + 5.00\ \text{b} = \boxed{18.3\ \text{b}}$$

(c)
$$\begin{aligned}\vec{\mathbf{R}} &= (-6.30 - 3.06 + 1.93)\ \text{b}\,\hat{\mathbf{i}} + (-2.57 - 2.30 - 5.00)\ \text{b}\,\hat{\mathbf{j}} \\ &= -7.44\ \text{b}\,\hat{\mathbf{i}} - 9.87\ \text{b}\,\hat{\mathbf{j}} \\ &= \sqrt{(7.44\ \text{b})^2 + (9.87\ \text{b})^2}\ \text{at}\ \tan^{-1}\left(\frac{9.87}{7.44}\right)\ \text{south of west} \\ &= 12.4\ \text{b at } 53.0°\ \text{south of west} \\ &= \boxed{12.4\ \text{b at } 233°\ \text{counterclockwise from east}}\end{aligned}$$

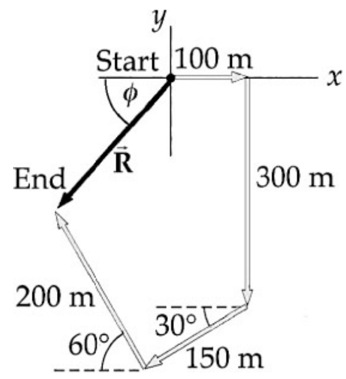
P3.50 To find the new speed and direction of the aircraft, we add the vector components of the wind to the vector velocity of the aircraft:

$$\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (300 + 100\cos 30.0°)\hat{\mathbf{i}} + (100\sin 30.0°)\hat{\mathbf{j}}$$

$$\vec{\mathbf{v}} = \left(387\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)\ \text{mi/h}$$

$$|\vec{\mathbf{v}}| = \boxed{390\ \text{mi/h at } 7.37°\ \text{N of E}}$$

P3.51 On our version of the diagram we have drawn in the resultant from the tail of the first arrow to the head of the last arrow. The resultant displacement $\vec{\mathbf{R}}$ is equal to the sum of the four individual displacements, $\vec{\mathbf{R}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 + \vec{\mathbf{d}}_4$. We translate from the pictorial representation to a mathematical representation by writing the individual displacements in unit-vector notation:



ANS. FIG. P3.51

$$\vec{\mathbf{d}}_1 = 100\hat{\mathbf{i}} \text{ m}$$
$$\vec{\mathbf{d}}_2 = -300\hat{\mathbf{j}} \text{ m}$$
$$\vec{\mathbf{d}}_3 = (-150 \cos 30°)\hat{\mathbf{i}} \text{ m} + (-150 \sin 30°)\hat{\mathbf{j}} \text{ m} = -130\hat{\mathbf{i}} \text{ m} - 75\hat{\mathbf{j}} \text{ m}$$
$$\vec{\mathbf{d}}_4 = (-200 \cos 60°)\hat{\mathbf{i}} \text{ m} + (200 \sin 60°)\hat{\mathbf{j}} \text{ m} = -100\hat{\mathbf{i}} \text{ m} + 173\hat{\mathbf{j}} \text{ m}$$

Summing the components together, we find

$$R_x = d_{1x} + d_{2x} + d_{3x} + d_{4x} = (100 + 0 - 130 - 100) \text{ m} = -130 \text{ m}$$
$$R_y = d_{1y} + d_{2y} + d_{3y} + d_{4y} = (0 - 300 - 75 + 173) \text{ m} = -202 \text{ m}$$

so altogether

$$\vec{\mathbf{R}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 + \vec{\mathbf{d}}_4 = \boxed{\left(-130\hat{\mathbf{i}} - 202\hat{\mathbf{j}}\right)\text{m}}$$

Its magnitude is

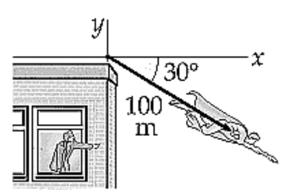
$$\left|\vec{\mathbf{R}}\right| = \sqrt{(-130)^2 + (-202)^2} = \boxed{240 \text{ m}}$$

We calculate the angle $\phi = \tan^{-1}\left(\dfrac{R_y}{R_x}\right) = \tan^{-1}\left(\dfrac{-202}{-130}\right) = 57.2°$.

The resultant points into the third quadrant instead of the first quadrant. The angle counterclockwise from the $+x$ axis is

$$\theta = 180 + \phi = \boxed{237°}$$

***P3.52** The superhero follows a straight-line path at 30.0° below the horizontal. If his displacement is 100 m, then the coordinates of the superhero are:



ANS. FIG. P3.52

$$x = (100 \text{ m})\cos(-30.0°) = \boxed{86.6 \text{ m}}$$
$$y = (100 \text{ m})\sin(-30.0°) = \boxed{-50.0 \text{ m}}$$

P3.53 (a) Take the x axis along the tail section of the snake. The displacement from tail to head is

$$\left(240\text{ m}\right)\hat{\mathbf{i}}+\left[(420-240)\text{ m}\right]\cos(180°-105°)\hat{\mathbf{i}}$$
$$-\left(180\text{ m}\right)\sin 75°\hat{\mathbf{j}}=287\text{ m}\hat{\mathbf{i}}-174\text{ m}\hat{\mathbf{j}}$$

Its magnitude is $\sqrt{\left(287\right)^2+\left(174\right)^2}\text{ m}=335\text{ m}$.

From $v=\dfrac{\text{distance}}{\Delta t}$, the time for each child's run is

$$\text{Inge: }\Delta t=\frac{\text{distance}}{v}=\frac{335\text{ m}\left(\text{h}\right)\left(1\text{ km}\right)\left(3600\text{ s}\right)}{\left(12\text{ km}\right)\left(1000\text{ m}\right)\left(1\text{ h}\right)}=101\text{ s}$$

$$\text{Olaf: }\Delta t=\frac{420\text{ m}\cdot\text{s}}{3.33\text{ m}}=126\text{ s}$$

Inge wins by $126-101=\boxed{25.4\text{ s}}$.

(b) Olaf must run the race in the same time:

$$v=\frac{d}{\Delta t}=\frac{420\text{ m}}{101\text{ s}}\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{\text{km}}{10^3\text{ m}}\right)=\boxed{15.0\text{ km/h}}$$

P3.54 The position vector from the ground under the controller of the first airplane is

$$\vec{\mathbf{r}}_1=(19.2\text{ km})(\cos 25°)\hat{\mathbf{i}}+(19.2\text{ km})(\sin 25°)\hat{\mathbf{j}}+(0.8\text{ km})\hat{\mathbf{k}}$$
$$=\left(17.4\hat{\mathbf{i}}+8.11\hat{\mathbf{j}}+0.8\hat{\mathbf{k}}\right)\text{ km}$$

The second is at

$$\vec{\mathbf{r}}_2=(17.6\text{ km})(\cos 20°)\hat{\mathbf{i}}+(17.6\text{ km})(\sin 20°)\hat{\mathbf{j}}+(1.1\text{ km})\hat{\mathbf{k}}$$
$$=\left(16.5\hat{\mathbf{i}}+6.02\hat{\mathbf{j}}+1.1\hat{\mathbf{k}}\right)\text{ km}$$

Now the displacement from the first plane to the second is

$$\vec{\mathbf{r}}_2-\vec{\mathbf{r}}_1=\left(-0.863\hat{\mathbf{i}}-2.09\hat{\mathbf{j}}+0.3\hat{\mathbf{k}}\right)\text{ km}$$

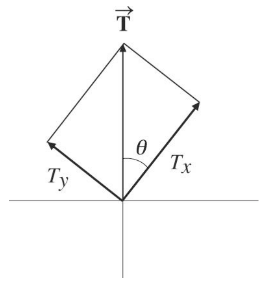
with magnitude

$$\sqrt{\left(0.863\right)^2+\left(2.09\right)^2+\left(0.3\right)^2}\text{ km}=\boxed{2.29\text{ km}}$$

P3.55 (a) The tensions T_x and T_y act as an equivalent tension T (see ANS. FIG. P3.55) which supports the downward weight; thus, the combination is equivalent to 0.150 N, upward. We know that T_x = 0.127 N, and the tensions are perpendicular to each other, so their combined magnitude is

$$\mathrm{T} = \sqrt{\mathrm{T}_x^2 + \mathrm{T}_y^2} = 0.150\ \mathrm{N} \rightarrow \mathrm{T}_y^2 = (0.150\ \mathrm{N})^2 - \mathrm{T}_x^2$$

$$\mathrm{T}_y^2 = (0.150\ \mathrm{N})^2 - (0.127\ \mathrm{N})^2 \rightarrow \mathrm{T}_y = 0.078\ \mathrm{N}$$



ANS. FIG. P3.55

(b) From the figure, $\theta = \tan^{-1}\left(\mathrm{T}_y/\mathrm{T}_x\right) = 32.1°$. The angle the x axis makes with the horizontal axis is $90° - \theta = \boxed{57.9°}$.

(c) From the figure, the angle the y axis makes with the horizontal axis is $\theta = \boxed{32.1°}$.

P3.56 (a) Consider the rectangle in the figure to have height H and width W. The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are related by $\vec{\mathbf{A}} + \overrightarrow{\mathbf{ab}} + \overrightarrow{\mathbf{bc}} = \vec{\mathbf{B}}$, where

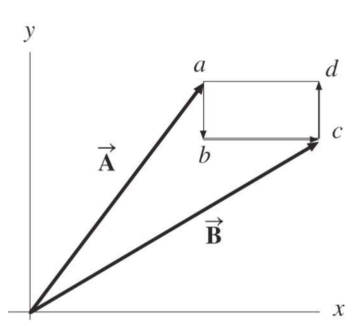
$$\vec{\mathbf{A}} = (10.0\ \mathrm{m})(\cos 50.0°)\hat{\mathbf{i}} + (10.0\ \mathrm{m})(\sin 50.0°)\hat{\mathbf{j}}$$

$$\vec{\mathbf{A}} = \left(6.42\hat{\mathbf{i}} + 7.66\hat{\mathbf{j}}\right)\ \mathrm{m}$$

$$\vec{\mathbf{B}} = (12.0\ \mathrm{m})(\cos 30.0°)\hat{\mathbf{i}} + (12.0\ \mathrm{m})(\sin 30.0°)\hat{\mathbf{j}}$$

$$\vec{\mathbf{B}} = \left(10.4\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)\ \mathrm{m}$$

$$\overrightarrow{\mathbf{ab}} = -\mathrm{H}\hat{\mathbf{j}} \text{ and } \overrightarrow{\mathbf{bc}} = \mathrm{W}\hat{\mathbf{i}}$$



ANS. FIG. P3.56

Therefore,

$$\vec{\mathbf{B}} - \vec{\mathbf{A}} = \overrightarrow{ab} + \overrightarrow{bc}$$

$$\left(3.96\hat{\mathbf{i}} - 1.66\hat{\mathbf{j}}\right) \text{ m} = W\hat{\mathbf{i}} - H\hat{\mathbf{j}} \to W = 3.96 \text{ m and } H = 1.66 \text{ m}$$

$\boxed{\text{The perimeter measures } 2(H + W) = 11.24 \text{ m.}}$

(b) The vector from the origin to the upper-right corner of the rectangle (point *d*) is

$$\vec{\mathbf{B}} + H\hat{\mathbf{j}} = 10.4 \text{ m}\hat{\mathbf{i}} + \left(6.00 \text{ m} + 1.66 \text{ m}\right)\hat{\mathbf{j}} = 10.4 \text{ m}\hat{\mathbf{i}} + 7.66 \text{ m}\hat{\mathbf{j}}$$

$$\boxed{\begin{array}{l} \text{magnitude: } \sqrt{(10.4 \text{ m})^2 + (7.66 \text{ m})^2} = 12.9 \text{ m} \\ \text{direction: } \tan^{-1}(7.66/10.4) = 36.4° \text{ above } +x \text{ axis (first quadrant)} \end{array}}$$

P3.57 (a) $R_x = \boxed{2.00}$, $R_y = \boxed{1.00}$, $R_z = \boxed{3.00}$

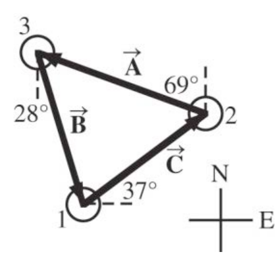
(b) $\left|\vec{\mathbf{R}}\right| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{4.00 + 1.00 + 9.00} = \sqrt{14.0} = \boxed{3.74}$

(c) $\cos\theta_x = \dfrac{R_x}{\left|\vec{\mathbf{R}}\right|} \Rightarrow \theta_x = \cos^{-1}\left(\dfrac{R_x}{\left|\vec{\mathbf{R}}\right|}\right) = \boxed{57.7° \text{ from } +x}$

$\cos\theta_y = \dfrac{R_y}{\left|\vec{\mathbf{R}}\right|} \Rightarrow \theta_y = \cos^{-1}\left(\dfrac{R_y}{\left|\vec{\mathbf{R}}\right|}\right) = \boxed{74.5° \text{ from } +y}$

$\cos\theta_z = \dfrac{R_z}{\left|\vec{\mathbf{R}}\right|} \Rightarrow \theta_z = \cos^{-1}\left(\dfrac{R_z}{\left|\vec{\mathbf{R}}\right|}\right) = \boxed{36.7° \text{ from } +z}$

P3.58 Let *A* represent the distance from island 2 to island 3. The displacement is $\vec{\mathbf{A}} = A$ at 159°. Represent the displacement from 3 to 1 as $\vec{\mathbf{B}} = B$ at 298°. We have 4.76 km at $37° + \vec{\mathbf{A}} + \vec{\mathbf{B}} = 0$.



ANS. FIG. P3.58

For the *x* components:

$$(4.76 \text{ km})\cos 37° + A\cos 159° + B\cos 298° = 0$$

$$3.80 \text{ km} - 0.934A + 0.470B = 0$$

$$B = -8.10 \text{ km} + 1.99A$$

For the *y* components:

$$(4.76 \text{ km})\sin 37° + A\sin 159° + B\sin 298° = 0$$

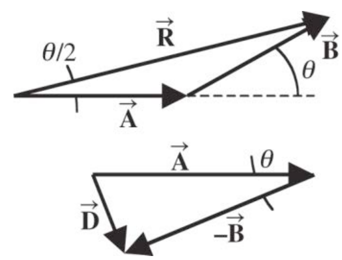
$$2.86 \text{ km} + 0.358A - 0.883B = 0$$

(a) We solve by eliminating B by substitution:

$$2.86 \text{ km} + 0.358A - 0.883(-8.10 \text{ km} + 1.99A) = 0$$
$$2.86 \text{ km} + 0.358A + 7.15 \text{ km} - 1.76A = 0$$
$$10.0 \text{ km} = 1.40A$$
$$A = \boxed{7.17 \text{ km}}$$

(b) $B = -8.10 \text{ km} + 1.99(7.17 \text{ km}) = \boxed{6.15 \text{ km}}$

P3.59 Let θ represent the angle between the directions of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitudes, $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ form an isosceles triangle in which the angles are $180° - \theta$, $\frac{\theta}{2}$, and $\frac{\theta}{2}$. The magnitude of $\vec{\mathbf{R}}$ is then $R = 2A\cos\left(\frac{\theta}{2}\right)$. This can be seen from applying the law of cosines to the isosceles triangle and using the fact that $B = A$.



ANS. FIG. P3.59

Again, $\vec{\mathbf{A}}$, $-\vec{\mathbf{B}}$, and $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$$

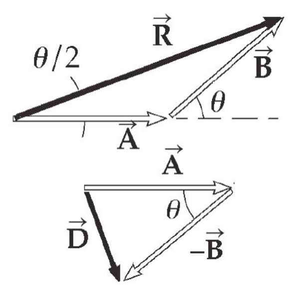
gives the magnitude of $\vec{\mathbf{D}}$ as $D = 2A\sin\left(\frac{\theta}{2}\right)$.

The problem requires that $R = 100D$.

Thus, $2A\cos\left(\frac{\theta}{2}\right) = 200A\sin\left(\frac{\theta}{2}\right)$. This gives

$$\tan\left(\frac{\theta}{2}\right) = 0.010 \text{ and } \boxed{\theta = 1.15°}$$

P3.60 Let θ represent the angle between the directions of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitudes, $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ form an isosceles triangle in which the angles are $180° - \theta$, $\frac{\theta}{2}$, and $\frac{\theta}{2}$. The magnitude of $\vec{\mathbf{R}}$ is then $R = 2A\cos\left(\frac{\theta}{2}\right)$. This can be seen by applying the



ANS. FIG. P3.60

law of cosines to the isosceles triangle and using the fact that $B = A$. Again, $\vec{\mathbf{A}}$, $-\vec{\mathbf{B}}$, and $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of $\vec{\mathbf{D}}$ as $D = 2A\sin\left(\frac{\theta}{2}\right)$.

The problem requires that $R = nD$ or

$$\cos\left(\frac{\theta}{2}\right) = n\sin\left(\frac{\theta}{2}\right) \text{ giving } \boxed{\theta = 2\tan^{-1}\left(\frac{1}{n}\right)}.$$

The larger R is to be compared to D, the smaller the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ becomes.

P3.61 (a) We write $\vec{\mathbf{B}}$ in terms of the sine and cosine of the angle θ, and add the two vectors:

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \left(-60 \text{ cm}\hat{\mathbf{j}}\right) + (80 \text{ cm} \cos\theta)\hat{\mathbf{i}} + (80 \text{ cm} \sin\theta)\hat{\mathbf{j}}$$

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (80 \text{ cm} \cos\theta)\hat{\mathbf{i}} + (80 \text{ cm} \sin\theta - 60 \text{ cm})\hat{\mathbf{j}}$$

Dropping units (cm), the magnitude is

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \left[(80 \cos\theta)^2 + (80 \sin\theta - 60)^2\right]^{1/2}$$

$$= \left[(80)^2\left(\cos^2\theta + \sin^2\theta\right) - 2(80)(60)\sin\theta + (60)^2\right]^{1/2}$$

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \left[(80)^2 + (60)^2 - 2(80)(60)\sin\theta\right]^{1/2}$$

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \boxed{\left[10{,}000 - (9600)\sin\theta\right]^{1/2} \text{ cm}}$$

(b) For $\theta = 270°$, $\sin\theta = -1$, and $\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \boxed{140 \text{ cm}}$.

(c) For $\theta = 90°$, $\sin\theta = 1$, and $\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \boxed{20.0 \text{ cm}}$.

(d) They do make sense. The maximum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in the same direction, and it is 60 cm + 80 cm. The minimum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in opposite directions, and it is 80 cm − 60 cm.

P3.62 We perform the integration:

$$\Delta\vec{\mathbf{r}} = \int_0^{0.380\text{ s}} \vec{\mathbf{v}}\,dt = \int_0^{0.380\text{ s}}\left(1.2\hat{\mathbf{i}}\text{ m/s} - 9.8t\hat{\mathbf{j}}\text{ m/s}^2\right)dt$$

$$= 1.2t\;\hat{\mathbf{i}}\text{ m/s}\Big|_0^{0.380\text{ s}} - \left(9.8\hat{\mathbf{j}}\text{ m/s}^2\right)\frac{t^2}{2}\Big|_0^{0.380\text{ s}}$$

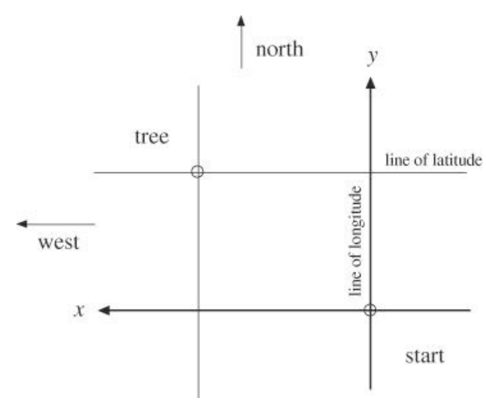
$$= \left(1.2\hat{\mathbf{i}}\text{ m/s}\right)(0.38\text{ s} - 0) - \left(9.8\hat{\mathbf{j}}\text{ m/s}^2\right)\left(\frac{(0.38\text{ s})^2 - 0}{2}\right)$$

$$= \boxed{0.456\hat{\mathbf{i}}\text{ m} - 0.708\hat{\mathbf{j}}\text{ m}}$$

P3.63 (a) $\dfrac{d\vec{\mathbf{r}}}{dt} = \dfrac{d\left(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2t\hat{\mathbf{k}}\right)}{dt} = -2\hat{\mathbf{k}} = \boxed{-(2.00\text{ m/s})\hat{\mathbf{k}}}$

(b) The position vector at $t = 0$ is $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$. At $t = 1$ s, the position is $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, and so on. The object is moving straight downward at 2 m/s, so $\dfrac{d\vec{\mathbf{r}}}{dt}$ represents $\boxed{\text{its velocity vector}}$.

P3.64 (a) The very small differences between the angles suggests we may consider this region of Earth to be small enough so that we may consider it to be flat (a plane); therefore, we may consider the lines of latitude and longitude to be parallel and perpendicular, so that we can use them as an *xy* coordinate system. Values of latitude, θ, increase as we travel north, so differences between latitudes can give the *y* coordinate. Values of longitude, ϕ, increase as we travel west, so differences between longitudes can give the *x* coordinate. Therefore, our coordinate system will have $+y$ to the north and $+x$ to the west.



ANS. FIG. P3.64

Since we are near the equator, each line of latitude and longitude may be considered to form a circle with a radius equal to the radius of Earth, $R = 6.36 \times 10^6$ m. Recall the length s of an arc of a circle of radius R that subtends an angle (in radians) $\Delta\theta$ (or $\Delta\phi$) is given by $s = R\Delta\theta$ (or $s = R\Delta\phi$). We can use this equation to find the components of the displacement from the starting point to the tree—these are parallel to the *x* and *y* coordinates axes. Therefore,

we can regard the origin to be the starting point and the displacements as the x and y coordinates of the tree.

The angular difference $\Delta\phi$ for longitude values is (west being positive)

$$\begin{aligned}\Delta\phi &= [75.64426° - 75.64238°] \\ &= (0.00188°)(\pi \text{ rad}/180°) \\ &= 3.28\times 10^{-5} \text{ rad}\end{aligned}$$

corresponding to the x coordinate (displacement west)

$$x = R\Delta\phi = (6.36\times 10^{6} \text{ m})(3.28\times 10^{-5} \text{ rad}) = 209 \text{ m}$$

The angular difference $\Delta\theta$ for latitude values is (north being positive)

$$\begin{aligned}\Delta\theta &= [0.00162° - (-0.00243°)] \\ &= (0.00405°)(\pi \text{ rad}/180°) \\ &= 7.07\times 10^{-5} \text{ rad}\end{aligned}$$

corresponding to the y coordinate (displacement north)

$$y = R\Delta\theta = (6.36\times 10^{6} \text{ m})(7.07\times 10^{-5} \text{ rad}) = 450 \text{ m}$$

The distance to the tree is

$$d = \sqrt{x^2+y^2} = \sqrt{(209 \text{ m})^2 + (450 \text{ m})^2} = \boxed{496 \text{ m}}$$

The direction to the tree is

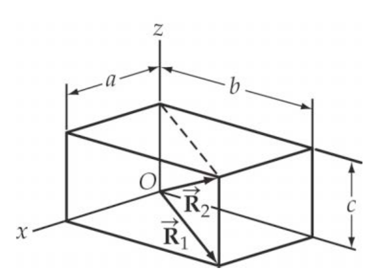
$$\tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{450 \text{ m}}{209 \text{ m}}\right) = 65.1° = \boxed{65.1° \text{ N of W}}$$

(b) Refer to the arguments above. They are justified because the distances involved are small relative to the radius of Earth.

P3.65 (a) From the picture, $\boxed{\vec{\mathbf{R}}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}.}$

(b) $\boxed{R_1 = \sqrt{a^2+b^2}}$

(c) $\boxed{\vec{\mathbf{R}}_2 = \vec{\mathbf{R}}_1 + c\hat{\mathbf{k}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}}$



ANS. FIG. P3.65

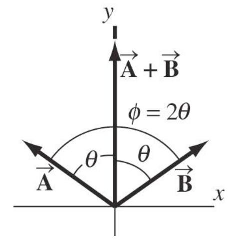
P3.66 Since

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = 6.00\hat{\mathbf{j}},$$

we have

$$\left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}} = 0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}$$

giving $A_x + B_x = 0 \rightarrow A_x = -B_x$.



ANS. FIG. P3.66

Because the vectors have the same magnitude and x components of equal magnitude but of opposite sign, the vectors are reflections of each other in the y axis, as shown in the diagram. Therefore, the two vectors have the same y components:

$$A_y = B_y = (1/2)(6.00) = 3.00$$

Defining θ as the angle between either $\vec{\mathbf{A}}$ or $\vec{\mathbf{B}}$ and the y axis, it is seen that

$$\cos\theta = \frac{A_y}{A} = \frac{B_y}{B} = \frac{3.00}{5.00} = 0.600 \rightarrow \theta = 53.1°$$

The angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is then $\boxed{\phi = 2\theta = 106°}$.

Challenge Problem

P3.67 (a) You start at point A: $\vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_A = \left(30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}}\right)$ m.

The displacement to B is

$$\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A = 60.0\hat{\mathbf{i}} + 80.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}} = 30.0\hat{\mathbf{i}} + 100\hat{\mathbf{j}}$$

You cover half of this, $\left(15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}}\right)$, to move to

$$\vec{\mathbf{r}}_2 = 30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}}$$

Now the displacement from your current position to C is

$$\vec{\mathbf{r}}_C - \vec{\mathbf{r}}_2 = -10.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} = -55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}$$

You cover one-third, moving to

$$\vec{\mathbf{r}}_3 = \vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_{23} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}} + \frac{1}{3}\left(-55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}}$$

The displacement from where you are to *D* is

$$\vec{\mathbf{r}}_D - \vec{\mathbf{r}}_3 = 40.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} - 26.7\hat{\mathbf{i}} - 16.7\hat{\mathbf{j}} = 13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}$$

You traverse one-quarter of it, moving to

$$\vec{\mathbf{r}}_4 = \vec{\mathbf{r}}_3 + \frac{1}{4}\left(\vec{\mathbf{r}}_D - \vec{\mathbf{r}}_3\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}} + \frac{1}{4}\left(13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}\right)$$
$$= 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}$$

The displacement from your new location to *E* is

$$\vec{\mathbf{r}}_E - \vec{\mathbf{r}}_4 = -70.0\hat{\mathbf{i}} + 60.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 55.0\hat{\mathbf{j}}$$

of which you cover one-fifth the distance, $-20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}}$, moving to

$$\vec{\mathbf{r}}_4 + \Delta\vec{\mathbf{r}}_{45} = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}} = 10.0\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}}$$

The treasure is at $\boxed{(10.0\text{ m}, 16.0\text{ m})}$.

(b) Following the directions brings you to the average position of the trees. The steps we took numerically in part (a) bring you to

$$\vec{\mathbf{r}}_A + \frac{1}{2}\left(\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A\right) = \left(\frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B}{2}\right)$$

then to

$$\frac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B\right)}{2} + \frac{\vec{\mathbf{r}}_C - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B\right)/2}{3} = \frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C}{3}$$

then to

$$\frac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C\right)}{3} + \frac{\vec{\mathbf{r}}_D - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C\right)/3}{4} = \frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D}{4}$$

and last to

$$\frac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D\right)}{4} + \frac{\vec{\mathbf{r}}_E - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D\right)/4}{5}$$
$$= \frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D + \vec{\mathbf{r}}_E}{5}$$

This center of mass of the tree distribution is the same location whatever order we take the trees in.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P3.2 (a) 2.31; (b) 1.15

P3.4 (a) (2.17, 1.25) m, (–1.90, 3.29) m; (b) 4.55m

P3.6 (a) r, 180° – θ; (b) 180° + θ; (c) $-\theta$

P3.8 $\vec{\mathbf{B}}$ is 43 units in the negative y direction

P3.10 9.5 N, 57° above the x axis

P3.12 (a) See ANS. FIG. P3.12; (b) The sum of a set of vectors is not affected by the order in which the vectors are added.

P3.14 310 km at 57° S of W

P3.16 A_x = 28.7 units, A_y = –20.1 units

P3.18 1.31 km north and 2.81 km east

P3.20 (a) 5.00 blocks at 53.1° N of E; (b) 13.00 blocks

P3.22 $\left(2.60\hat{\mathbf{i}} + 4.50\hat{\mathbf{j}}\right)$ m

P3.24 788 miles at 48.0° northeast of Dallas

P3.26 (a) See ANS. FIG. P3.24; (b) $5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}, -1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}$; (c) 6.40 at 38.7°, 8.06 at 97.2°

P3.28 (a) Its component parallel to the surface is (1.50 m) cos 141° = –1.17 m, or 1.17 m toward the top of the hill; (b) Its component perpendicular to the surface is (1.50 m) sin 141° = 0.944 m, or 0.944 m away from the snow.

P3.30 42.7 yards

P3.32 C_x = 7.30 cm; C_y = –7.20 cm

P3.34 59.2° with the x axis, 39.8° with the y axis, 67.4° with the z axis

P3.36 (a) $5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}$, 5.92 m; (b) $(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}})$ m, 19.0) m

P3.38 (a) $49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}$; (b) 56.4, 28.7°

P3.40 magnitude: 170.1 cm, direction: 57.2° above +x axis (first quadrant); magnitude: 145.7 cm, direction: 58.6° above +x axis (first quadrant)

P3.42 (a) $\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right)$ km; (b) 6.31 km

P3.44 Impossible because 12.4 m is greater than 5.00 m

P3.46 (a) $\left(5 = 11\,\mathrm{f}\right)\hat{\mathbf{i}} + \left(3 + 9\,\mathrm{f}\right)\hat{\mathbf{j}}$ meters; (b) $(5+0)\hat{\mathbf{i}} + (3+0)\hat{\mathbf{j}}$ meters; (c) This is reasonable because it is the location of the starting point, $5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ meters. (d) $16\hat{\mathbf{i}} + 12\hat{\mathbf{j}}$ meters; (e) This is reasonable because we have completed the trip, and this is the position vector of the endpoint.

P3.48 2.24 m, 26.6°

P3.50 390 mi/h at 7.37° N of E

P3.52 86.6 m, –50.0 m

P3.54 2.29 km

P3.56 (a) The perimeter measures $2(H + W) = 11.24$ m; (b) magnitude: 12.9 m, direction: 36.4° above $+x$ axis (first quadrant)

P3.58 (a) 7.17 km; (b) 6.15 km

P3.60 $\theta = 2\tan^{-1}\left(\frac{1}{\mathrm{n}}\right)$

P3.62 $0.456\hat{\mathbf{i}}\ \mathrm{m} - 0.708\hat{\mathbf{j}}\ \mathrm{m}$

P3.64 (a) 496 m, 65.1° N of W; (b) The arguments are justified because the distances involved are small relative to the radius of the Earth.

P3.66 $\phi = 2\theta = 106°$

4

Motion in Two Dimensions

CHAPTER OUTLINE

* An asterisk indicates an item new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ4.1 The car's acceleration must have an inward component and a forward component: answer (e). Another argument: Draw a final velocity vector of two units west. Add to it a vector of one unit south. This represents subtracting the initial velocity from the final velocity, on the way to finding the acceleration. The direction of the resultant is that of vector (e).

OQ4.2 (i) The 45° angle means that at point *A* the horizontal and vertical velocity components are equal. The horizontal velocity component is the same at *A*, *B*, and *C*. The vertical velocity component is zero at *B* and negative at *C*. The assembled answer is $a = b = c > d = 0 > e$.

(ii) The *x component* of acceleration is everywhere zero and the *y component* is everywhere $-9.80\ \text{m/s}^2$. Then we have $a = c = 0 > b = d = e$.

OQ4.3 Because gravity pulls downward, the horizontal and vertical motions of a projectile are independent of each other. Both balls have zero initial vertical components of velocity, and both have the same vertical accelerations, $-g$; therefore, both balls will have identical vertical motions: they will reach the ground at the same time. Answer (b).

OQ4.4 The projectile on the moon is in flight for a time interval six times larger, with the same range of vertical speeds and with the same constant horizontal speed as on Earth. Then its maximum altitude is (d) six times larger.

OQ4.5 The acceleration of a car traveling at constant speed in a circular path is directed toward the center of the circle. Answer (d).

OQ4.6 The acceleration of gravity near the surface of the Moon acts the same way as on Earth, it is constant and it changes only the vertical component of velocity. Answers (b) and (c).

OQ4.7 The projectile on the Moon is in flight for a time interval six times larger, with the same range of vertical speeds and with the same constant horizontal speed as on Earth. Then its range is (d) six times larger.

OQ4.8 Let the positive x direction be that of the girl's motion. The x component of the velocity of the ball relative to the ground is (+5 – 12) m/s = –7 m/s. The x-velocity of the ball relative to the girl is (–7 – 8) m/s = –15 m/s. The relative speed of the ball is +15 m/s, answer (d).

OQ4.9 Both wrench and boat have identical horizontal motions because gravity influences the vertical motion of the wrench only. Assuming neither air resistance nor the wind influences the horizontal motion of the wrench, the wrench will land at the base of the mast. Answer (b).

OQ4.10 While in the air, the baseball is a projectile whose velocity always has a constant horizontal component ($v_x = v_{xi}$) and a vertical component that changes at a constant rate ($\Delta v_y / \Delta t = a_y = -g$). At the highest point on the path, the vertical velocity of the ball is momentarily zero. Thus, at this point, the resultant velocity of the ball is horizontal and its acceleration continues to be directed downward ($a_x = 0$, $a_y = -g$). The only correct choice given for this question is (c).

OQ4.11 The period $T = 2\pi r/v$ changes by a factor of 4/4 = 1. The answer is (a).

OQ4.12 The centripetal acceleration $a = v^2/r$ becomes $(3v)^2/(3r) = 3v^2/r$, so it is 3 times larger. The answer is (b).

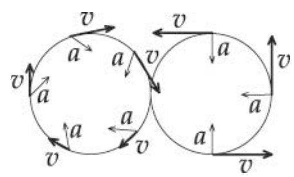
OQ4.13 (a) Yes (b) No: The escaping jet exhaust exerts an extra force on the plane. (c) No (d) Yes (e) No: The stone is only a few times more dense than water, so friction is a significant force on the stone. The answer is (a) and (d).

OQ4.14 With radius half as large, speed should be smaller by a factor of $1/\sqrt{2}$, so that $a = v^2/r$ can be the same. The answer is (d).

ANSWERS TO CONCEPTUAL QUESTIONS

CQ4.1 A parabola results, because the originally forward velocity component stays constant and the rocket motor gives the spacecraft constant acceleration in a perpendicular direction. These are the same conditions for a projectile, for which the velocity is constant in the horizontal direction and there is a constant acceleration in the perpendicular direction. Therefore, a curve of the same shape is the result.

CQ4.2 The skater starts at the center of the eight, goes clockwise around the left circle and then counterclockwise around the right circle.



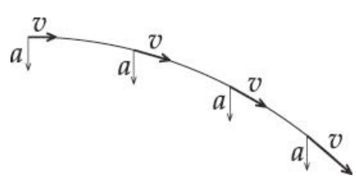
CQ4.3 No, you cannot determine the instantaneous velocity because the points could be separated by a finite displacement, but you can determine the average velocity. Recall the definition of average velocity:

$$\vec{\mathbf{v}}_{\text{avg}} = \frac{\Delta\vec{\mathbf{x}}}{\Delta t}$$

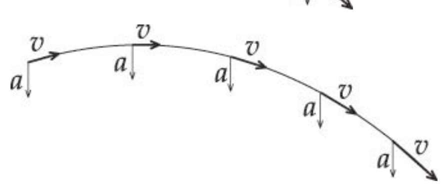
CQ4.4 (a) On a straight and level road that does not curve to left or right.
(b) Either in a circle or straight ahead on a level road. The acceleration magnitude can be constant either with a nonzero or with a zero value.

CQ4.5 (a) Yes, the projectile is in free fall. (b) Its vertical component of acceleration is the downward acceleration of gravity. (c) Its horizontal component of acceleration is zero.

CQ4.6 (a)



(b)

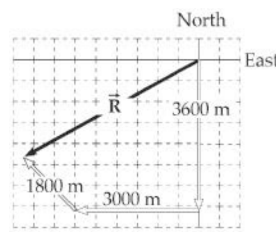


CQ4.7 (a) No. Its velocity is constant in magnitude and direction. (b) Yes. The particle is continuously changing the direction of its velocity vector.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 4.1 The Position, Velocity, and Acceleration Vectors

P4.1 We must use the method of vector addition and the definitions of average velocity and of average speed.



ANS. FIG. P4.1

(a) For each segment of the motion we model the car as a particle under constant velocity. Her displacements are

$$\vec{\mathbf{R}} = (20.0\text{ m/s})(180\text{ s})\textbf{ south} + (25.0\text{ m/s})(120\text{ s})\textbf{ west} + (30.0\text{ m/s})(60.0\text{ s})\textbf{ northwest}$$

Choosing $\hat{\mathbf{i}}$ = east and $\hat{\mathbf{j}}$ = north, we have

$$\vec{\mathbf{R}} = (3.60\text{ km})(-\hat{\mathbf{j}}) + (3.00\text{ km})(-\hat{\mathbf{i}}) + (1.80\text{ km})\cos 45°(-\hat{\mathbf{i}}) + (1.80\text{ km})\sin 45°(\hat{\mathbf{j}})$$

$$\vec{\mathbf{R}} = (3.00 + 1.27)\text{ km}(-\hat{\mathbf{i}}) + (1.27 - 3.60)\text{ km}(\hat{\mathbf{j}}) = (-4.27\hat{\mathbf{i}} - 2.33\hat{\mathbf{j}})\text{ km}$$

The answer can also be written as

$$\vec{\mathbf{R}} = \sqrt{(-4.27\text{ km})^2 + (-2.33\text{ km})^2} = 4.87\text{ km}$$

$$\text{at } \tan^{-1}\left(\frac{2.33}{4.27}\right) = 28.6°$$

or $\boxed{4.87\text{ km at }28.6°\text{S of W}}$

(b) The total distance or path length traveled is (3.60 + 3.00 + 1.80) km = 8.40 km, so

$$\text{average speed} = \left(\frac{8.40\text{ km}}{6.00\text{ min}}\right)\left(\frac{1.00\text{ min}}{60.0\text{ s}}\right)\left(\frac{1\,000\text{ m}}{\text{km}}\right) = \boxed{23.3\text{ m/s}}$$

(c) $$\text{Average velocity} = \frac{4.87\times 10^3\text{ m}}{360\text{ s}} = \boxed{13.5\text{ m/s along }\vec{\mathbf{R}}}$$

P4.2 The sun projects onto the ground the x component of the hawk's velocity:

$$(5.00\text{ m/s})\cos(-60.0°) = \boxed{2.50\text{ m/s}}$$

***P4.3** (a) For the average velocity, we have

$$\vec{\mathbf{v}}_{\text{avg}} = \left(\frac{x(4.00) - x(2.00)}{4.00\text{ s} - 2.00\text{ s}}\right)\hat{\mathbf{i}} + \left(\frac{y(4.00) - y(2.00)}{4.00\text{ s} - 2.00\text{ s}}\right)\hat{\mathbf{j}}$$
$$= \left(\frac{5.00\text{ m} - 3.00\text{ m}}{2.00\text{ s}}\right)\hat{\mathbf{i}} + \left(\frac{3.00\text{ m} - 1.50\text{ m}}{2.00\text{ s}}\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{v}}_{\text{avg}} = \boxed{\left(1.00\hat{\mathbf{i}} + 0.750\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) For the velocity components, we have

$$v_x = \frac{dx}{dt} = a = 1.00\text{ m/s}$$

and $v_y = \dfrac{dy}{dt} = 2ct = \left(0.250\text{ m/s}^2\right)t$

Therefore,

$$\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (1.00\text{ m/s})\hat{\mathbf{i}} + \left(0.250\text{ m/s}^2\right)t\,\hat{\mathbf{j}}$$

$$\boxed{\vec{\mathbf{v}}(t = 2.00\text{ s}) = (1.00\text{ m/s})\hat{\mathbf{i}} + (0.500\text{ m/s})\hat{\mathbf{j}}}$$

and the speed is

$$|\vec{\mathbf{v}}(t = 2.00\text{ s})| = \sqrt{(1.00\text{ m/s})^2 + (0.500\text{ m/s})^2} = \boxed{1.12\text{ m/s}}$$

P4.4 (a) From $x = -5.00\sin\omega t$, we determine the components of the velocity by taking the time derivatives of *x* and *y*:

$$v_x = \frac{dx}{dt} = \left(\frac{d}{dt}\right)(-5.00\sin\omega t) = -5.00\omega\cos\omega t$$

and $v_y = \dfrac{dy}{dt} = \left(\dfrac{d}{dt}\right)(4.00 - 5.00\cos\omega t) = 0 + 5.00\omega\sin\omega t$

At $t = 0$,

$$\vec{\mathbf{v}} = \left(-5.00\omega\cos 0\right)\hat{\mathbf{i}} + \left(5.00\omega\sin 0\right)\hat{\mathbf{j}} = \boxed{-5.00\omega\,\hat{\mathbf{i}}\text{ m/s}}$$

(b) Acceleration is the time derivative of the velocity, so

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(-5.00\omega\cos\omega t\right) = +5.00\omega^2\sin\omega t$$

and $a_y = \dfrac{dv_y}{dt} = \left(\dfrac{d}{dt}\right)(5.00\omega\sin\omega t) = 5.00\omega^2\cos\omega t$

At $t = 0$,

$$\vec{\mathbf{a}} = \left(5.00\omega^2\sin 0\right)\hat{\mathbf{i}} + \left(5.00\omega^2\cos 0\right)\hat{\mathbf{j}} = \boxed{5.00\omega^2\hat{\mathbf{j}}\text{ m/s}^2}$$

(c) $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = \boxed{(4.00\text{ m})\hat{\mathbf{j}} + (5.00\text{ m})(-\sin\omega t\hat{\mathbf{i}} - \cos\omega t\hat{\mathbf{j}})}$

$$\vec{\mathbf{v}} = \boxed{(5.00\text{ m})\omega\left[-\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}}\right]}$$

$$\vec{\mathbf{a}} = \boxed{(5.00\text{ m})\omega^2\left[\sin\omega t\hat{\mathbf{i}} + \cos\omega t\hat{\mathbf{j}}\right]}$$

(d) the object moves in a circle of radius 5.00 m centered at (0, 4.00 m)

P4.5 (a) The x and y equations combine to give us the expression for $\vec{\mathbf{r}}$:

$\vec{\mathbf{r}} = 18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}$, where $\vec{\mathbf{r}}$ is in meters and t is in seconds.

(b) We differentiate the expression for $\vec{\mathbf{r}}$ with respect to time:

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{d}{dt}\left[18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}\right]$$

$$= \frac{d}{dt}(18.0t)\hat{\mathbf{i}} + \frac{d}{dt}\left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}$$

$\vec{\mathbf{v}} = 18.0\hat{\mathbf{i}} + \left[4.00 - (9.80)t\right]\hat{\mathbf{j}}$, where $\vec{\mathbf{v}}$ is in meters per second and t is in seconds.

(c) We differentiate the expression for $\vec{\mathbf{v}}$ with respect to time:

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \frac{d}{dt}\left\{18.0\hat{\mathbf{i}} + \left[4.00 - (9.80)t\right]\hat{\mathbf{j}}\right\}$$

$$= \frac{d}{dt}(18.0)\hat{\mathbf{i}} + \frac{d}{dt}\left[4.00 - (9.80)t\right]\hat{\mathbf{j}}$$

$$\boxed{\vec{\mathbf{a}} = -9.80\hat{\mathbf{j}}\text{ m/s}^2}$$

(d) By substitution,

$$\vec{\mathbf{r}}(3.00\text{ s}) = \boxed{(54.0\text{ m})\hat{\mathbf{i}} - (32.1\text{ m})\hat{\mathbf{j}}}$$

$$\vec{\mathbf{v}}(3.00\text{ s}) = \boxed{(18.0\text{ m/s})\hat{\mathbf{i}} - (25.4\text{ m/s})\hat{\mathbf{j}}}$$

$$\vec{\mathbf{a}}(3.00\text{ s}) = \boxed{(-9.80\text{ m/s}^2)\hat{\mathbf{j}}}$$

Section 4.2 Two-Dimensional Motion with Constant Acceleration

P4.6 We use the vector versions of the kinematic equations for motion in two dimensions. We write the initial position, initial velocity, and acceleration of the particle in vector form:

$$\vec{\mathbf{a}} = 3.00\hat{\mathbf{j}}\ \text{m/s}^2;\ \vec{\mathbf{v}}_i = 5.00\hat{\mathbf{i}}\ \text{m/s};\ \vec{\mathbf{r}}_i = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$$

(a) The position of the particle is given by Equation 4.9:

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2 = (5.00\ \text{m/s})t\hat{\mathbf{i}} + \frac{1}{2}(3.00\ \text{m/s}^2)t^2\hat{\mathbf{j}}$$
$$= \boxed{5.00t\hat{\mathbf{i}} + 1.50t^2\hat{\mathbf{j}}}$$

where r is in m and t in s.

(b) The velocity of the particle is given by Equation 4.8:

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \boxed{5.00\hat{\mathbf{i}} + 3.00t\,\hat{\mathbf{j}}}$$

where v is in m/s and t in s.

(c) To obtain the particle's position at t = 2.00 s, we plug into the equation obtained in part (a):

$$\vec{\mathbf{r}}_f = (5.00\ \text{m/s})(2.00\ \text{s})\hat{\mathbf{i}} + (1.50\ \text{m/s}^2)(2.00\ \text{s})^2\hat{\mathbf{j}}$$
$$= (10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}})\ \text{m}$$

so $x_f = \boxed{10.0\ \text{m}}$, $y_f = \boxed{6.00\ \text{m}}$

(d) To obtain the particle's speed at t = 2.00 s, we plug into the equation obtained in part (b):

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = (5.00\ \text{m/s})\hat{\mathbf{i}} + (3.00\ \text{m/s}^2)(2.00\ \text{s})\hat{\mathbf{j}}$$
$$= (5.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}})\ \text{m/s}$$

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00\ \text{m/s})^2 + (6.00\ \text{m/s})^2} = \boxed{7.81\ \text{m/s}}$$

P4.7 (a) We differentiate the equation for the vector position of the particle with respect to time to obtain its velocity:

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \left(\frac{d}{dt}\right)(3.00\hat{\mathbf{i}} - 6.00t^2\hat{\mathbf{j}}) = \boxed{-12.0t\hat{\mathbf{j}}\ \text{m/s}}$$

(b) Differentiating the expression for velocity with respect to time gives the particle's acceleration:

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \left(\frac{d}{dt}\right)\left(-12.0t\hat{\mathbf{j}}\right) = \boxed{-12.0\hat{\mathbf{j}}\ \text{m/s}^2}$$

(c) By substitution, when t = 1.00 s,

$$\boxed{\vec{\mathbf{r}} = \left(3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}\right)\text{m};\ \vec{\mathbf{v}} = -12.0\hat{\mathbf{j}}\ \text{m/s}}$$

***P4.8** (a) For the x component of the motion we have $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$.

$$0.01\ \text{m} = 0 + \left(1.80\times10^7\ \text{m/s}\right)t + \frac{1}{2}\left(8\times10^{14}\ \text{m/s}^2\right)t^2$$

$$\left(4\times10^{14}\ \text{m/s}^2\right)t^2 + \left(1.80\times10^7\ \text{m/s}\right)t - 10^{-2}\ \text{m} = 0$$

$$t = \left(\frac{1}{2\left(4\times10^{14}\,\text{m/s}^2\right)}\right)\Big[-1.80\times10^7\ \text{m/s} \pm\sqrt{\left(1.8\times10^7\ \text{m/s}\right)^2 - 4\left(4\times10^{14}\ \text{m/s}^2\right)\left(-10^{-2}\,\text{m}\right)}\Big]$$

$$= \frac{-1.8\times10^7 \pm 1.84\times10^7\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2}$$

We choose the + sign to represent the physical situation:

$$t = \frac{4.39\times10^5\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2} = 5.49\times10^{-10}\ \text{s}$$

Here

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$= 0 + 0 + \frac{1}{2}\left(1.6\times10^{15}\,\text{m/s}^2\right)\left(5.49\times10^{-10}\ \text{s}\right)^2$$

$$= 2.41\times10^{-4}\ \text{m}$$

So, $\boxed{\vec{\mathbf{r}}_f = \left(10.0\,\hat{\mathbf{i}} + 0.241\,\hat{\mathbf{j}}\right)\ \text{mm}}$

(b) $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t$

$$= 1.80\times10^7\,\hat{\mathbf{i}}\ \text{m/s} + \left(8\times10^{14}\,\hat{\mathbf{i}}\ \text{m/s}^2 + 1.6\times10^{15}\,\hat{\mathbf{j}}\ \text{m/s}^2\right)\left(5.49\times10^{-10}\ \text{s}\right)$$

$$= \left(1.80\times10^7\ \text{m/s}\right)\hat{\mathbf{i}} + \left(4.39\times10^5\ \text{m/s}\right)\hat{\mathbf{i}} + \left(8.78\times10^5\ \text{m/s}\right)\hat{\mathbf{j}}$$

$$= \boxed{\left(1.84\times10^7\ \text{m/s}\right)\hat{\mathbf{i}} + \left(8.78\times10^5\ \text{m/s}\right)\hat{\mathbf{j}}}$$

(c) $\left|\vec{\mathbf{v}}_f\right| = \sqrt{\left(1.84\times10^7\text{ m/s}\right)^2 + \left(8.78\times10^5\text{ m/s}\right)^2} = \boxed{1.85\times10^7\text{ m/s}}$

(d) $\theta = \tan^{-1}\left(\dfrac{v_y}{v_x}\right) = \tan^{-1}\left(\dfrac{8.78\times10^5}{1.84\times10^7}\right) = \boxed{2.73°}$

P4.9 Model the fish as a particle under constant acceleration. We use our old standard equations for constant-acceleration straight-line motion, with *x* and *y* subscripts to make them apply to parts of the whole motion. At $t = 0$,

$$\vec{\mathbf{v}}_i = \left(4.00\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}\right)\text{ m/s and } \hat{\mathbf{r}}_i = (10.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})\text{ m}$$

At the first "final" point we consider, 20.0 s later,

$$\vec{\mathbf{v}}_f = (20.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}})\text{ m/s}$$

(a) $a_x = \dfrac{\Delta v_x}{\Delta t} = \dfrac{20.0\text{ m/s} - 4.00\text{ m/s}}{20.0\text{ s}} = \boxed{0.800\text{ m/s}^2}$

$a_y = \dfrac{\Delta v_y}{\Delta t} = \dfrac{-5.00\text{ m/s} - 1.00\text{ m/s}}{20.0\text{ s}} = \boxed{-0.300\text{ m/s}^2}$

(b) $\theta = \tan^{-1}\left(\dfrac{-0.300\text{ m/s}^2}{0.800\text{ m/s}^2}\right) = -20.6° = \boxed{339°\text{ from }+x\text{ axis}}$

(c) At $t = 25.0$ s the fish's position is specified by its coordinates and the direction of its motion is specified by the direction angle of its velocity:

$$\begin{aligned} x_f &= x_i + v_{xi}t + \frac{1}{2}a_x t^2 \\ &= 10.0\text{ m} + \left(4.00\text{ m/s}\right)(25.0\text{ s}) + \frac{1}{2}(0.800\text{ m/s}^2)(25.0\text{ s})^2 \\ &= \boxed{360\text{ m}} \end{aligned}$$

$$\begin{aligned} y_f &= y_i + v_{yi}t + \frac{1}{2}a_y t^2 \\ &= -4.00\text{ m} + \left(1.00\text{ m/s}\right)(25.0\text{ s}) + \frac{1}{2}(-0.300\text{ m/s}^2)(25.0\text{ s})^2 \\ &= \boxed{-72.7\text{ m}} \end{aligned}$$

$$v_{xf} = v_{xi} + a_x t = 4.00\text{ m/s} + \left(0.800\text{ m/s}^2\right)(25.0\text{ s}) = 24\text{ m/s}$$
$$v_{yf} = v_{yi} + a_y t = 1.00\text{ m/s} - \left(0.300\text{ m/s}^2\right)(25.0\text{ s}) = -6.50\text{ m/s}$$
$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-6.50\text{ m/s}}{24.0\text{ m/s}}\right) = \boxed{-15.2°}$$

P4.10 The directions of the position, velocity, and acceleration vectors are given with respect to the x axis, and we know that the components of a vector with magnitude A and direction θ are given by $A_x = A\cos\theta$ and $A_y = A\sin\theta$; thus we have

$$\vec{\mathbf{r}}_i = 29.0\cos 95.0°\,\hat{\mathbf{i}} + 29.0\sin 95.0°\,\hat{\mathbf{j}} = -2.53\,\hat{\mathbf{i}} + 28.9\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{v}}_i = 4.50\cos 40.0°\,\hat{\mathbf{i}} + 4.50\sin 40.0°\,\hat{\mathbf{j}} = 3.45\,\hat{\mathbf{i}} + 2.89\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{a}} = 1.90\cos 200°\,\hat{\mathbf{i}} + 1.90\sin 200°\,\hat{\mathbf{j}} = -1.79\,\hat{\mathbf{i}} + -0.650\,\hat{\mathbf{j}}$$

where $\vec{\mathbf{r}}$ is in m, $\vec{\mathbf{v}}$ in m/s, $\vec{\mathbf{a}}$ in m/s^2, and t in s.

(a) From $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t$,

$$\boxed{\vec{\mathbf{v}}_f = (3.45 - 1.79t)\hat{\mathbf{i}} + (2.89 - 0.650t)\hat{\mathbf{j}}}$$

where $\vec{\mathbf{v}}$ in m/s and t in s.

(b) The car's position vector is given by

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$= \left(-2.53 + 3.45t + \frac{1}{2}(-1.79)t^2\right)\hat{\mathbf{i}} + \left(28.9 + 2.89t + \frac{1}{2}(-0.650)t^2\right)\hat{\mathbf{j}}$$

$$\boxed{\vec{\mathbf{r}}_f = (-2.53 + 3.45t - 0.893t^2)\hat{\mathbf{i}} + (28.9 + 2.89t - 0.325t^2)\hat{\mathbf{j}}}$$

where $\vec{\mathbf{r}}$ is in m and t in s.

Section 4.3 Projectile Motion

P4.11 At the maximum height $v_y = 0$, and the time to reach this height is found from

$$v_{yf} = v_{yi} + a_y t \text{ as } t = \frac{v_{yf} - v_{yi}}{a_y} = \frac{0 - v_{yi}}{-g} = \frac{v_{yi}}{g}.$$

The vertical displacement that has occurred during this time is

$$(\Delta y)_{\max} = v_{y,\text{avg}}t = \left(\frac{v_{yf} + v_{yi}}{2}\right)t = \left(\frac{0 + v_{yi}}{2}\right)\left(\frac{v_{yi}}{g}\right) = \frac{v_{yi}^2}{2g}$$

Thus, if $(\Delta y)_{\max} = 12\text{ ft}\left(\dfrac{1\text{ m}}{3.281\text{ ft}}\right) = 3.66\text{ m}$, then

$$v_{yi} = \sqrt{2g(\Delta y)_{\max}} = \sqrt{2(9.80\text{ m/s}^2)(3.66\text{ m})} = 8.47\text{ m/s}$$

and if the angle of projection is $\theta = 45°$, the launch speed is

$$v_i = \frac{v_{yi}}{\sin\theta} = \frac{8.47 \text{ m/s}}{\sin 45°} = \boxed{12.0 \text{ m/s}}$$

***P4.12** From Equation 4.13 with $R = 15.0$ m, $v_i = 3.00$ m/s, and $\theta_{max} = 45.0°$:

$$g_{planet} = \frac{v_i^2 \sin 2\theta}{R} = \frac{v_i^2 \sin 90°}{R} = \frac{9.00 \text{ m}^2/\text{s}^2}{15.0 \text{ m}} = \boxed{0.600 \text{ m/s}^2}$$

P4.13 (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If t is the time at which it hits the ground, then since there is no horizontal acceleration,

$$x_f = v_{xi}t \rightarrow t = x_f/v_{xi} \rightarrow t = (1.40 \text{ m}/v_{xi})$$

At time t, it has fallen a distance of 1.22 m with a downward acceleration of 9.80 m/s^2. Then

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$0 = 1.22 \text{ m} - (4.90 \text{ m/s}^2)(1.40 \text{ m}/v_{xi})^2$$

Thus,

$$v_{xi} = \sqrt{\frac{(4.90 \text{ m/s}^2)(1.40 \text{ m})^2}{1.22 \text{ m}}} = \boxed{2.81 \text{ m/s}}$$

(b) The vertical velocity component with which it hits the floor is

$$v_{yf} = v_{yi} + a_y t \rightarrow v_{yf} = v_{yi} + (-g)(1.40 \text{ m}/v_{xi})$$
$$v_{yf} = 0 + (-9.80 \text{ m/s}^2)(1.40 \text{ m}/2.81 \text{ m/s}) = -4.89 \text{ m/s}$$

Hence, the angle θ at which the mug strikes the floor is given by

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-4.89 \text{ m/s}}{2.81 \text{ m/s}}\right) = -60.2°$$

$\boxed{\text{The mug's velocity is } 60.2° \text{ below the horizontal when it strikes the ground.}}$

P4.14 The mug is a projectile from just after leaving the counter until just before it reaches the floor. Taking the origin at the point where the mug leaves the bar, the coordinates of the mug at any time t are

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \rightarrow x_f = 0 + v_{xi}t \rightarrow x_f = v_{xi}t$$

and

$$y_f = y_i + v_{yi}t + \frac{1}{2}at^2 \rightarrow y_f = -0 + 0 - \frac{1}{2}gt^2 \rightarrow y_f = -\frac{1}{2}gt^2$$

(a) When the mug reaches the floor, $y_f = h$ and $x_f = d$, so

$$-h = -\frac{1}{2}gt^2 \rightarrow h = \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2h}{g}}$$

is the time of impact, and

$$x_f = v_{xi}t \rightarrow d = v_{xi}t \rightarrow v_{xi} = \frac{d}{t}$$

$$\boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}$$

(b) Just before impact, the x component of velocity is still

$v_{xf} = v_{xi}$

while the y component is

$$v_{yf} = v_{yi} + at \rightarrow v_{yf} = 0 - gt = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}$$

Then the direction of motion just before impact is below the horizontal at an angle of

$$\theta = \tan^{-1}\frac{v_{yf}}{v_{xf}} = \tan^{-1}\left(\frac{-\sqrt{2gh}}{d\sqrt{\frac{g}{2h}}}\right)$$

$$\theta = \tan^{-1}\left(\frac{-2h}{d}\right) = -\tan^{-1}\left(\frac{2h}{d}\right)$$

because the x component of velocity is positive (forward) and the y component is negative (downward).

$\boxed{\text{The direction of the mug's velocity is } \tan^{-1}(2h/d) \text{ below the horizontal.}}$

P4.15 We ignore the trivial case where the angle of projection equals zero degrees.

$$h = \frac{v_i^2 \sin^2\theta_i}{2g}; \quad R = \frac{v_i^2(\sin 2\theta_i)}{g}; \quad 3h = R$$

so $$\frac{3v_i^2\sin^2\theta_i}{2g} = \frac{v_i^2(\sin 2\theta_i)}{g}$$

or $$\frac{2}{3} = \frac{\sin^2\theta_i}{\sin 2\theta_i} = \frac{\tan\theta_i}{2}$$

thus, $$\theta_i = \tan^{-1}\left(\frac{4}{3}\right) = \boxed{53.1°}$$

P4.16 The horizontal range of the projectile is found from $x = v_{xi}t = v_i\cos\theta_i t$. Plugging in numbers,

$$x = (300\text{ m/s})(\cos 55.0°)(42.0\text{ s})$$

$$x = \boxed{7.23\times 10^3\text{ m}}$$

The vertical position of the projectile is found from

$$y = v_{yi}t - \frac{1}{2}gt^2 = v_i\sin\theta_i t - \frac{1}{2}gt^2$$

Plugging in numbers,

$$y = (300\text{ m/s})(\sin 55.0°)(42.0\text{ s}) - \frac{1}{2}(9.80\text{ m/s}^2)(42.0\text{ s})^2$$

$$= \boxed{1.68\times 10^3\text{ m}}$$

P4.17 (a) The vertical component of the salmon's velocity as it leaves the water is

$$v_{yi} = +v_i\sin\theta = +(6.26\text{ m/s})\sin 45.0° \simeq +4.43\text{ m/s}$$

When the salmon returns to water level at the end of the leap, the vertical component of velocity will be

$$v_{yf} = -v_{yi} \simeq -4.43\text{ m/s}$$

If the salmon jumps out of the water at $t = 0$, the time interval required for it to return to the water is

$$\Delta t_1 = \frac{v_{yf} - v_{yi}}{a_y} = \frac{-4.43\text{ m/s} - 4.43\text{ m/s}}{-9.80\text{ m/s}^2} \simeq 0.903\text{ s}$$

The horizontal distance traveled during the leap is

$$L = v_{xi}\Delta t_i = (v_i\cos\theta)\Delta t_i$$
$$= (6.26\text{ m/s})\cos 45.0°(0.903\text{ s}) = 4.00\text{ m}$$

To travel this same distance underwater, at speed v = 3.58 m/s, requires a time interval of

$$\Delta t_2 = \frac{L}{v} = \frac{4.00\text{ m}}{3.58\text{ m/s}} \simeq 1.12\text{ s}$$

The average horizontal speed for the full porpoising maneuver is then

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{2L}{\Delta t_1 + \Delta t_2} = \frac{2(4.00\text{ m})}{0.903\text{ s} + 1.12\text{ s}} = \boxed{3.96\text{ m/s}}$$

(b) From (a), the total time interval for the porpoising maneuver is

$$\Delta t = 0.903\text{ s} + 1.12\text{ s} = 2.02\text{ s}$$

Without porpoising, the time interval to travel distance 2L is

$$\Delta t_2 = \frac{2L}{v} = \frac{8.00\text{ m}}{3.58\text{ m/s}} \simeq 2.23\text{ s}$$

The percentage difference is

$$\frac{\Delta t_1 - \Delta t_2}{\Delta t_2} \times 100\% = -9.6\%$$

$\boxed{\text{Porpoising reduces the time interval by 9.6\%.}}$

P4.18 (a) We ignore the trivial case where the angle of projection equals zero degrees. Because the projectile motion takes place over level ground, we can use Equations 4.12 and 4.13:

$$R = h \rightarrow \frac{v_i^2 \sin 2\theta_i}{g} = \frac{v_i^2 \sin^2\theta_i}{2g}$$

Expanding,

$$2\sin 2\theta_i = \sin^2\theta_i$$
$$4\sin\theta_i\cos\theta_i = \sin^2\theta_i$$
$$\tan\theta_i = 4$$
$$\theta_i = \tan^{-1}(4) = \boxed{76.0°}$$

(b) The maximum range is attained for θ_i = 45°:

$$R = \frac{v_i^2 \sin\left[2(76.0°)\right]}{g} \text{ and } R_{\text{max}} = \frac{v_i^2 \sin\left[2(45.0°)\right]}{g} = \frac{v_i^2}{g}$$

then

$$R_{max} = \frac{v_i^2 \sin[2(76.0°)]}{g \sin[2(76.0°)]} = \frac{R}{\sin[2(76.0°)]}$$

$$R_{max} = \boxed{2.13R}$$

(c) Since g divides out, the answer is the same on every planet.

***P4.19** Consider the motion from original zero height to maximum height *h:*

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i) \text{ gives } 0 = v_{yi}^2 - 2g(h - 0)$$

or $$v_{yi} = \sqrt{2gh}$$

Now consider the motion from the original point to half the maximum height:

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i) \text{ gives } v_{yh}^2 = 2gh + 2(-g)\left(\frac{1}{2}h - 0\right)$$

so $$v_{yh} = \sqrt{gh}$$

At maximum height, the speed is $v_x = \frac{1}{2}\sqrt{v_x^2 + v_{yh}^2} = \frac{1}{2}\sqrt{v_x^2 + gh}$

Solving,

$$v_x = \sqrt{\frac{gh}{3}}$$

Now the projection angle is

$$\theta_i = \tan^{-1}\frac{v_{yi}}{v_x} = \tan^{-1}\frac{\sqrt{2gh}}{\sqrt{gh/3}} = \tan^{-1}\sqrt{6} = \boxed{67.8°}$$

P4.20 (a) $x_f = v_{xi}t = (8.00 \text{ m/s})\cos 20.0° (3.00 \text{ s}) = \boxed{22.6 \text{ m}}$

(b) Taking y positive downwards,

$$y_f = v_{yi}t + \frac{1}{2}gt^2$$

$$y_f = (8.00 \text{ m/s})\sin 20.0°(3.00 \text{ s}) + \frac{1}{2}(9.80 \text{ m/s}^2)(3.00 \text{ s})^2$$

$$= \boxed{52.3 \text{ m}}$$

(c) $$10.0 \text{ m} = (8.00 \text{ m/s})(\sin 20.0°)t + \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

Suppressing units,

$$4.90t^2 + 2.74t - 10.0 = 0$$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18 \text{ s}}$$

P4.21 The horizontal component of displacement is $x_f = v_{xi}t = (v_i \cos\theta_i)t$. Therefore, the time required to reach the building a distance d away is $t = \frac{d}{v_i \cos\theta_i}$. At this time, the altitude of the water is

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = v_i \sin\theta_i\left(\frac{d}{v_i \cos\theta_i}\right) - \frac{g}{2}\left(\frac{d}{v_i \cos\theta_i}\right)^2$$

Therefore, the water strikes the building at a height h above ground level of

$$h = y_f = \boxed{d\tan\theta_i - \frac{gd^2}{2v_i^2 \cos^2\theta_i}}$$

P4.22 (a) The time of flight of a water drop is given by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = y_1 - \frac{1}{2}gt^2$$

For $t_1 > 0$, the root is $t_1 = \sqrt{\frac{2y_1}{g}} = \sqrt{\frac{2(2.35 \text{ m})}{9.8 \text{ m/s}^2}} = 0.693$ s.

The horizontal range of a water drop is

$$x_{f1} = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$= 0 + 1.70 \text{ m/s } (0.693 \text{ s}) + 0 = 1.18 \text{ m}$$

This is about the width of a town sidewalk, so $\boxed{\text{there is}}$ space for a walkway behind the waterfall. Unless the lip of the channel is well designed, water may drip on the visitors. A tall or inattentive person may get his or her head wet.

(b) Now the flight time t_2 is given by

$$0 = y_2 + 0 - \frac{1}{2}gt_2^2, \text{ where } y_2 = \frac{y_1}{12}:$$

$$t_2 = \sqrt{\frac{2y_2}{g}} = \sqrt{\frac{2y_1}{g(12)}} = \frac{1}{\sqrt{12}} \times \sqrt{\frac{2y_1}{g}} = \frac{t_1}{\sqrt{12}}$$

From the same equation as in part (a) for horizontal range, $x_2 = v_2 t_2$, where $x_2 = x_1/12$:

$$x_2 = v_2 t_2 \rightarrow \frac{x_1}{12} = v_2 \frac{t_1}{\sqrt{12}}$$

$$v_2 = \frac{x_1}{t_1\sqrt{12}} = \frac{v_1}{\sqrt{12}} = \frac{1.70 \text{ m/s}}{\sqrt{12}} = \boxed{0.491 \text{ m/s}}$$

The rule that the scale factor for speed is the square root of the scale factor for distance is Froude's law, published in 1870.

P4.23 (a) From the particle under constant velocity model in the x direction, find the time at which the ball arrives at the goal:

$$x_f = x_i + v_i t \rightarrow t = \frac{x_f - x_i}{v_{xi}} = \frac{36.0 \text{ m} - 0}{(20 \text{ m/s}) \cos 53.0°} = 2.99 \text{ s}$$

From the particle under constant acceleration model in the y direction, find the height of the ball at this time:

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

$$y_f = 0 + (20.0 \text{ m/s}) \sin 53.0°(2.99 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(2.99 \text{ s})^2$$

$$y_f = 3.94 \text{ m}$$

Therefore, the ball clears the crossbar by

$$3.94 \text{ m} - 3.05 \text{ m} = \boxed{0.89 \text{ m}}$$

(b) Use the particle under constant acceleration model to find the time at which the ball is at its highest point in its trajectory:

$$v_{yf} = v_{yi} - gt \rightarrow t = \frac{v_{yf} - v_{yi}}{g} = \frac{(20.0 \text{ m/s})\sin 53.0° - 0}{9.80 \text{ m/s}^2} = 1.63 \text{ s}$$

Because this is earlier than the time at which the ball reaches the goal, the ball clears the goal on its way down.

P4.24 From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2\left(-9.80 \text{ m/s}^2\right)(1.85 - 1.02) \text{ m}$. From this, $v_{yi} = 4.03$ m/s. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation gives $v_{yf}^2 = 0 + 2(-9.80\,\text{m/s}^2)(0.900 - 1.85)$ m. Thus, the vertical velocity just before he lands is $v_{yf} = -4.32$ m/s.

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$:

$$-4.32 \text{ m/s} = 4.03 \text{ m/s} + (-9.80 \text{ m/s}^2)t$$

or $t = \boxed{0.852 \text{ s}}$.

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes

$$2.80 \text{ m} = v_{xi}\,(0.852 \text{ s})$$

which yields $v_{xi} = \boxed{3.29 \text{ m/s}}$.

(c) $v_{yi} = \boxed{4.03 \text{ m/s}}$. See above for proof.

(d) The takeoff angle is: $\theta = \tan^{-1}\dfrac{v_{yi}}{v_{xi}} = \tan^{-1}\left(\dfrac{4.03 \text{ m/s}}{3.29 \text{ m/s}}\right) = \boxed{50.8°}$

(e) Similarly for the deer, the upward part of the flight gives

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

$$0 = v_{yi}^2 + 2\left(-9.80 \text{ m/s}^2\right)(2.50 - 1.20) \text{ m}$$

so $v_{yi} = 5.04$ m/s

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ yields

$$v_{yf}^2 = 0 + 2\left(-9.80\,\text{m/s}^2\right)(0.700 - 2.50) \text{ m and } v_{yf} = -5.94 \text{ m/s.}$$

The hang time is then found as $v_{yf} = v_{yi} + a_y t$:

$$-5.94 \text{ m/s} = 5.04 \text{ m/s} + (-9.80 \text{ m/s}^2)t \text{ and}$$

$$\boxed{t = 1.12 \text{ s}}$$

P4.25 (a) For the horizontal motion, we have $x_f = d = 24$ m:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$24 \text{ m} = 0 + v_i\left(\cos 53°\right)\left(2.2 \text{ s}\right) + 0$$

$$v_i = \boxed{18.1 \text{ m/s}}$$

(b) As it passes over the wall, the ball is above the street by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$y_f = 0 + (18.1\ \text{m/s})(\sin\ 53°)(2.2\ \text{s}) + \frac{1}{2}(-9.8\ \text{m/s}^2)(2.2\ \text{s})^2 = 8.13\ \text{m}$$

So it clears the parapet by 8.13 m – 7 m = $\boxed{1.13\ \text{m}}$.

(c) Note that the highest point of the ball's trajectory is not directly above the wall. For the whole flight, we have from the trajectory equation:

$$y_f = (\tan\theta_i)x_f - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2$$

or $$6\ \text{m} = (\tan 53°)x_f - \left(\frac{9.8\ \text{m/s}^2}{2(18.1\ \text{m/s})^2\cos^2 53°}\right)x_f^2$$

Solving,

$$(0.041\ 2\ \text{m}^{-1})x_f^2 - 1.33x_f + 6\ \text{m} = 0$$

and, suppressing units,

$$x_f = \frac{1.33 \pm \sqrt{1.33^2 - 4(0.041\ 2)(6)}}{2(0.041\ 2)}$$

This yields two results:

x_f = 26.8 m or 5.44 m

The ball passes twice through the level of the roof.

It hits the roof at distance from the wall

26.8 m – 24 m = $\boxed{2.79\ \text{m}}$

P4.26 We match the given equations:

$$x_f = 0 + (11.2\ \text{m/s})(\cos\ 18.5°)t$$

$$0.360\ \text{m} = 0.840\ \text{m} + (11.2\ \text{m/s})(\sin\ 18.5°)t - \frac{1}{2}(9.80\ \text{m/s}^2)t^2$$

to the equations for the coordinates of the final position of a projectile:

$$x_f = x_i + v_{xi}t$$

$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$$

For the equations to represent the same functions of time, all coefficients must agree: $x_i = 0$, $y_i = 0.840$ m, $v_{xi} = (11.2 \text{ m/s}) \cos 18.5°$, $v_{yi} = (11.2 \text{ m/s}) \sin 18.5°$, and $g = 9.80 \text{ m/s}^2$.

(a) Then the original position of the athlete's center of mass is the point with coordinates $(x_i, y_i) = \boxed{(0, 0.840 \text{ m})}$. That is, his original position has position vector $\vec{\mathbf{r}} = 0\hat{\mathbf{i}} + 0.840\hat{\mathbf{j}}$ m.

(b) His original velocity is $\vec{\mathbf{v}}_i = (11.2 \text{ m/s})(\cos 18.5°)\hat{\mathbf{i}} + (11.2 \text{ m/s})(\sin 18.5°)\hat{\mathbf{j}} = \boxed{11.2 \text{ m/s at } 18.5°}$ above the x axis.

(c) From $(4.90 \text{ m/s}^2)t^2 - (3.55 \text{ m/s})t - 0.48 \text{ m} = 0$, we find the time of flight, which must be positive. Suppressing units,

$$t = \frac{-(-3.55) + \sqrt{(-3.55)^2 - 4(4.90)(-0.48)}}{2(4.90)} = 0.841 \text{ s}$$

Then $x_f = (11.2 \text{ m/s}) \cos 18.5°(0.841 \text{ s}) = \boxed{8.94 \text{ m}}$.

P4.27 Model the rock as a projectile, moving with constant horizontal velocity, zero initial vertical velocity, and with constant vertical acceleration. Note that the sound waves from the splash travel in a straight line to the soccer player's ears. The time of flight of the rock follows from

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-40.0 \text{ m} = 0 + 0 + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)t^2$$

$$t = 2.86 \text{ s}$$

The extra time 3.00 s – 2.86 s = 0.140 s is the time required for the sound she hears to travel straight back to the player. It covers distance

$$(343 \text{ m/s})\ 0.143 \text{ s} = 49.0 \text{ m} = \sqrt{x^2 + (40.0 \text{ m})^2}$$

where x represents the horizontal distance the rock travels. Solving for x gives $x = 28.3$ m. Since the rock moves with constant speed in the x direction and travels horizontally during the 2.86 s that it is in flight,

$$x = 28.3 \text{ m} = v_{xi}t + 0t^2$$

$$\therefore v_{xi} = \frac{28.3 \text{ m}}{2.86 \text{ s}} = \boxed{9.91 \text{ m/s}}$$

P4.28 The initial velocity components of the projectile are

$$x_i = 0 \quad \text{and} \quad y_i = h$$

$$v_{xi} = v_i \cos\theta \quad \text{and} \quad v_{yi} = v_i \sin\theta$$

while the constant acceleration components are

$$a_x = 0 \quad \text{and} \quad a_y = -g$$

The coordinates of the projectile are

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = (v_i \cos\theta)t \quad \text{and}$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = h + (v_i \sin\theta)t - \frac{1}{2}gt^2$$

and the components of velocity are

$$v_{xf} = v_{xi} + a_x t = v_i \cos\theta \quad \text{and}$$

$$v_{yf} = v_{yi} + a_y t = v_i \sin\theta - gt$$

(a) We know that when the projectile reaches its maximum height, $v_{yf} = 0$:

$$v_{yf} = v_i \sin\theta - gt = 0 \rightarrow \boxed{t = \frac{v_i \sin\theta}{g}}$$

(b) At the maximum height, $y = h_{\text{max}}$:

$$h_{\text{max}} = h + (v_i \sin\theta)t - \frac{1}{2}gt^2$$

$$h_{\text{max}} = h + v_i \sin\theta \frac{v_i \sin\theta}{g} - \frac{1}{2}g\left(\frac{v_i \sin\theta}{g}\right)^2$$

$$\boxed{h_{\text{max}} = h + \frac{(v_i \sin\theta)^2}{2g}}$$

P4.29 (a) Initial coordinates: $\boxed{x_i = 0.00 \text{ m}, y_i = 0.00 \text{ m}}$

(b) Components of initial velocity: $\boxed{v_{xi} = 18.0 \text{ m/s}, \ v_{yi} = 0}$

(c) $\boxed{\text{Free fall motion, with constant downward acceleration } g = 9.80 \text{ m/s}^2.}$

(d) $\boxed{\text{Constant velocity motion in the horizontal direction.}}$ There is no horizontal acceleration from gravity.

(e) $v_{xf} = v_{xi} + a_x t \quad \rightarrow \quad \boxed{v_{xf} = v_{xi}}$

$v_{yf} = v_{yi} + a_y t \quad \rightarrow \quad \boxed{v_{yf} = -gt}$

(f) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \quad \rightarrow \quad \boxed{x_f = v_{xi}t}$

$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \quad \rightarrow \quad \boxed{y_f = -\frac{1}{2}gt^2}$

(g) We find the time of impact:

$$y_f = -\frac{1}{2}gt^2$$

$$-h = -\frac{1}{2}gt^2 \quad \rightarrow \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(50.0\text{ m})}{9.80\text{ m/s}^2}} = \boxed{3.19\text{ s}}$$

(h) At impact, $v_{xf} = v_{xi} = 18.0$ m/s, and the vertical component is

$$v_{yf} = -gt$$

$$= -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh} = -\sqrt{2(9.80\text{ m/s}^2)(50.0\text{ m})} = -31.3\text{ m/s}$$

Thus,

$$v_f = \sqrt{v_{xf}^{\ 2} + v_{yf}^{\ 2}} = \sqrt{(18.0\text{ m/s})^2 + (-31.3\text{ m/s})^2} = \boxed{36.1\text{ m/s}}$$

and

$$\theta_f = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-31.3}{18.0}\right) = \boxed{-60.1°}$$

which in this case means the velocity points into the fourth quadrant because its y component is negative.

P4.30 (a) When a projectile is launched with speed v_i at angle θ_i above the horizontal, the initial velocity components are $v_{xi} = v_i \cos\theta_i$ and $v_{yi} = v_i \sin\theta_i$. Neglecting air resistance, the vertical velocity when the projectile returns to the level from which it was launched (in this case, the ground) will be $v_y = -v_{yi}$. From this information, the total time of flight is found from $v_{yf} = v_{yi} + a_y t$ to be

$$t_{total} = \frac{v_{yf} - v_{yi}}{a_y} = \frac{-v_{yi} - v_{yi}}{-g} = \frac{2v_{yi}}{g} \quad \text{or} \quad t_{total} = \frac{2v_i \sin\theta_i}{g}$$

Since the horizontal velocity of a projectile with no air resistance is constant, the horizontal distance it will travel in this time (i.e., its range) is given by

$$R = v_{xi}t_{\text{total}} = (v_i \cos\theta_i)\left(\frac{2v_i \sin\theta_i}{g}\right) = \frac{v_i^2}{g}(2\sin\theta_i \cos\theta_i)$$
$$= \frac{v_i^2 \sin(2\theta_i)}{g}$$

Thus, if the projectile is to have a range of $R = 81.1$ m when launched at an angle of $\theta_i = 45.0°$, the required initial speed is

$$v_i = \sqrt{\frac{Rg}{\sin(2\theta_i)}} = \sqrt{\frac{(81.1\text{ m})(9.80\text{ m/s}^2)}{\sin(90.0°)}} = \boxed{28.2\text{ m/s}}$$

(b) With $v_i = 28.2$ m/s and $\theta_i = 45.0°$ the total time of flight (as found above) will be

$$t_{\text{total}} = \frac{2v_i \sin\theta_i}{g} = \frac{2(28.2\text{ m/s})\sin(45.0°)}{9.80\text{ m/s}^2} = \boxed{4.07\text{ s}}$$

(c) Note that at $\theta_i = 45.0°$, and that sin $(2\theta_i)$ will decrease as θ_i is increased above this optimum launch angle. Thus, if the range is to be kept constant while the launch angle is increased above 45.0°, we see from $v_i = \sqrt{Rg/\sin(2\theta_i)}$ that $\boxed{\text{the required initial velocity will increase}}$.

Observe that for $\theta_i < 90°$, the function sinθ_i increases as θ_i is increased. Thus, increasing the launch angle above 45.0° while keeping the range constant means that both v_i and sinθ_i will increase. Considering the expression for t_{total} given above, we see that $\boxed{\text{the total time of flight will increase}}$.

P4.31 We first consider the vertical motion of the stone as it falls toward the water. The initial y velocity component of the stone is

$$v_{yi} = v_i \sin\theta = -(4.00\text{ m/s})\sin 60.0° = -3.46\text{ m/s}$$

and its y coordinate is

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = h + (v_i \sin\theta)t - \frac{1}{2}gt^2$$
$$y_f = 2.50 - 3.46t - 4.90t^2$$

where y is in m and t in s. We have taken the water's surface to be at $y = 0$. At the water,

$$4.90t^2 + 3.46t - 2.50 = 0$$

Solving for the positive root of the equation, we get

$$t = \frac{-3.46 + \sqrt{(3.46)^2 - 4(4.90)(-2.50)}}{2(4.90)}$$

$$t = \frac{-3.46 + 7.81}{9.80}$$

$$t = 0.443 \text{ s}$$

The y component of velocity of the stone when it reaches the water at this time t is

$$v_{yf} = v_{yi} + a_y t = -3.46 - gt = -7.81 \text{ m/s}$$

After the stone enters to water, its speed, and therefore the magnitude of each velocity component, is reduced by one-half. Thus, the y component of the velocity of the stone in the water is

$$v_{yi} = (-7.81 \text{ m/s})/2 = -3.91 \text{ m/s},$$

and this component remains constant until the stone reaches the bottom. As the stone moves through the water, its y coordinate is

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$y_f = -3.91t$$

The stone reaches the bottom of the pool when $y_f = -3.00$ m:

$$y_f = -3.91t = -3.00 \rightarrow t = 0.767 \text{ s}$$

The total time interval the stone takes to reach the bottom of the pool is

$$\Delta t = 0.443 \text{ s} + 0.767 \text{ s} = \boxed{1.21 \text{ s}}$$

***P4.32** (a) The time for the ball to reach the fence is

$$t = \frac{\Delta x}{v_{xi}} = \frac{130 \text{ m}}{v_i \cos 35.0°} = \frac{159 \text{ m}}{v_i}$$

At this time, the ball must be $\Delta y = 21.0 \text{ m} - 1.00 \text{ m} = 20.0 \text{ m}$ above its launch position, so

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$$

gives

$$20.0\text{ m} = (v_i \sin 35.0°)\left(\frac{159\text{ m}}{v_i}\right) - \frac{1}{2}(9.80\text{ m/s}^2)\left(\frac{159\text{ m}}{v_i}\right)^2$$

or

$$(159\text{ m})\sin 35.0° - 20.0\text{ m} = \frac{(4.90\text{ m/s}^2)(159\text{ m})^2}{v_i^2}$$

from which we obtain

$$v_i = \sqrt{\frac{(4.90\text{ m/s}^2)(159\text{ m})^2}{(159\text{ m})\sin 35.0° - 20.0\text{ m}}} = \boxed{41.7\text{ m/s}}$$

(b) From our equation for the time of flight above,

$$t = \frac{159\text{ m}}{v_i} = \frac{159\text{ m}}{41.7\text{ m/s}} = \boxed{3.81\text{ s}}$$

(c) When the ball reaches the wall (at $t = 3.81$ s),

$$v_x = v_i \cos 35.0° = (41.7\text{ m/s})\cos 35.0° = \boxed{34.1\text{ m/s}}$$

$$\begin{aligned} v_y &= v_i \sin 35.0° + a_y t \\ &= (41.7\text{ m/s})\sin 35.0° - (9.80\text{ m/s}^2)(3.81\text{ s}) \\ &= \boxed{-13.4\text{ m/s}} \end{aligned}$$

and $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(34.1\text{ m/s})^2 + (-13.4\text{ m/s})^2} = \boxed{36.7\text{ m/s}}$

Section 4.4 Analysis Model: Particle in Uniform Circular Motion

P4.33 Model the discus as a particle in uniform circular motion. We evaluate its centripetal acceleration from the standard equation proved in the text.

$$a_c = \frac{v^2}{r} = \frac{(20.0\text{ m/s})^2}{1.06\text{ m}} = \boxed{377\text{ m/s}^2}$$

The mass is unnecessary information.

P4.34 Centripetal acceleration is given by $a = \frac{v^2}{R}$. To find the velocity of a point at the equator, we note that this point travels through $2\pi R_E$ (where $R_E = 6.37 \times 10^6$ m is Earth's radius) in 24.0 hours. Then,

$$v = \frac{2\pi R_E}{T} = \frac{2\pi\left(6.37 \times 10^6 \text{ m}\right)}{(24 \text{ h})(3\,600 \text{ s/h})} = 463 \text{ m/s}$$

and,

$$a = \frac{v^2}{R} = \frac{(463 \text{ m/s})^2}{6.37 \times 10^6 \text{ m}}$$

$$= \boxed{0.033\,7 \text{ m/s}^2 \text{ directed toward the center of Earth}}$$

***P4.35** Centripetal acceleration is given by $a_c = \frac{v^2}{r}$. Let *f* represent the rotation rate. Each revolution carries each bit of metal through distance $2\pi r$, so $v = 2\pi r f$ and

$$a_c = \frac{v^2}{r} = 4\pi^2 r f^2 = 100g$$

A smaller radius implies smaller acceleration. To meet the criterion for each bit of metal we consider the minimum radius:

$$f = \left(\frac{100g}{4\pi^2 r}\right)^{1/2} = \left(\frac{100 \cdot 9.8 \text{ m/s}^2}{4\pi^2 (0.021 \text{ m})}\right)^{1/2}$$

$$= 34.4 \ \frac{1}{s}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{2.06 \times 10^3 \text{ rev/min}}$$

***P4.36** The radius of the tire is $r = 0.500$ m. The speed of the stone on its outer edge is

$$v_t = \frac{2\pi r}{T} = \frac{2\pi(0.500 \text{ m})}{(60.0 \text{ s}/200 \text{ rev})} = \boxed{10.5 \text{ m/s}}$$

and its acceleration is

$$a = \frac{v^2}{R} = \frac{(10.5 \text{ m/s})^2}{0.500 \text{ m}} = \boxed{219 \text{ m/s}^2 \text{ inward}}$$

P4.37 Centripetal acceleration is $a_c = \frac{v^2}{r} \rightarrow v = \sqrt{a_c r}$, where $a_c = 20.0g$, and speed v is in meters per second if r is in meters.

We can convert the speed into a rotation rate, in rev/min, by using the relations 1 revolution = $2\pi r$, and 1 min = 60 s:

$$v = \sqrt{a_c r} = \sqrt{a_c r}\left(\frac{1\text{ rev}}{2\pi r}\right) = \frac{1\text{ rev}}{2\pi}\sqrt{\frac{a_c}{r}}$$

$$= \frac{1\text{ rev}}{2\pi}\sqrt{\frac{20.0\left(9.80\text{ m/s}^2\right)}{29.0\text{ ft}}\left(\frac{3.281\text{ ft}}{1\text{ m}}\right)}\left(\frac{60\text{ s}}{1\text{ min}}\right)$$

$$= \boxed{45.0\text{ rev/min}}$$

P4.38 (a) Using the definition of speed and noting that the ball travels in a circular path,

$$v = \frac{d}{\Delta t} = \frac{2\pi R}{T}$$

where R is the radius of the circle and T is the period, that is, the time interval required for the ball to go around once. For the periods given in the problem,

$$8.00\text{ rev/s} \rightarrow T = \frac{1}{8.00\text{ rev/s}} = 0.125\text{ s}$$

$$6.00\text{ rev/s} \rightarrow T = \frac{1}{6.00\text{ rev/s}} = 0.167\text{ s}$$

Therefore, the speeds in the two cases are:

$$8.00\text{ rev/s} \rightarrow v = \frac{2\pi(0.600\text{ m})}{0.125\text{ s}} = 30.2\text{ m/s}$$

$$6.00\text{ rev/s} \rightarrow v = \frac{2\pi(0.900\text{ m})}{0.167\text{ s}} = 33.9\text{ m/s}$$

Therefore, $\boxed{6.00\text{ rev/s}}$ gives the greater speed of the ball.

(b) Acceleration $= \dfrac{v^2}{r} = \dfrac{(9.60\pi\text{ m/s})^2}{0.600\text{ m}} = \boxed{1.52\times 10^3\text{ m/s}^2}$.

(c) At 6.00 rev/s, acceleration $= \dfrac{(10.8\pi\text{ m/s})^2}{0.900\text{ m}} = \boxed{1.28\times 10^3\text{ m/s}^2}$. So 8 rev/s gives the higher acceleration.

***P4.39** The satellite is in free fall. Its acceleration is due to gravity and is by effect a centripetal acceleration: $a_c = g$. So

$$\frac{v^2}{r} = g$$

Solving for the velocity,

$$v = \sqrt{rg} = \sqrt{(6{,}400+600)\left(10^3\ \text{m}\right)\left(8.21\ \text{m/s}^2\right)}$$
$$= \boxed{7.58\times10^3\ \text{m/s}}$$

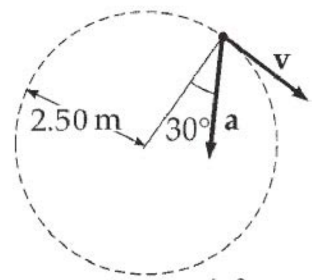
$$v = \frac{2\pi r}{T}$$

and

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(7{,}000\times10^3\ \text{m}\right)}{7.58\times10^3\ \text{m/s}} = \boxed{5.80\times10^3\ \text{s}}$$
$$T = 5.80\times10^3\ \text{s}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 96.7\ \text{min}$$

Section 4.5 Tangential and Radial Acceleration

P4.40 From the given magnitude and direction of the acceleration we can find both the centripetal and the tangential components. From the centripetal acceleration and radius we can find the speed in part (b). $r = 2.50$ m, $a = 15.0$ m/s^2.



$a = 15.0\ \text{m/s}^2$

ANS. FIG. P4.40

(a) The acceleration has an inward radial component:

$$a_c = a\cos 30.0° = \left(15.0\ \text{m/s}^2\right)\left(\cos 30°\right)$$
$$= \boxed{13.0\ \text{m/s}^2}$$

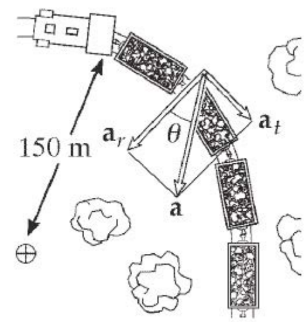
(b) The speed at the instant shown can be found by using

$$a_c = \frac{v^2}{r}$$
$$v^2 = ra_c = 2.50\ \text{m}\left(13.0\ \text{m/s}\right)$$
$$= 32.5\ \text{m}^2/\text{s}^2$$
$$v = \sqrt{32.5}\ \text{m/s} = \boxed{5.70\ \text{m/s}}$$

(c) $a^2 = a_t^2 + a_r^2$

so $a_t = \sqrt{a^2 - a_r^2} = \sqrt{\left(15.0\ \text{m/s}^2\right)^2 - \left(13.0\ \text{m/s}^2\right)^2} = \boxed{7.50\ \text{m/s}^2}$

P4.41 Since the train is changing both its speed and direction, the acceleration vector will be the vector sum of the tangential and radial acceleration components. The tangential acceleration can be found from the changing speed and elapsed time, while the radial acceleration can be found from the radius of curvature and the train's speed.



ANS. FIG. P4.41

First, let's convert the speed units from km/h to m/s:

$$v_i = 90.0 \text{ km/h} = (90.0 \text{ km/h})(10^3 \text{ m/km})(1 \text{ h}/3600 \text{ s})$$
$$= 25.0 \text{ m/s}$$
$$v_f = 50.0 \text{ km/h} = (50.0 \text{ km/h})(10^3 \text{ m/km})(1 \text{ h}/3600 \text{ s})$$
$$= 13.9 \text{ m/s}$$

The tangential acceleration and radial acceleration are, respectively,

$$a_t = \frac{\Delta v}{\Delta t} = \frac{13.9 \text{ m/s} - 25.0 \text{ m/s}}{15.0 \text{ s}} = -0.741 \text{ m/s}^2 \quad \text{(backward)}$$

and $$a_r = \frac{v^2}{r} = \frac{(13.9 \text{ m/s})^2}{150 \text{ m}} = 1.29 \text{ m/s} \quad \text{(inward)}$$

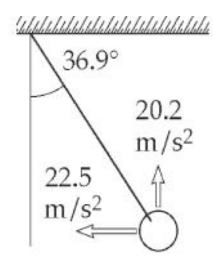
so $$a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(1.29 \text{ m/s}^2\right)^2 + \left(-0.741 \text{ m/s}^2\right)^2} = 1.48 \text{ m/s}^2$$

at an angle of

$$\tan^{-1}\left(\frac{|a_t|}{a_c}\right) = \tan^{-1}\left(\frac{0.741 \text{ m/s}^2}{1.29 \text{ m/s}^2}\right) = 29.9°$$

therefore, $\vec{a} = \boxed{1.48 \text{ m/s}^2 \text{ inward and } 29.9° \text{ backward}}$

P4.42 (a) See ANS. FIG. P4.42.



ANS. FIG. P4.42

(b) The components of the 20.2 m/s^2 and the 22.5 m/s^2 accelerations along the rope together constitute the centripetal acceleration:

$$a_c = \left(22.5 \text{ m/s}^2\right) \cos\left(90.0° - 36.9°\right)$$
$$+ \left(20.2 \text{ m/s}^2\right) \cos 36.9° = \boxed{29.7 \text{ m/s}^2}$$

(c) $a_c = \dfrac{v^2}{r}$ so $v = \sqrt{a_c r} = \sqrt{29.7 \text{ m/s}^2 (1.50 \text{ m})} = 6.67 \text{ m/s}$ tangent to the circle.

P4.43 The particle's centripetal acceleration is $v^2/r = (3\text{ m/s})^2/2\text{ m} = 4.50\text{ m/s}^2$. The total acceleration magnitude can be larger than or equal to this, but not smaller.

(a) $\boxed{\text{Yes. The particle can be either speeding up or slowing down, with a tangential component of acceleration of magnitude } \sqrt{6^2 - 4.5^2} = 3.97\text{ m/s.}}$

(b) $\boxed{\text{No. The magnitude of the acceleration cannot be less than } v^2/r = 4.5\text{ m/s}^2.}$

Section 4.6 Relative Velocity and Relative Acceleration

***P4.44** The westward speed of the airplane is the horizontal component of its velocity vector, and the northward speed of the wind is the vertical component of its velocity vector, which has magnitude and direction given by

$$v = \sqrt{(150\text{ km/h})^2 + (30.0\text{ km/h})^2} = \boxed{153\text{ km/h}}$$

$$\theta = \tan^{-1}\left(\frac{30.0\text{ km/h}}{150\text{ km/h}}\right) = \boxed{11.3°\text{ north of west}}$$

P4.45 The airplane (AP) travels through the air (W) that can move relative to the ground (G). The airplane is to make a displacement of 750 km north. Treat north as positive y and west as positive x.

(a) The wind (W) is blowing at 35.0 km/h, south. The northern component of the airplane's velocity relative to the ground is

$$\begin{aligned}(v_{AP,G})_y &= (v_{AP,W})_y + (v_{W,G})_y = 630\text{ km/h} - 35.0\text{ km/h} \\ &= 595\text{ km/h}\end{aligned}$$

We can find the time interval the airplane takes to travel 750 km north:

$$\boxed{\begin{aligned}&\Delta y = \left(v_{AP,G}\right)_y \Delta t \rightarrow \\ &\Delta t = \frac{\Delta y}{\left(v_{AP,G}\right)_y} = \frac{750\text{ km}}{595\text{ km/h}} = 1.26\text{ h}\end{aligned}}$$

(b) The wind (W) is blowing at 35.0 km/h, north. The northern component of the airplane's velocity relative to the ground is

$$\begin{aligned}(v_{AP,G})_y &= (v_{AP,W})_y + (v_{W,G})_y = 630\ \text{km/h} + 35.0\ \text{km/h} \\ &= 665\ \text{km/h}\end{aligned}$$

We can find the time interval the airplane takes to travel 750 km north:

$$\boxed{\Delta t = \frac{\Delta y}{\left(v_{AP,G}\right)_y} = \frac{750\ \text{km}}{665\ \text{km/h}} = 1.13\ \text{h}}$$

(c) Now, the wind (W) is blowing at 35.0 km/h, east. The airplane must travel directly north to reach its destination, so it must head somewhat west and north so that the east component of the wind's velocity is cancelled by the airplane's west component of velocity. If the airplane heads at an angle θ measured west of north, then

$$\begin{aligned}(v_{AP,G})_x &= (v_{AP,W})_x + (v_{W,G})_x \\ &= (630\ \text{km/h})\sin\theta + (-35.0\ \text{km/h}) = 0\end{aligned}$$

$$\sin\theta = 35.0/630 \to \theta = 3.18°$$

The northern component of the airplane's velocity relative to the ground is

$$\begin{aligned}(v_{AP,G})_y &= (v_{AP,W})_y + (v_{W,G})_y = (630\ \text{km/h})\cos 3.18° + 0 \\ &= 629\ \text{km/h}\end{aligned}$$

We can find the time interval the airplane takes to travel 750 km north:

$$\boxed{\Delta t = \frac{\Delta y}{\left(v_{AP,G}\right)_y} = \frac{750\ \text{km}}{629\ \text{km/h}} = 1.19\ \text{h}}$$

P4.46 Consider the direction the first beltway (B1) moves to be the positive direction. The first beltway moves relative to the ground (G) with velocity $v_{B1,G} = v_1$.

(a) The woman's velocity relative to the ground is $v_{WG} = v_{W,B1} + v_{B1,G} = v_1 + 0 = v_1$. The time interval required for the woman to travel distance L relative to the ground is

$$\boxed{\Delta t_{woman} = \frac{L}{v_1}}$$

(b) The man's (M) velocity relative to the ground is $v_{MG} = v_{M,B1} + v_{B1,G} = v_2 + v_1$. The time interval required for the man to travel distance L relative to the ground is

$$\boxed{\Delta t_{man} = \frac{L}{v_1 + v_2}}$$

(c) The second beltway (B2) moves in the negative direction; its velocity is $v_{B2,G} = -v_1$, and the child (C) rides on the second beltway; his velocity relative to the ground is

$$v_{CG} = v_{C,B2} + v_{B2,G} = 0 - v_1 = -v_1$$

The man's velocity relative to the child is

$$v_{MC} = v_{M,B1} + v_{B1,G} + v_{G,B2} + v_{B2,C}$$
$$v_{MC} = v_{M,B1} + v_{B1,G} - v_{B2,G} - v_{C,B2}$$
$$v_{MC} = v_2 + v_1 - (-v_1) + 0 = v_1 + 2\,v_2$$

so, the time interval required for the man to travel distance L relative to the child is

$$\boxed{\Delta t_{man} = \frac{L}{v_1 + 2v_2}}$$

P4.47 Both police car (P) and motorist (M) move relative to the ground (G). Treating west as the positive direction, the components of their velocities (in km/h) are:

$$v_{PG} = 95.0\text{ km/h (west)} \qquad v_{PG} = 80\text{ km/h (west)}$$

(a) $v_{MP} = v_{MG} + v_{GP} = v_{MG} - v_{PG} = 80.0\text{ km/h} - 95.0\text{ km/h} = -15.0$
$= \boxed{15.0\text{ km/h, east}}$

(b) $v_{PM} = -v_{MP} + \boxed{15.0\text{ km/h, west}}$

(c) Relative to the motorist, the police car approaches at 15.0 km/h:

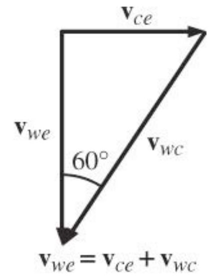
$$d = v\Delta t$$
$$\rightarrow \Delta t = \frac{d}{v} = \frac{0.250\text{ km}}{15.0\text{ km/h}} = \left(1.67\times 10^{-2}\text{ h}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = \boxed{60.0\text{ s}}$$

We define the following velocity vectors:

$\vec{\mathbf{v}}_{ce}$ = the velocity of the car relative to the Earth

$\vec{\mathbf{v}}_{wc}$ = the velocity of the water relative to the car

$\vec{\mathbf{v}}_{we}$ = the velocity of the water relative to the Earth



ANS. FIG. P4.48

These velocities are related as shown in ANS. FIG. P4.48

(a) Since $\vec{\mathbf{v}}_{we}$ is vertical, $v_{wc}\sin 60.0° = v_{ce} = 50.0$ km/h or

$$\vec{\mathbf{v}}_{wc} = \boxed{57.7\text{ km/h at }60.0°\text{ west of vertical}}$$

(b) Since $\vec{\mathbf{v}}_{ce}$ has zero vertical component,

$$v_{we} = v_{wc}\cos 60.0° = (57.7\text{ km/h})\cos 60.0°$$
$$= \boxed{28.9\text{ km/h downward}}$$

P4.49 (a) To an observer at rest in the train car, the bolt accelerates downward and toward the rear of the train.

$$a = \sqrt{(2.50\text{ m/s})^2 + (9.80\text{ m/s})^2} = \boxed{10.1\text{ m/s}^2}$$

$$\tan\theta = \frac{2.50\text{ m/s}^2}{9.80\text{ m/s}^2} = 0.255$$

$$\theta = \boxed{14.3°\text{ to the south from the vertical}}$$

To this observer, the bolt moves as if it were in a gravitational field of 9.80 m/s^2 down + 2.50 m/s^2 south.

(b) $a = \boxed{9.80\text{ m/s}^2\text{ vertically downward}}$

(c) If it is at rest relative to the ceiling at release, the bolt moves on a straight line download and southward at 14.3 degrees from the vertical.

(d) The bolt moves on a parabola with a vertical axis.

P4.50 The total time interval in the river is the longer time spent swimming upstream (against the current) plus the shorter time swimming downstream (with the current). For each part, we will use the basic equation $t = d/v$, where v is the speed of the student relative to the shore.

(a) Total time = time upstream plus time downstream:

$$t_{up} = \frac{1\,000 \text{ m}}{1.20 \text{ m/s} - 0.500 \text{ m/s}} = 1.43 \times 10^3 \text{ s}$$

$$t_{down} = \frac{1\,000 \text{ m}}{1.20 \text{ m/s} + 0.500 \text{ m/s}} = 588 \text{ s}$$

Therefore, $t_{total} = 1.43 \times 10^3 \text{ s} + 588 \text{ s} = \boxed{2.02 \times 10^3 \text{ s}}$.

(b) Total time in still water $t = \frac{d}{v} = \frac{2\,000}{1.20} = \boxed{1.67 \times 10^3 \text{ s}}$.

(c) Swimming with the current does not compensate for the time lost swimming against the current.

P4.51 The student must swim faster than the current to travel upstream.

(a) The speed of the student relative to shore is $v_{up} = c - v$ while swimming upstream (against the current), and $v_{down} = c + v$ while swimming downstream (with the current).

Note, The student must swim faster than the current to travel upstream. The time interval required to travel distance d upstream is then

$$\Delta t_{up} = \frac{d}{v_{up}} = \frac{d}{c - v}$$

and the time interval required to swim the same distance d downstream is

$$\Delta t_{down} = \frac{d}{v_{down}} = \frac{d}{c + v}$$

The time interval for the round trip is therefore

$$\Delta t = \Delta t_{up} + \Delta t_{down} = \frac{d}{c - v} + \frac{d}{c + v} = d\frac{(c + v) + (c - v)}{(c - v)(c + v)}$$

$$\boxed{\Delta t = \frac{2dc}{c^2 - v^2}}$$

(b) In still water, $v = 0$, so $v_{up} = v_{down} = c$; the equation for the time interval for the complete trip reduces to

$$\boxed{\Delta t = \frac{2d}{c}}$$

(c) The equation for the time interval for the complete trip can be written as

$$\Delta t = \frac{2dc}{c^2 - v^2} = \frac{2d}{c\left(1 - \frac{v^2}{c^2}\right)}$$

Because the denominator is always smaller than c, $\boxed{\text{swimming with and against the current is always longer than in still water}}$.

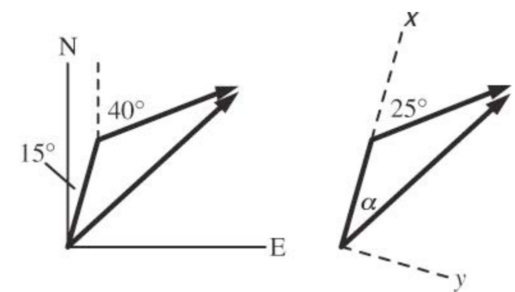
P4.52 Choose the x axis along the 20-km distance. The y components of the displacements of the ship and the speedboat must agree:

$$(26\text{ km/h})t\sin(40.0° - 15.0°) = (50\text{ km/h})t\sin\alpha$$

$$\alpha = \sin^{-1}\left(\frac{11.0\text{ km/h}}{50\text{ km/h}}\right) = 12.7°$$

The speedboat should head

$$15.0° + 12.7° = \boxed{27.7°\text{ E of N}}$$



ANS. FIG. P4.52

P4.53 Identify the student as the S' observer and the professor as the S observer. For the initial motion in S', we have

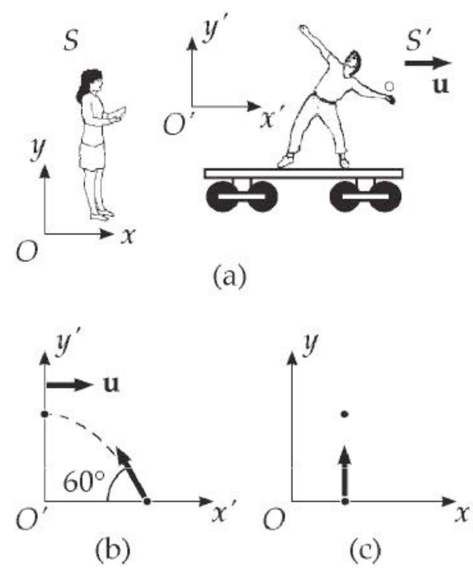
$$\frac{v'_y}{v'_x} = \tan 60.0° = \sqrt{3}$$

Let u represent the speed of S' relative to S. Then because there is no x motion in S, we can write $v_x = v'_x + u = 0$ so that $v'_x = -u = -10.0$ m/s. Hence the ball is thrown backwards in S'. Then,

$$v'_y = v'_y = \sqrt{3}\left|v'_x\right| = 10.0\sqrt{3}\text{ m/s}$$

Using $v_y^2 = 2gh$ we find

$$h = \frac{\left(10.0\sqrt{3}\text{ m/s}\right)^2}{2\left(9.80\text{ m/s}^2\right)} = \boxed{15.3\text{ m}}$$



ANS. FIG. P4.53

The motion of the ball as seen by the student in S' is shown in ANS. FIG. P4.53(b). The view of the professor in S is shown in ANS. FIG. P4.53(c).

P4.54 (a) For the boy to catch the can at the same location on the truck bed, he must throw it $\boxed{\text{straight up, at } 0° \text{ to the vertical}}$.

(b) We find the time of flight of the can by considering its horizontal motion:

$$16.0 \text{ m} = (9.50 \text{ m/s})t + 0 \rightarrow t = 1.68 \text{ s}$$

For the free fall of the can, $y_f = y_i + v_{yi}t - \frac{1}{2}a_y t^2$:

$$0 = 0 + v_{yi}(1.68 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(1.68 \text{ s})^2$$

which gives $v_{yi} = \boxed{8.25 \text{ m/s}}$.

(c) The boy sees the can always over his head, traveling in $\boxed{\text{a straight up and down line}}$.

(d) The ground observer sees the can move as a projectile traveling in $\boxed{\text{a symmetric parabola opening downward}}$.

(e) Its initial velocity is

$$\sqrt{(9.50 \text{ m/s})^2 + (8.25 \text{ m/s})^2} = \boxed{12.6 \text{ m/s north}}$$

at an angle of

$$\tan^{-1}\left(\frac{8.25 \text{ m/s}}{9.50 \text{ m/s}}\right) = \boxed{41.0° \text{ above the horizontal}}$$

Additional Problems

***P4.55** After the string breaks the ball is a projectile, and reaches the ground at time t:

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2$$

$$-1.20 \text{ m} = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2$$

so t = 0.495 s. Its constant horizontal speed is

$$v_x = \frac{x}{t} = \frac{2.00\ \text{m}}{0.495\ \text{s}} = 4.04\ \text{m/s}$$

so before the string breaks

$$a_c = \frac{v_x^2}{r} = \frac{(4.04\ \text{m/s})^2}{0.300\ \text{m}} = \boxed{54.4\ \text{m/s}^2}$$

***P4.56** The maximum height of the ball is given by Equation 4.12:

$$h = \frac{v_i^2 \sin^2\theta_i}{2g}$$

Equation 4.13 then gives the horizontal range of the ball:

$$R = \frac{v_i^2 \sin 2\theta_i}{g} = \frac{2v_i^2 \sin\theta_i \cos\theta_i}{g}$$

If $h = \frac{R}{6}$, Equation 4.12 yields

$$v_i \sin\theta_i = \sqrt{\frac{gR}{3}} \qquad \textbf{[1]}$$

Substituting equation [1] above into Equation 4.13 gives

$$R = \frac{2\left(\sqrt{gR/3}\right)v_i \cos\theta_i}{g}$$

which reduces to

$$v_i \cos\theta_i = \frac{1}{2}\sqrt{3gR} \qquad \textbf{[2]}$$

(a) From $v_{yf} = v_{yi} + a_y t$, the time to reach the peak of the path $\left(\text{where } v_{yf} = 0\right)$ is found to be

$$t_{peak} = v_i \sin\theta_i / g$$

Using equation [1], this gives

$$t_{peak} = \sqrt{\frac{R}{3g}}$$

The total time of the ball's flight is then

$$\boxed{t_{flight} = 2t_{peak} = 2\sqrt{\frac{R}{3g}}}$$

(b) At the path's peak, the ball moves horizontally with speed

$$v_{peak} = v_{xi} = v_i \cos\theta_i$$

Using equation [2], this becomes

$$v_{peak} = \boxed{\frac{1}{2}\sqrt{3gR}}$$

(c) The initial vertical component of velocity is $v_{yi} = v_i \sin\theta_i$. From equation [1],

$$v_{yi} = \boxed{\sqrt{\frac{gR}{3}}}$$

(d) Squaring equations [1] and [2] and adding the results,

$$v_i^2\left(\sin^2\theta_i + \cos^2\theta_i\right) = \frac{gR}{3} + \frac{3gR}{4} = \frac{13gR}{12}$$

Thus, the initial speed is

$$v_i = \boxed{\sqrt{\frac{13gR}{12}}}$$

(e) Dividing equation [1] by [2] yields

$$\tan\theta_i = \frac{v_i \sin\theta_i}{v_i \cos\theta_i} = \left[\frac{\left(\sqrt{gR/3}\right)}{\left(\frac{1}{2}\sqrt{3gR}\right)}\right] = \frac{2}{3}$$

Therefore,

$$\theta_i = \tan^{-1}\left(\frac{2}{3}\right) = \boxed{33.7°}$$

(f) For a given initial speed, the projection angle yielding maximum peak height is $\theta_i = 90.0°$. With the speed found in (d), Equation 4.12 then yields

$$h_{max} = \frac{(13gR/12)\sin^2 90.0°}{2g} = \boxed{\frac{13}{24}R}$$

(g) For a given initial speed, the projection angle yielding maximum range is $\theta_i = 45.0°$. With the speed found in (d), Equation 4.13 then gives

$$R_{max} = \frac{(13gR/12)\sin 90.0°}{g} = \boxed{\frac{13}{12}R}$$

P4.57 We choose positive y to be in the downward direction. The ball when released has velocity components $v_{xi} = v$ and $v_{yi} = 0$, where v is the speed of the man. We can find the length of the time interval the ball takes to fall the distance h using

$$\Delta y = \frac{1}{2}a_y(\Delta t)^2 = \frac{1}{2}g(\Delta t)^2 \to \Delta t = \sqrt{\frac{2h}{g}}$$

The horizontal displacement of the ball during this time interval is

$$\Delta x = v_{xi}\ \Delta t = v\sqrt{\frac{2h}{g}} = 7.00h$$

Solve for the speed:

$$v = \sqrt{\frac{49.0gh}{2}} = \sqrt{\frac{49.0(9.80\ \text{m/s}^2)h}{2}} = 15.5\sqrt{h}$$

where h is in m and v in m/s.

If we express the height as a function of speed, we have

$$h = (4.16 \times 10^{-2})v^2$$

where h is in m and v is in m/s.

For a normally proportioned adult, h is about 0.50 m, which would mean that $v = 15.5\sqrt{0.50} = 11$ m/s, which is about 39 km/h; no normal adult could walk "briskly" at that speed. If the speed were a realistic typical speed of 4 km/h, from our equation for h, we find that the height would be about 4 cm, much too low for a normal adult.

P4.58 (a) From $\vec{\mathbf{a}} = d\vec{\mathbf{v}}/dt$, we have

$$\int_i^f d\vec{\mathbf{v}} = \int_i^f \vec{\mathbf{a}}\,dt = \Delta\vec{\mathbf{v}}$$

Then

$$\vec{\mathbf{v}} - 5\hat{\mathbf{i}}\ \text{m/s} = \int_0^t 6\,t^{1/2}dt\,\hat{\mathbf{j}} = 6\left.\frac{t^{3/2}}{3/2}\right|_0^t \hat{\mathbf{j}} = 4\,t^{3/2}\,\hat{\mathbf{j}}\ \text{m/s}$$

so $\vec{\mathbf{v}} = \boxed{\left(5\,\hat{\mathbf{i}} + 4\,t^{3/2}\,\hat{\mathbf{j}}\right)\ \text{m/s}}$.

(b) From $\vec{\mathbf{v}} = d\vec{\mathbf{r}}/dt$, we have

$$\int_i^f d\vec{\mathbf{r}} = \int_i^f \vec{\mathbf{v}}\,dt = \Delta\vec{\mathbf{r}}$$

Then

$$\vec{\mathbf{r}} - 0 = \int_0^t \left(5\,\hat{\mathbf{i}} + 4\,t^{3/2}\,\hat{\mathbf{j}}\right)dt = \left(5t\,\hat{\mathbf{i}} + 4\frac{t^{5/2}}{5/2}\hat{\mathbf{j}}\right)\Bigg|_0^t$$

$$= \boxed{\left(5t\,\hat{\mathbf{i}} + 1.6\,t^{5/2}\,\hat{\mathbf{j}}\right)\text{ m}}$$

P4.59 (a) The speed at the top is

$$v_x = v_i \cos\theta_i = (143\text{ m/s})\cos 45° = \boxed{101\text{ m/s}}$$

(b) In free fall the plane reaches altitude given by

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

$$0 = \left(143\text{ m/s }\sin 45°\right)^2 + 2\left(-9.80\text{ m/s}^2\right)\left(y_f - 31\ 000\text{ ft}\right)$$

$$y_f = 31\ 000\text{ ft } + 522\text{ m}\left(\frac{3.28\text{ ft}}{1\text{ m}}\right) = \boxed{3.27\times10^4\text{ ft}}$$

(c) For the whole free-fall motion $v_{yf} = v_{yi} + a_y t$:

$$-101\text{ m/s} = +101\text{ m/s} - \left(9.80\text{ m/s}^2\right)t$$

$$t = \boxed{20.6\text{ s}}$$

P4.60 (a) The acceleration is that of gravity: $\boxed{9.80\text{ m/s}^2\text{, downward.}}$

(b) The horizontal component of the initial velocity is $v_{xi} = v_i \cos 40.0° = 0.766v_i$, and the time required for the ball to move 10.0 m horizontally is

$$t = \frac{\Delta x}{v_{xi}} = \frac{10.0\text{ m}}{0.766\ v_i} = \frac{13.1\text{ m}}{v_i}$$

At this time, the vertical displacement of the ball must be

$$\Delta y = y_f - y_i = 3.05\text{ m} - 2.00\text{ m} = 1.05\text{ m}$$

Thus, $\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$ becomes

$$1.05\text{ m} = \left(\cancel{v_i}\ \sin 40.0°\right)\frac{13.1\text{ m}}{\cancel{v_i}} + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)\frac{(13.1\text{ m})^2}{v_i^2}$$

or $$1.05\text{ m} = 8.39\text{ m} - \frac{835\text{ m}^3/\text{s}^2}{v_i^2}$$

which yields

$$v_i = \sqrt{\frac{835\ \text{m}^3/\text{s}^2}{8.39\ \text{m} - 1.05\ \text{m}}} = \boxed{10.7\ \text{m/s}}$$

P4.61 Both Lisa and Jill start from rest. Their accelerations are

$$\vec{\mathbf{a}}_L = (3.00\,\hat{\mathbf{i}} - 2.00\,\hat{\mathbf{j}})\ \text{m/s}^2$$

$$\vec{\mathbf{a}}_J = (1.00\,\hat{\mathbf{i}} + 3.00\,\hat{\mathbf{j}})\ \text{m/s}^2$$

Integrating these, and knowing that they start from rest, we find their velocities:

$$\vec{\mathbf{v}}_L = (3.00t\,\hat{\mathbf{i}} - 2.00t\,\hat{\mathbf{j}})\ \text{m/s}$$

$$\vec{\mathbf{v}}_J = (1.00t\,\hat{\mathbf{i}} + 3.00t\,\hat{\mathbf{j}})\ \text{m/s}$$

Integrating again, and knowing that they start from the origin, we find their positions:

$$\vec{\mathbf{r}}_L = (1.50t^2\,\hat{\mathbf{i}} - 1.00t^2\,\hat{\mathbf{j}})\ \text{m}$$

$$\vec{\mathbf{r}}_J = (0.50\,t^2\,\hat{\mathbf{i}} + 1.50t^2\,\hat{\mathbf{j}})\ \text{m}$$

All of the above are with respect to the ground (G).

(a) In general, Lisa's velocity with respect to Jill is

$$\vec{\mathbf{v}}_{LJ} = \vec{\mathbf{v}}_{LG} + \vec{\mathbf{v}}_{GJ} = \vec{\mathbf{v}}_{LG} - \vec{\mathbf{v}}_{JG}$$
$$\vec{\mathbf{v}}_{LJ} = \vec{\mathbf{v}}_L - \vec{\mathbf{v}}_J = (3.00t\,\hat{\mathbf{i}} - 2.00t\,\hat{\mathbf{j}}) - (1.00t\,\hat{\mathbf{i}} + 3.00t\,\hat{\mathbf{j}})$$
$$\vec{\mathbf{v}}_{LJ} = (2.00t\,\hat{\mathbf{i}} - 5.00t\,\hat{\mathbf{j}})$$

When $t = 5.00$ s, $\vec{\mathbf{v}}_{LJ} = (10.0\,\hat{\mathbf{i}} - 25.0\,\hat{\mathbf{j}})$ m/s, so the speed (magnitude) is

$$\boxed{v = \sqrt{(10.0)^2 + (25.0)^2} = 26.9\ \text{m/s}}$$

(b) In general, Lisa's position with respect to Jill is

$$\vec{\mathbf{r}}_{LJ} = \vec{\mathbf{r}}_L - \vec{\mathbf{r}}_J = (1.50t^2\,\hat{\mathbf{i}} - 1.00t^2\,\hat{\mathbf{j}}) - (0.50t^2\,\hat{\mathbf{i}} + 1.50t^2\,\hat{\mathbf{j}})$$
$$\vec{\mathbf{r}}_{LJ} = (1.00t^2\,\hat{\mathbf{i}} - 2.50t^2\,\hat{\mathbf{j}})$$

When $t = 5.00$ s, $\vec{\mathbf{r}}_{LJ} = (25.0\,\hat{\mathbf{i}} - 62.5\,\hat{\mathbf{j}})$ m, and their distance apart is

$$\boxed{d = \sqrt{(25.0\ \text{m})^2 + (62.5\ \text{m})^2} = 67.3\ \text{m}}$$

(c) In general, Lisa's acceleration with respect to Jill is

$$\vec{\mathbf{a}}_{LJ} = \vec{\mathbf{a}}_L - \vec{\mathbf{a}}_J = (3.00\,\hat{\mathbf{i}} - 2.00\,\hat{\mathbf{j}}) - (1.00\,\hat{\mathbf{i}} + 3.00\,\hat{\mathbf{j}})$$

$$\vec{\mathbf{a}}_{LJ} = \boxed{(2.00\,\hat{\mathbf{i}} - 5.00\,\hat{\mathbf{j}})\text{ m/s}^2}$$

P4.62 (a) The stone's initial velocity components (at $t = 0$) are v_{xi} and $v_{yi} = 0$, and the stone falls through a vertical displacement $\Delta y = -h$. We find the time t when the stone strikes the ground using

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2 \rightarrow -h = 0 - \frac{1}{2}gt^2 \rightarrow \boxed{t = \sqrt{\frac{2h}{g}}}$$

(b) To find the stone's initial horizontal component of velocity, we know at the above time t, the stone's horizontal displacement is $\Delta x = d$:

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2 \rightarrow d = v_{xi}t \rightarrow v_{ox} = \frac{d}{t} \rightarrow \boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}$$

(c) The vertical component of velocity at time t is

$$v_{yf} = v_{yi} + a_y t = 0 - gt \rightarrow v_{yf} = -g\sqrt{\frac{2h}{g}} \rightarrow v_{yf} = -\sqrt{2gh}$$

and the horizontal component does not change; therefore, the speed of the stone as it reaches the ocean is

$$\boxed{v_f = \sqrt{{v_{xf}}^2 + {v_{yf}}^2} = \sqrt{\left(\frac{d^2 g}{2h}\right) + (2gh)}}$$

(d) From above,

$$\theta_f = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-\sqrt{2gh}}{d\sqrt{\dfrac{g}{2h}}}\right)$$

$$\theta_f = -\tan^{-1}\left(\frac{2h}{d}\right)$$

which means the velocity points below the horizontal by angle

$$\boxed{\theta_f = \tan^{-1}\left(\frac{2h}{d}\right)}$$

P4.63 We use a fixed coordinate system that, viewed from above, has its positive x axis passing through point A when the flea jumps, and its positive y axis 90° counterclockwise from its x axis. Its positive z axis is upward. The turntable rotates clockwise. At $t = 0$, the flea jumps straight up relative to the turntable, but the turntable is spinning, so the flea has both horizontal and vertical components of velocity relative to the fixed coordinate axes. Because the turntable is spinning clockwise, the horizontal velocity of the flea is in the negative y directon:

$$v_y = \left(-33.3\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi(10.0\text{ cm})}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = -34.9\text{ cm/s}$$

The vertical motion of the flea is independent of its horizontal motion. The time interval the flea takes to rise to a height h of 5.00 cm is the same time interval the flea takes to drop back to the turntable. We find the interval to drop using

$$z_f = z_i + v_{zi}t + \frac{1}{2}a_z t^2 \rightarrow 0 = h - \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2h}{g}}$$

where h is in m and t in s. Substituting, we find

$$t = \sqrt{\frac{2(0.050\ 0\text{ m})}{9.80\text{ m/s}^2}} = 0.101\text{ s}$$

The total time interval for the flea to leave the surface of the turntable and return is twice this: $\Delta t = 0.202$ s.

(a) Find the clockwise angle the turntable rotates through in the time interval Δt:

$$\begin{aligned}\Delta\theta &= \left(\frac{33.3\text{ rev}}{\text{min}}\right)(0.202\text{ s}) \\ &= \left[\left(\frac{33.3\text{ rev}}{\text{min}}\right)\left(\frac{360°}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right](0.202\text{ s}) \\ &= 40.4°\end{aligned}$$

Point A lies 10.0 cm from the origin. When the flea jumps, the line passing from the origin to point A coincides with the positive x axis, but when the flea lands, the line makes an angle of −40.4° with the positive x axis:

$$\vec{\mathbf{r}}_A = [10.0\cos(-40.4°)]\hat{\mathbf{i}} + [10.0\sin(-40.4°)]\hat{\mathbf{j}}$$

$$\vec{\mathbf{r}}_A = \boxed{(7.61\hat{\mathbf{i}} - 6.48\hat{\mathbf{j}})\text{cm}}$$

(b) During this time interval, the flea goes through a horizontal y displacement

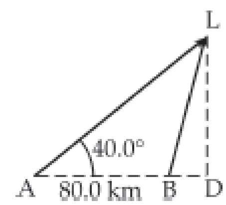
$$\Delta y = v_y \Delta t = (-34.9 \text{ cm/s})(0.202 \text{ s}) = -7.05 \text{ cm}.$$

The flea has no motion parallel to the x axis; therefore, the position of point B where the flea lands is

$$\vec{\mathbf{r}}_B = \boxed{(10.0\hat{\mathbf{i}} - 7.05\hat{\mathbf{j}}) \text{ cm}}$$

***P4.64** ANS. FIG. P4.64 shows the triangles ALB and ALD. To find the length $\overline{AL}$, we write

$$\overline{AL} = v_1 t = (90.0 \text{ km/h})(2.50 \text{ h}) = 225 \text{ km}$$



ANS. FIG. P4.64

To find the distance travelled by the second couple, we need to determine the length $\overline{BD}$:

$$\overline{BD} = \overline{AD} - \overline{AB}$$
$$= \overline{AL}\cos 40.0° - 80.0 \text{ km} = 92.4 \text{ km}$$

Then, from the triangle BLD in ANS. FIG. P4.64,

$$\overline{BL} = \sqrt{\left(\overline{BD}\right)^2 + \left(\overline{DL}\right)^2}$$
$$= \sqrt{(92.4 \text{ km})^2 + \left(\overline{AL}\sin 40.0°\right)^2} = 172 \text{ km}$$

Note that the law of cosines can also be used for the triangle ABL to solve for the length BD. Since Car 2 travels this distance in 2.50 h, its constant speed is

$$v_2 = \frac{172 \text{ km}}{2.5 \text{ h}} = \boxed{68.8 \text{ km/h}}$$

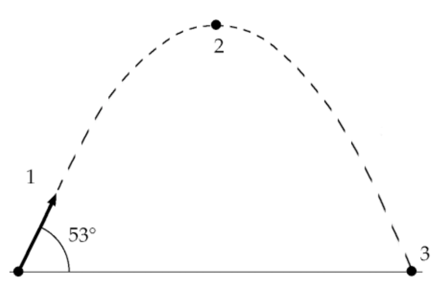
***P4.65** Consider the rocket's trajectory in 3 parts as shown in the diagram on the right. Our initial conditions give:

$$a_y = (30.0 \text{ m/s}^2)\sin 53.0° = 24.0 \text{ m/s}^2$$

$$a_x = (30.0 \text{ m/s}^2)\cos 53.0° = 18.1 \text{ m/s}^2$$

$$v_{yi} = (100 \text{ m/s})\sin 53.0° = 79.9 \text{ m/s}$$

$$v_{xi} = (100 \text{ m/s})\cos 53.0° = 60.2 \text{ m/s}$$



ANS. FIG. P4.65

The distances traveled during each phase of the motion are given in Table P4.65 below.

Path Part #1:

$$v_{yf} = v_{yi} + a_y t$$
$$= 79.9\text{ m/s} + \left(24.0\text{ m/s}^2\right)\left(3.00\text{ s}\right)$$
$$= 152\text{ m/s}$$
$$v_{xf} = v_{xi} + a_x t$$
$$= 60.2\text{ m/s} + \left(18.1\text{ m/s}^2\right)\left(3.00\text{ s}\right)$$
$$= 114\text{ m/s}$$
$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$$
$$= \left(79.9\text{ m/s}\right)\left(3.00\text{ s}\right) + \frac{1}{2}\left(24.0\text{ m/s}^2\right)\left(3.00\text{ s}\right)^2$$
$$= 347\text{ m}$$
$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$
$$= \left(60.2\text{ m/s}\right)\left(3.00\text{ s}\right) + \frac{1}{2}\left(18.1\text{ m/s}^2\right)\left(3.00\text{ s}\right)^2$$
$$= 262\text{ m}$$

Path Part #2:

Now $a_x = 0$, $a_y = -9.80\text{ m/s}^2$, $v_{xf} = v_{xi} = 114\text{ m/s}$, $v_{yi} = 152\text{ m/s}$, and $v_{yf} = 0$, so

$$v_{yf} = v_{yi} + a_y t$$
$$0 = 152\text{ m/s} - \left(9.80\text{ m/s}^2\right)t$$

which gives $t = 15.5$ s

$$\Delta x = v_{xf}t = \left(114\text{ m/s}\right)\left(15.5\text{ s}\right) = 1.77 \times 10^3\text{ m}$$

$$\Delta y = \left(152\text{ m/s}\right)\left(15.5\text{ s}\right) - \frac{1}{2}\left(9.80\text{ m/s}^2\right)\left(15.5\text{ s}\right)^2 = 1.17 \times 10^3\text{ m}$$

Path Path #3:

With $v_{yi} = 0$, $a_x = 0$, $a_y = -9.80\text{ m/s}^2$, and $v_{xf} = v_{xi} = 114\text{ m/s}$, then

$$\left(v_{yf}\right)^2 - \left(v_{yi}\right)^2 = 2a\Delta y$$
$$\left(v_{yf}\right)^2 - 0 = 2\left(-9.80\text{ m/s}^2\right)\left(-1.52 \times 10^3\text{ m}\right)$$

which gives $v_{yf} = -173\text{ m/s}$

We find the time from $v_{yf} = v_{yi} - gt$, which gives

$-173\text{ m/s} - 0 = -(9.80\text{ m/s}^2)t$, or $t = 17.6$ s

$\Delta x = v_{xf}t = 114(17.6) = 2.02 \times 10^3\text{ m}$

(a) $\Delta y(\text{max}) = \boxed{1.52 \times 10^3\text{ m}}$

(b) $t(\text{net}) = 3.00\text{ s} + 15.5\text{ s} + 17.6\text{ s} = \boxed{36.1\text{ s}}$

(c) $\Delta x(\text{net}) = 262\text{ m} + 1.77 \times 10^3\text{ m} + 2.02 \times 10^3\text{ m}$

$\Delta x(\text{net}) = \boxed{4.05 \times 10^3\text{ m}}$

	Path Part		
	#1	#2	#3
a_y	24.0	–9.80	–9.80
a_x	18.1	0.0	0.0
v_{yf}	152	0.0	–173
v_{xf}	114	114	114
v_{yi}	79.9	152	0.0
v_{xi}	60.2	114	114
Δy	347	1.17×10^3	-1.52×10^3
Δx	262	1.77×10^3	2.02×10^3
t	3.00	15.5	17.6

Table P4.65

***P4.66** Take the origin at the mouth of the cannon. We have $x_f = v_{xi}t$, which gives

$$2\,000\text{ m} = (1\,000\text{ m/s})\cos\theta_i t$$

Therefore,

$$t = \frac{2.00\text{ s}}{\cos\theta_i}$$

From $y_f = v_{yi}t + \frac{1}{2}a_y t^2$:

$$800\text{ m} = (1\,000\text{ m/s})\sin\theta_i t + \frac{1}{2}(-9.80\text{ m/s}^2)t^2$$

$$800\text{ m} = (1\,000\text{ m/s})\sin\theta_i\left(\frac{2.00\text{ s}}{\cos\theta_i}\right) - \frac{1}{2}(9.80\text{ m/s}^2)\left(\frac{2.00\text{ s}}{\cos\theta_i}\right)^2$$

$$800\text{ m}(\cos^2\theta_i) = 2\,000\text{ m}(\sin\theta_i\cos\theta_i) - 19.6\text{ m}$$

$$19.6\text{ m} + 800\text{ m}(\cos^2\theta_i) = 2\,000\text{ m}\sqrt{1-\cos^2\theta_i}(\cos\theta_i)$$

$$384 + (31\ 360)\cos^2\theta_i + (640\ 000)\cos^4\theta_i$$
$$= (4\ 000\ 000)\cos^2\theta_i - (4\ 000\ 000)\cos^4\theta_i$$

$$4\ 640\ 000\cos^4\theta_i - 3\ 968\ 640\cos^2\theta_i + 384 = 0$$

$$\cos^2\theta_i = \frac{3\ 968\ 640 \pm \sqrt{(3\ 968\ 640)^2 - 4(4\ 640\ 000)(384)}}{9\ 280\ 000}$$

$$\cos\theta_i = 0.925 \text{ or } \cos\theta_i = 0.009\ 84$$

$$\theta_i = \boxed{22.4° \text{ or } 89.4°} \qquad \text{(Both solutions are valid.)}$$

P4.67 Given the initial velocity, we can calculate the height change of the ball as it moves 130 m horizontally. So this is what we do, expecting the answer to be inconsistent with grazing the top of the bleachers. We assume the ball field is horizontal. We think of the ball as a particle in free fall (moving with constant acceleration) between the point just after it leaves the bat until it crosses above the cheap seats.

The initial components of velocity are

$$v_{xi} = v_i\cos\theta = 41.7\cos 35.0° = 34.2\text{ m/s}$$
$$v_{yi} = v_i\sin\theta = 41.7\sin 35.0° = 23.9\text{ m/s}$$

We find the time when the ball has traveled through a horizontal displacement of 130 m:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \rightarrow x_f = x_i + v_{xi}t \rightarrow t = (x_f - x_i)/v_{xi}$$

$$t = \frac{130\text{ m} - 0}{34.2\text{ m/s}} = 3.80\text{ s}$$

Now we find the vertical position of the ball at this time:

$$y_f = y_i + v_{yi}t + \frac{1}{2}at^2 = 0 + v_{yi}t - \frac{1}{2}t^2$$

$$y_f = (23.9 \text{ m/s})(3.80 \text{ s}) - (4.90 \text{ m/s}^2)(3.80 \text{ s})^2 = 20.1 \text{ m}$$

$\boxed{\text{The ball would not be high enough to have cleared the 24.0-m-high bleachers.}}$

P4.68 At any time *t*, the two drops have identical *y* coordinates. The distance between the two drops is then just twice the magnitude of the horizontal displacement either drop has undergone. Therefore,

$$d = 2|x(t)| = 2(v_{xi}t) = 2(v_i \cos\theta_i)t = \boxed{2v_i t \cos\theta_i}$$

P4.69 (a) The Moon's gravitational acceleration is the probe's centripetal acceleration: (For the Moon's radius, see end papers of text.)

$$a = \frac{v^2}{r}$$

$$\frac{1}{6}\left(9.80 \text{ m/s}^2\right) = \frac{v^2}{1.74 \times 10^6 \text{ m}}$$

$$v = \sqrt{2.84 \times 10^6 \text{ m}^2/\text{s}^2} = \boxed{1.69 \text{ km/s}}$$

(b) The time interval can be found from

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(1.74 \times 10^6 \text{ m}\right)}{1.69 \times 10^3 \text{ m/s}} = 6.47 \times 10^3 \text{ s} = \boxed{1.80 \text{ h}}$$

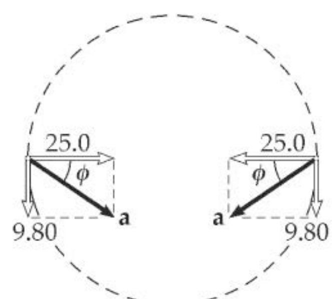
P4.70 (a) The length of the cord is given as $r = 1.00$ m. At the positions with $\theta = 90.0°$ and 270°,

$$a_c = \frac{v^2}{r} = \frac{(5.00 \text{ m/s})^2}{1.00 \text{ m}} = \boxed{25.0 \text{ m/s}^2}$$

(b) The tangential acceleration is only the acceleration due to gravity,

$$a_t = g = \boxed{9.80 \text{ m/s}^2}$$

(c) See ANS. FIG. P4.70.



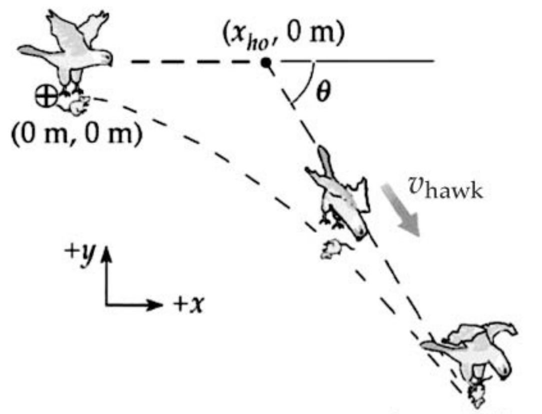
ANS. FIG. P4.70

(d) The magnitude and direction of the total acceleration at these positions is given by

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(25.0\ \text{m/s}^2\right)^2 + \left(9.80\ \text{m/s}^2\right)^2} = \boxed{26.8\ \text{m/s}^2}$$

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{9.80\ \text{m/s}^2}{25.0\ \text{m/s}^2}\right) = \boxed{21.4°}$$

P4.71 We know the distance that the mouse and hawk move down, but to find the diving speed of the hawk, we must know the time interval of descent, so we will solve part (c) first. If the hawk and mouse both maintain their original horizontal velocity of 10 m/s (as the mouse should without air resistance), then the hawk only needs to think about diving straight down, but to a ground-based observer, the path will appear to be a straight line angled less than 90° below horizontal.



ANS. FIG. P4.71

We begin with the simple calculation of the free-fall time interval for the mouse.

(c) The mouse falls a total vertical distance y = 200 m – 3.00 m = 197 m. The time interval of fall is found from (with $v_{yi} = 0$)

$$y = v_{yi}t - \frac{1}{2}gt^2 \quad \rightarrow \quad t = \sqrt{\frac{2(197\ \text{m})}{9.80\ \text{m/s}^2}} = \boxed{6.34\ \text{s}}$$

(a) To find the diving speed of the hawk, we must first calculate the total distance covered from the vertical and horizontal components. We already know the vertical distance y; we just need the horizontal distance during the same time interval (minus the 2.00-s late start).

$$x = v_{xi}(t - 2.00\ \text{s}) = (10.0\ \text{m/s})(6.34\ \text{s} - 2.00\ \text{s}) = 43.4\ \text{m}$$

The total distance is

$$d = \sqrt{x^2 + y^2} = \sqrt{(43.4\ \text{m})^2 + (197\ \text{m})^2} = 202\ \text{m}$$

So the hawk's diving speed is

$$v = \frac{\Delta d}{\Delta t} = \frac{\sqrt{(197\ \text{m})^2 + (43.4\ \text{m})^2}}{4.34\ \text{s}} = \boxed{46.5\ \text{m/s}}$$

(b) at an angle below the horizontal of

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{197\text{ m}}{43.4\text{ m}}\right) = \boxed{77.6°}$$

P4.72 (a) We find the x coordinate from $x = 12t$. We find the y coordinate from $49t - 4.9t^2$. Then we find the projectile's distance from the origin as $(x^2 + y^2)^{1/2}$, with these results:

***t* (s)**	0	1	2	3	4	5	6	7	8	9	10
***r* (m)**	0	45.7	82.0	109	127	136	138	133	124	117	120

(b) From the table, it looks like the magnitude of r is largest at a bit less than 6 s.

The vector $\vec{\mathbf{v}}$ tells how $\vec{\mathbf{r}}$ is changing. If $\vec{\mathbf{v}}$ at a particular point has a component along $\vec{\mathbf{r}}$, then $\vec{\mathbf{r}}$ will be increasing in magnitude (if $\vec{\mathbf{v}}$ is at an angle less than 90° from $\vec{\mathbf{r}}$) or decreasing (if the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{r}}$ is more than 90°). To be at a maximum, the distance from the origin must be momentarily staying constant, and the only way this can happen is for the angle between velocity and displacement to be a right angle. Then $\vec{\mathbf{r}}$ will be changing in direction at that point, but not in magnitude.

(c) When $t = 5.70$ s, $r = \boxed{138\text{ m.}}$

(d) $\boxed{\text{We can require } dr^2/dt = 0 = (d/dt)[(12t)^2 + (49t - 4.9t^2)^2]\text{, which results in the solution.}}$

P4.73 (a) The time of flight must be positive. It is determined by

$$y_f = y_i + v_{yi}t + (1/2)a_y t^2 \rightarrow \quad 0 = 1.20 + v_i \sin 35.0°t - 4.90t^2$$

From the quadratic formula, and suppressing units, we find

$$t = \frac{0.574v_i + \sqrt{0.329v_i^2 + 23.52}}{9.80}$$

Then the range follows from $x = v_{xi}t + 0 = v_0 t$ as

$$\boxed{x(v_i) = v_i\sqrt{0.164\,3 + 0.002\,299v_i^2} + 0.047\,94v_i^2}$$

where x is in meters and v_i is in meters per second.

(b) Substituting $v_i = 0.100$ gives $x(v_i = 0.100) = \boxed{0.041\,0\text{ m}}$

(c) Substituting $v_i = 100$ gives $x(v_i = 100) = \boxed{961\text{ m}}$

(d) When v_i is small, v_i^2 becomes negligible. The expression $x(v_i)$ simplifies to $v_i\sqrt{0.164\ 3+0}+0=\boxed{0.405\ v_i}$. Note that this gives nearly the answer to part (b).

(e) When v_i is large, v_i is negligible in comparison to v_i^2. Then $x(v_i)$ simplifies to

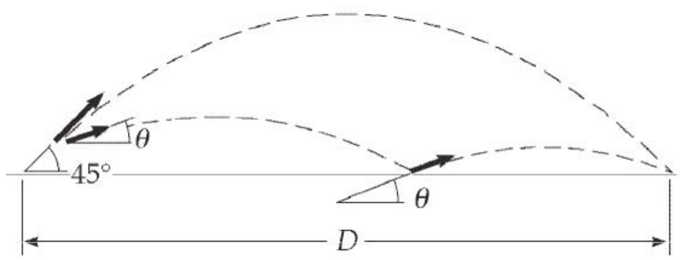
$$x(v_i)\cong v_i\sqrt{0+0.002\ 299\ v_i^2}+0.047\ 94\ v_i^2=\boxed{0.0959\ v_i^2}$$

This nearly gives the answer to part (c).

(f) $\boxed{\text{The graph of } x \text{ versus } v_i \text{ starts from the origin as a straight line with slope 0.405 s. Then it curves upward above this tangent line, getting closer and closer to the parabola } x=(0.095\ 9\ \mathrm{s^2/m})v_i^2.}$

P4.74 The special conditions allowing use of the horizontal range equation applies. For the ball thrown at 45°,

$$D=R_{45}=\frac{v_i^2\sin 90°}{g}$$



ANS. FIG. P4.74

For the bouncing ball,

$$D=R_1+R_2=\frac{v_i^2\sin 2\theta}{g}+\frac{(v_i/2)^2\sin 2\theta}{g}$$

where θ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:

$$\frac{v_i^2}{g}=\frac{v_i^2\sin 2\theta}{g}+\frac{v_i^2\sin 2\theta}{4g}$$

$$\sin 2\theta=\frac{4}{5}$$

$$\boxed{\theta=26.6°}$$

(b) The time for any symmetric parabolic flight is given by

$$y_f = v_{yi}t - \frac{1}{2}gt^2$$
$$0 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

If $t = 0$ is the time the ball is thrown, then $t = \frac{2v_i \sin\theta_i}{g}$ is the time at landing. So for the ball thrown at 45.0°:

$$t_{45} = \frac{2v_i \sin 45.0°}{g}$$

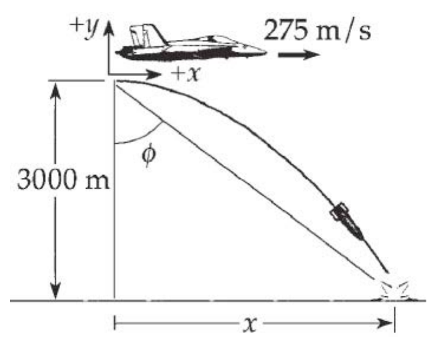
For the bouncing ball,

$$t = t_1 + t_2 = \frac{2v_i \sin 26.6°}{g} + \frac{2(v_i/2)\sin 26.6°}{g} = \frac{3v_i \sin 26.6°}{g}$$

The ratio of this time to that for no bounce is

$$\frac{3v_i \sin 26.6°/g}{2v_i \sin 45.0°/g} = \frac{1.34}{1.41} = \boxed{0.949}$$

P4.75 We model the bomb as a particle with constant acceleration, equal to the downward free-fall acceleration, from the moment after release until the moment before impact. After we find its range it will be a right-triangle problem to find the bombsight angle.



ANS. FIG. P4.75

(a) We take the origin at the point under the plane at bomb release. In its horizontal flight, the bomb has $v_{yi} = 0$ and $v_{xi} = 275$ m/s. We represent the height of the plane as y.

Then, $\Delta y = -\frac{1}{2}gt^2$; $\Delta x = v_i t$

Combining the equations to eliminate t gives:

$$\Delta y = -\frac{1}{2}g\left(\frac{\Delta x}{v_i}\right)^2$$

From this, $(\Delta x)^2 = \left(\frac{-2\Delta y}{g}\right)v_i^2$. Thus

$$\Delta x = v_i\sqrt{\frac{-2\Delta y}{g}} = (275 \text{ m/s})\sqrt{\frac{-2(-3\,000 \text{ m})}{9.80 \text{ m/s}^2}}$$
$$= 6.80\times 10^3 \text{ m} = \boxed{6.80 \text{ km}}$$

(b) The plane has the same velocity as the bomb in the *x* direction. Therefore, the plane will be $\boxed{\text{3 000 m directly above the bomb}}$ when it hits the ground.

(c) When ϕ is measured from the vertical, $\tan\phi = \frac{\Delta x}{\Delta y}$;

therefore, $\phi = \tan^{-1} = \left(\frac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\frac{6\,800 \text{ m}}{3\,000 \text{ m}}\right) = \boxed{66.2°}$.

P4.76 Equation of bank: $y^2 = 16x$ **[1]**

Equations of motion: $x = v_i t$ **[2]**

$$y = -\frac{1}{2}gt^2 \quad \textbf{[3]}$$

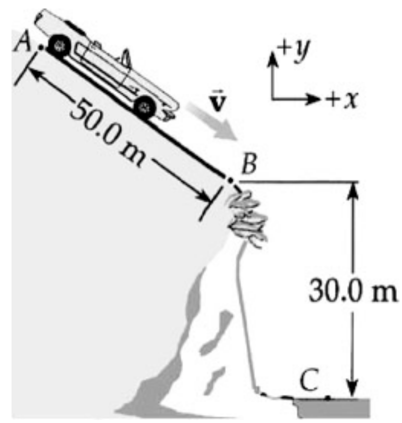
Substitute for *t* from [2] into [3]: $y = -\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right)$. Equate *y* from the bank equation to *y* from the equations of motion:

$$16x = \left[-\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right)\right]^2 \Rightarrow \frac{g^2x^4}{4v_i^4} - 16x = x\left(\frac{g^2x^3}{4v_i^4} - 16\right) = 0$$

From this, $x = 0$ or $x^3 = \frac{64v_i^4}{g^2}$ and $x = 4\left(\frac{10^4}{9.80^2}\right)^{1/3}$ m $= \boxed{18.8 \text{ m}}$.

Also, $y = -\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right) = -\frac{1}{2}\frac{(9.80 \text{ m/s}^2)(18.8 \text{ m})^2}{(10.0 \text{ m/s})^2} = \boxed{-17.3 \text{ m}}$

P4.77 The car has one acceleration while it is on the slope and a different acceleration when it is falling, so we must take the motion apart into two different sections. Our standard equations only describe a chunk of motion during which acceleration stays constant. We imagine the acceleration to change instantaneously at the brink of the cliff, but the velocity and the position must be the same just before point *B* and just after point *B*.



ANS. FIG. P4.77

(a) From point *A* to point *B* (along the incline), the car can be modeled as a particle under constant acceleration in one dimension, starting from rest ($v_i = 0$). Therefore, taking Δx to be the position along the incline,

$$v_f^2 - v_i^2 = 2a\Delta x$$
$$v_f^2 - 0 = 2(4.00\text{ m/s}^2)(50.0\text{ m})$$
$$v_f = \boxed{20.0\text{ m/s}}$$

(b) We can find the elapsed time interval from

$$v_f = v_i + at$$
$$20.0\text{ m/s} = 0 + \left(4.00\text{ m/s}^2\right)t$$
$$t = \boxed{5.00\text{ s}}$$

(c) Initial free-fall conditions give us $v_{xi} = 20.0\cos 37.0° = 16.0$ m/s and $v_{yi} = -20.0\sin 37.0° = -12.0$ m/s. Since $a_x = 0$, $v_{xf} = v_{xi}$ and

$$v_{yf} = -\sqrt{2a_y\Delta y + v_{yi}^2}$$
$$= -\sqrt{2\left(-9.80\text{ m/s}^2\right)\left(-30.0\text{ m}\right) + \left(-12.0\text{ m/s}\right)^2}$$
$$= -27.1\text{ m/s}$$
$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{\left(16.0\text{ m/s}\right)^2 + \left(-27.1\text{ m/s}\right)^2}$$
$$= \boxed{31.5\text{ m/s at }59.4°\text{ below the horizontal}}$$

(d) From point *B* to *C*, the time is

$$t_1 = 5\text{ s};\; t_2 = \frac{v_{yf} - v_{yi}}{a_y} = \frac{-27.1\text{ m/s} + 12.0\text{ m/s}}{-9.80\text{ m/s}^2} = 1.53\text{ s}$$

The total elapsed time interval is

$$t = t_1 + t_2 = \boxed{6.53\ \text{s}}$$

(e) The horizontal distance covered is

$$\Delta x = v_{xi}t_2 = (16.0\ \text{m/s})(1.53\ \text{s}) = \boxed{24.5\ \text{m}}$$

P4.78 (a) Coyote: $\Delta x = \frac{1}{2}at^2 \rightarrow 70.0\ \text{m} = \frac{1}{2}(15.0\ \text{m/s}^2)\,t^2$

Roadrunner: $\Delta x = v_{xi}t \rightarrow 70.0\ \text{m} = v_{xi}t$

Solving the above, we get

$$v_{xi} = \boxed{22.9\ \text{m/s}} \text{ and } t = \boxed{3.06\ \text{s}}$$

(b) At the edge of the cliff, $v_{xi} = at = (15.0\ \text{m/s}^2)(3.06\ \text{s}) = 45.8\ \text{m/s}$

Substituting $\Delta y = -100\ \text{m}$ into $\Delta y = \frac{1}{2}a_yt^2$, we find

$$-100\ \text{m} = \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t^2$$
$$t = 4.52\ \text{s}$$
$$\Delta x = v_xt + \frac{1}{2}a_xt^2$$
$$= (45.8\ \text{m/s})(4.52\ \text{s}) + \frac{1}{2}\left(15.0\ \text{m/s}^2\right)(4.52\ \text{s})^2$$

Solving, $\Delta x = \boxed{360\ \text{m}}$.

(c) For the Coyote's motion through the air,

$$v_{xf} = v_{xi} + a_xt = 45.8\ \text{m/s} + \left(15\ \text{m/s}^2\right)(4.52\ \text{s}) = \boxed{114\ \text{m/s}}$$
$$v_{yf} = v_{yi} + a_yt = 0 - \left(9.80\ \text{m/s}^2\right)(4.52\ \text{s}) = \boxed{-44.3\ \text{m/s}}$$

P4.79 (a) Reference frame: Earth

The ice chest floats downstream 2 km in time interval Δt, so

$$2\ \text{km} = v_{ow}\Delta t \rightarrow \Delta t = 2\ \text{km}/v_{ow}$$

The upstream motion of the boat is described by

$$d = (v - v_{ow})(15\ \text{min})$$

and the downstream motion is described by

$$d + 2\ \text{km} = (v - v_{ow})(\Delta t - 15\ \text{min})$$

We substitute the above expressions for Δt and d:

$$(v - v_{ow})(15 \text{ min}) + 2 \text{ km} = (v + v_{ow})\left(\frac{2 \text{ km}}{v_{ow}} - 15 \text{ min}\right)$$

$$v(15 \text{ min}) - v_{ow}(15 \text{ min}) + 2 \text{ km}$$

$$= \frac{v}{v_{ow}}(2 \text{ km}) + 2 \text{ km} - v(15 \text{ min}) - v_{ow}(15 \text{ min})$$

$$v(30 \text{ min}) = \frac{v}{v_{ow}}(2 \text{ km})$$

$$v_{ow} = \boxed{4.00 \text{ km/h}}$$

(b) Reference frame: water

After the boat travels so that it and its starting point are 2 km apart, the chest enters the water, where, in the frame of the water, it is motionless. The boat then travels upstream for 15 min at speed v, and then downstream at the same speed, to return to the same point where the chest is at rest in the water. Thus, the boat travels for a total time interval of 30 min. During this same time interval, the starting point approaches the chest at speed v_{ow}, traveling 2 km. Thus,

$$v_{ow} = \frac{\Delta x}{\Delta t_{\text{total}}} = \frac{2 \text{ km}}{30 \text{ min}} = \boxed{4.00 \text{ km/h}}$$

P4.80 Think of shaking down the mercury in an old fever thermometer. Swing your hand through a circular arc, quickly reversing direction at the bottom end. Suppose your hand moves through one-quarter of a circle of radius 60 cm in 0.1 s. Its speed is

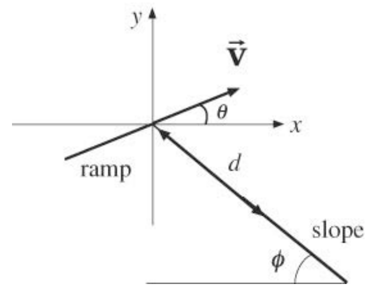
$$\frac{\frac{1}{4}(2\pi)(0.6 \text{ m})}{0.1 \text{ s}} \approx 9 \text{ m/s}$$

and its centripetal acceleration is $\dfrac{v^2}{r} \approx \dfrac{(9 \text{ m/s})^2}{0.6 \text{ m}} \boxed{\sim 10^2 \text{ m/s}^2}$.

The tangential acceleration of stopping and reversing the motion will make the total acceleration somewhat larger, but will not affect its order of magnitude.

Challenge Problems

P4.81 ANS. FIG. P4.81 indicates that a line extending along the slope will past through the end of the ramp, so we may take the position of the skier as she leaves the ramp to be the origin of our coordinate system.



ANS. FIG. P4.81

(a) Measured from the end of the ramp, the skier lands a distance *d* down the slope at time *t*:

$$\Delta x = v_{xi}t$$
$$\rightarrow d\cos 50.0° = (10.0 \text{ m/s})(\cos 15.0°)t$$

and

$$\Delta y = v_{yi}t + \frac{1}{2}gt^2 \rightarrow$$
$$-d\sin 50.0° = (10.0 \text{ m/s})(\sin 15.0°)t + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2$$

Solving, $d = \boxed{43.2 \text{ m}}$ and $t = 2.88$ s.

(b) Since $a_x = 0$,

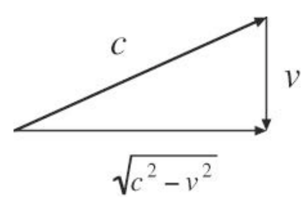
$$v_{xf} = v_{xi} = (10.0 \text{ m/s})\cos 15.0° = \boxed{9.66 \text{ m/s}}$$
$$v_{yf} = v_{yi} + a_y t = (10.0 \text{ m/s})\sin 15.0° - (9.80 \text{ m/s}^2)(2.88 \text{ s})$$
$$= \boxed{-25.6 \text{ m/s}}$$

(c) Air resistance would ordinarily decrease the values of the range and landing speed. As an airfoil, she can deflect air downward so that the air deflects her upward. This means she can get some lift and increase her distance.

P4.82 (a) For Chris, his speed downstream is $c + v$, while his speed upstream is $c - v$.

Therefore, the total time for Chris is

$$\boxed{\Delta t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2\ L/c}{1 - v^2/c^2}}$$



ANS. FIG. P4.82

(b) Sarah must swim somewhat upstream to counteract the effect from the current. As is shown in the diagram, the magnitude of her cross-stream velocity is $\sqrt{c^2 - v^2}$.

Thus, the total time for Sarah is

$$\boxed{\Delta t_2 = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2\ L/c}{\sqrt{1 - v^2/c^2}}}$$

(c) Since the term $\left(1 - v^2/c^2\right) < 1, \Delta t_1 > \Delta t_2$, so Sarah, who swims cross-stream, returns first.

***P4.83** Let the river flow in the *x* direction.

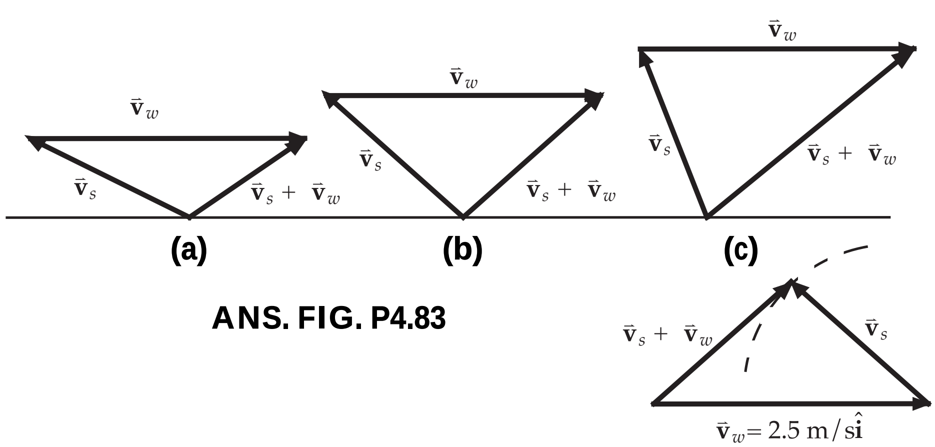
(a) To minimize time, $\boxed{\text{swim perpendicular to the banks}}$ in the *y* direction. You are in the water for time *t* in $\Delta y = v_y t$,

$$t = \frac{80\ \text{m}}{1.5\ \text{m/s}} = 53.3\ \text{s}$$

(b) The water carries you downstream by

$$\Delta x = v_x t = (2.50\ \text{m/s})53.3\ \text{s} = \boxed{133\ \text{m}}$$

(c) To minimize downstream drift, you should swim so that your resultant velocity $\vec{\mathbf{v}}_s + \vec{\mathbf{v}}_w$ is perpendicular to your swimming velocity $\vec{\mathbf{v}}_s$ relative to the water. This is shown graphically in the upper row of ANS. FIG. P4.83. Unlike the situations shown in ANS. FIG. P4.83(a) and ANS. FIG. P4.83(b), this condition (shown in ANS. FIG. P4.83(b)) maximizes the angle between the resultant velocity and the shore. The angle between $\vec{\mathbf{v}}_s$ and the shore is given by $\cos\theta = \dfrac{1.5\ \text{m/s}}{2.5\ \text{m/s}}$, $\boxed{\theta = 53.1°}$.



ANS. FIG. P4.83

(d) See ANS. FIG. P4.83(d). Now,
$v_y = v_s \sin\theta = (1.5 \text{ m/s})\sin 53.1° = 1.20 \text{ m/s}$

$$t = \frac{\Delta y}{v_y} = \frac{80 \text{ m}}{1.2 \text{ m/s}} = 66.7 \text{ s}$$

$$\Delta x = v_x t = [2.5 \text{ m/s} - (1.5 \text{ m/s})\cos 53.1°](66.7 \text{ s}) = \boxed{107 \text{ m}}$$

P4.84 Measure heights above the level ground. The elevation y_b of the ball follows

$$y_b = R + 0 - \frac{1}{2}gt^2$$

with $x = v_i t$ so $y_b = R - \dfrac{gx^2}{2v_i^2}$.

(a) The elevation y_r of points on the rock is described by

$$y_r^2 + x^2 = R^2$$

We will have $y_b = y_r$ at $x = 0$, but for all other x we require the ball to be above the rock's surface as in $y_b > y_r$. Then $y_b^2 + x^2 > R^2$:

$$\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 > R^2$$

$$R^2 - \frac{gx^2R}{v_i^2} + \frac{g^2x^4}{4v_i^4} + x^2 > R^2$$

$$\frac{g^2x^4}{4v_i^4} + x^2 > \frac{gx^2R}{v_i^2}$$

If this inequality is satisfied for x approaching zero, it will be true for all x. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock:

$1 > \dfrac{gR}{v_i^2}$, so

$$\boxed{v_i > \sqrt{gR}}$$

(b) With $v_i = \sqrt{gR}$ and $y_b = 0$, we have $0 = R - \dfrac{gx^2}{2gR}$

or $x = R\sqrt{2}$. The distance from the rock's base is

$$x - R = \boxed{\left(\sqrt{2} - 1\right)R}$$

P4.85 When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance x_f given by

$$x_f = \sqrt{(3.25\text{ km})^2 - (2.15\text{ km})^2} = 2.437\text{ km}$$

The vertical displacement of the bomb is

$$y_f = x_f \tan\theta_i - \frac{g x_f^2}{2v_i^2\cos^2\theta_i}$$

Substituting,

$$-2\,150\text{ m} = (2\,437\text{ m})\tan\theta_i - \frac{(9.8\text{ m/s}^2)(2\,437\text{ m})^2}{2(280\text{ m/s})^2\cos^2\theta_i}$$

or

$$-2\,150\text{ m} = (2\,437\text{ m})\tan\theta_i - (371.19\text{ m})(1+\tan^2\theta_i)$$

$$\therefore \tan^2\theta_i - 6.565\tan\theta_i - 4.792 = 0$$

$$\therefore \tan\theta_i = \frac{1}{2}\left(6.565 \pm \sqrt{(6.565)^2 - 4(1)(-4.792)}\right) = 3.283 \pm 3.945$$

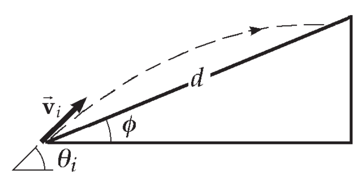
We select the negative solution, since θ_i is below the horizontal.

$$\therefore \tan\theta_i = -0.662,\ \boxed{\theta_i = -33.5°}$$

P4.86 (a) The horizontal distance traveled by the projectile is given by

$$x_f = v_{xi}t = (v_i\cos\theta_i)t$$

$$\rightarrow t = \frac{x_f}{v_i\cos\theta_i}$$



ANS. FIG. P4.86

We substitute this into the equation for the displacement in y:

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = (\tan\theta_i)(x_f) - \frac{g}{2v_i^2\cos^2\theta_i}x_f^2$$

Now setting $x_f = d\cos\phi$ and $y_f = d\sin\phi$, we have

$$d\sin\phi = (\tan\theta_i)(d\cos\phi) - \frac{g}{2v_i^2\cos^2\theta_i}(d\cos\phi)^2$$

Solving for d yields

$$d = \frac{2v_i^2\cos\theta_i\left[\sin\theta_i\cos\phi - \sin\phi\cos\theta_i\right]}{g\cos^2\phi}$$

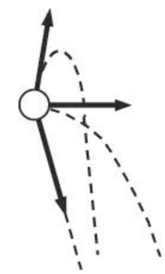
or $\quad d = \boxed{\dfrac{2v_i^2 \cos\theta_i \sin(\theta_i - \phi)}{g\cos^2\phi}}$

(b) Setting $\dfrac{d}{d\theta_i}(d) = 0$ leads to

$$\boxed{\theta_i = 45° + \frac{\phi}{2}} \text{ and } \boxed{d_{max} = \frac{v_i^2(1-\sin\phi)}{g\cos^2\phi}}$$

P4.87 For the smallest impact angle

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right)$$


ANS. FIG. P4.87

we want to minimize v_{yf} and maximize $v_{xf} = v_{xi}$. The final y component of velocity is related to v_{yi} by $v_{yf}^2 = v_{yi}^2 + 2gh$, so we want to minimize v_{yi} and maximize v_{xi}. Both are accomplished by making the initial velocity horizontal. Then $v_{xi} = v$, $v_{yi} = 0$, and $v_{yf} = \sqrt{2gh}$. At last, the impact angle is

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \boxed{\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)}$$

P4.88 We follow the steps outlined in Example 4.5, eliminating $t = \dfrac{d\cos\phi}{v_i\cos\theta}$ to find

$$\frac{v_i \sin\theta\, d\cos\phi}{v_i\cos\theta} - \frac{gd^2\cos^2\phi}{2v_i^2\cos^2\theta} = -d\sin\phi$$

Clearing the fractions gives

$$2v_i^2\cos\theta\sin\theta\cos\phi - gd\cos^2\phi = -2v_i^2\cos^2\theta\sin\phi$$

To maximize d as a function of θ, we differentiate through with respect to θ and set $\dfrac{d}{d\theta}(d) = 0$:

$$2v_i^2\cos\theta\cos\theta\cos\phi + 2v_i^2\sin\theta(-\sin\theta)\cos\phi$$
$$- g\left[\frac{d}{d\theta}(d)\right]\cos^2\phi = -2v_i^2 2\cos\theta(-\sin\theta)\sin\phi$$

We use the trigonometric identities from Appendix B4:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \quad \text{and} \quad \sin 2\theta = 2\sin\theta\cos\theta$$

to find

$$\cos\phi\cos 2\theta = \sin 2\theta \sin\phi$$

Next, $\dfrac{\sin\phi}{\cos\phi} = \tan\phi$ and $\cot 2\theta = \dfrac{1}{\tan 2\theta}$ give $\cot 2\theta = \tan\phi$ so

$$\phi = 90° - 2\theta \quad \text{and} \quad \theta = 45° - \frac{\phi}{2}$$

P4.89 Find the highest firing angle θ_H for which the projectile will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest firing angle; this will yield the maximum range under these conditions if both θ_H and θ_L are > 45°, x = 2 500 m, y = 1 800 m, and v_i = 250 m/s.

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = v_i(\sin\theta)t - \frac{1}{2}gt^2$$
$$x_f = v_{xi}t = v_i(\cos\theta)t$$

Thus,

$$t = \frac{x_f}{v_i\cos\theta}$$

Substitute into the expression for y_f:

$$y_f = v_i(\sin\theta)\frac{x_f}{v_i\cos\theta} - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta}\right)^2 = x_f\tan\theta - \frac{gx_f^2}{2v_i^2\cos^2\theta}$$

but $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$, so $y_f = x_f\tan\theta - \dfrac{gx_f^2}{2v_i^2}\left(\tan^2\theta + 1\right)$ and

$$0 = \frac{gx_f^2}{2v_i^2}\tan^2\theta - x_f\tan\theta + \frac{gx_f^2}{2v_i^2} + y_f$$

Substitute values, use the quadratic formula, and find

$\tan\theta = 3.905$ or 1.197, which gives $\theta_H = 75.6°$ and $\theta_L = 50.1°$.

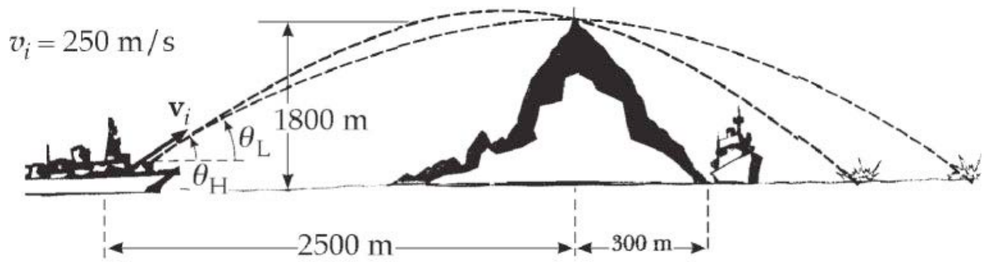
$$\text{Range}\left(\text{at } \theta_H\right) = \frac{v_i^2\sin 2\theta_H}{g} = 3.07\times10^3 \text{ m from enemy ship}$$

$$3.07\times10^3 \text{ m} - 2\,500 \text{ m} - 300 \text{ m} = 270 \text{ m from shore}$$

$$\text{Range}\left(\text{at } \theta_L\right) = \frac{v_i^2 \sin 2\theta_L}{g} = 6.28 \times 10^3 \text{ m from enemy ship}$$

$$6.28 \times 10^3 \text{ m} - 2\,500 \text{ m} - 300 \text{ m} = 3.48 \times 10^3 \text{ m from shore}$$

Therefore, the safe distance is $\boxed{< 270 \text{ m}}$ or $\boxed{> 3.48 \times 10^3 \text{ m}}$ from the shore.



ANS. FIG. P4.89

ANSWERS TO EVEN-NUMBERED PROBLEMS

P4.2 2.50 m/s

P4.4 (a) $-5.00\omega\ \hat{\mathbf{i}}$ m/s; (b) $-5.00\omega^2\ \hat{\mathbf{j}}$ m/s;
(c) $(4.00\text{ m})\hat{\mathbf{j}}+(5.00\text{ m})\left(-\sin\omega t\hat{\mathbf{i}}-\cos\omega t\hat{\mathbf{j}}\right)$,
$(5.00\text{ m})\omega\left[-\cos\omega\hat{\mathbf{i}}+\sin\omega t\hat{\mathbf{j}}\right]$, $(5.00\text{ m})\omega^2\left[\sin\omega t\hat{\mathbf{i}}+\cos\omega\hat{\mathbf{j}}\right]$; (d) a circle of radius 5.00 m centered at (0, 4.00 m)

P4.6 (a) $5.00t\hat{\mathbf{i}}+1.50t^2\hat{\mathbf{j}}$; (b) $5.00\hat{\mathbf{i}}+3.00t\ \hat{\mathbf{j}}$; (c) 10.0 m, 6.00 m; (d) 7.81 m/s

P4.8 (a) $\left(10.0\ \hat{\mathbf{i}}+0.241\ \hat{\mathbf{j}}\right)$ mm; (b) $\left(1.84\times10^7\text{ m/s}\right)\hat{\mathbf{i}}+\left(8.78\times10^5\text{ m/s}\right)\hat{\mathbf{j}}$;
(c) 1.85×10^7 m/s; (d) 2.73°

P4.10 (a) $\vec{\mathbf{v}}_f=(3.45-1.79t)\hat{\mathbf{i}}+\left(2.89-0.650t\right)\hat{\mathbf{j}}$;
(b) $\vec{\mathbf{r}}_f=(-25.3+3.45t-0.893t^2)\hat{\mathbf{i}}+\left(28.9+2.89t-0.325t^2\right)\hat{\mathbf{j}}$

P4.12 0.600 m/s^2

P4.14 (a) $v_{xi}=d\sqrt{\dfrac{g}{2h}}$, (b) The direction of the mug's velocity is $\tan^{-1}(2h/d)$ below the horizontal.

P4.16 $x=7.23\times10^3$ m, $y=1.68\times10^3$ m

P4.18 (a) 76.0°, (b) $R_{max}=2.13R$, (c) the same on every planet

P4.20 (a) 22.6 m; (b) 52.3 m; (c) 1.18 s

P4.22 (a) there is; (b) 0.491 m/s

P4.24 (a) 0.852 s; (b) 3.29 m/s; (c) 4.03 m/s; (d) 50.8°; (e) $t=1.12$ s

P4.26 (a) (0, 0.840 m); (b) 11.2 m/s at 18.5°; (c) 8.94 m

P4.28 (a) $t=v_i\sin\theta/g$; (b) $h_{max}=h+\dfrac{\left(v_i\sin\theta\right)^2}{2g}$

P4.30 (a) 28.2 m/s; (b) 4.07 s; (c) the required initial velocity will increase, the total time of flight will increase

P4.32 (a) 41.7 m/s; (b) 3.81 s; (c) $v_x=34.1$ m/s, $v_y=-13.4$ m/s, $v=36.7$ m/s

P4.24 0.033 7 m/s^2 directed toward the center of Earth

P4.36 10.5 m/s, 219 m/s^2 inward

P4.38 (a) 6.00 rev/s; (b) $1.52\times10^3\ \text{m/s}^2$; (c) $1.28\times10^3\ \text{m/s}^2$

P4.40 (a) 13.0 m/s^2; (b) 5.70 m/s; (c) 7.50 m/s^2

P4.42 (a) See ANS. FIG. P4.42; (b) 29.7 m/s^2; (c) 6.67 m/s tangent to the circle

P4.44 153 km/h at 11.3° north of west

P4.46 (a) $\Delta t_{woman} = \frac{L}{v_1}$; (b) $\Delta t_{man} = \frac{L}{v_1 + v_2}$; (c) $\Delta t_{man} = \frac{L}{v_1 + 2v_2}$

P4.48 (a) 57.7 km/h at 60.0° west of vertical; (b) 28.9 km/h downward

P4.50 (a) 2.02×10^3 s; (b) 1.67×10^3 s; (c) Swimming with the current does not compensate for the time lost swimming against the current.

P4.52 27.7° E of N

P4.54 (a) straight up, at 0° to the vertical; (b) 8.25 m/s; (c) a straight up and down line; (d) a symmetric parabola opening downward; (e) 12.6 m/s north at $\tan^{-1}(8.25/9.5) = 41.0°$ above the horizontal

P4.56 (a) $2\sqrt{\frac{R}{3g}}$; (b) $\frac{1}{2}\sqrt{3gR}$; (c) $\sqrt{\frac{gR}{3}}$; (d) $\sqrt{\frac{13gR}{12}}$; (e) 33.7°; (f) $\frac{13}{24}R$; (g) $\frac{13}{12}R$

P4.58 (a) $5\hat{\mathbf{i}} + 4t^{3/2}\hat{\mathbf{j}}$; (b) $5t\hat{\mathbf{i}} + 1.6t^{5/2}\hat{\mathbf{j}}$

P4.60 (a) 9.80 m/s^2, downward; (b) 10.7 m/s

P4.62 (a) $t = \sqrt{\frac{2h}{g}}$; (b) $v_{xi} = d\sqrt{\frac{g}{2h}}$; (c) $v_f = \sqrt{v_{xf}^{\ 2} + v_{yf}^{\ 2}} = \sqrt{\left(\frac{d^2 g}{2h}\right) + \left(2gh\right)}$; (d) $\theta_f = \tan^{-1}\left(\frac{2h}{d}\right)$

P4.64 68.8 km/h

P4.66 22.4° or 89.4°

P4.68 $2v_i t\cos\theta_i$

P4.70 (a) 25.0 m/s^2; (b) 9.80 m/s^2; (c) See ANS. FIG. P4.70; (d) 26.8 m/s^2, 21.4°

P4.72 (a) See table in P4.72(a); (b) From the table, it looks like the magnitude of *r* is largest at a bit less than 6 s; (c) 138 m; (d) We can require $dr^2/dt = 0 = (d/dt)[(12t)^2 + (49t - 4.9t^2)^2]$, which results in the solution.

P4.74 (a) $\theta = 26.6°$; (b) 0.949

P4.76 18.8 m, −17.3 m

P4.78 (a) 22.9 m/s and 3.06 s; (b) 360 m; (c) 114 m/s, −44.3 m/s

P4.80 $\sim 10^2\ \text{m/s}^2$

P4.82 (a) $\Delta t_1 = \dfrac{L}{c+v} + \dfrac{L}{c-v} = \dfrac{2L/c}{1-v^2/c^2}$; (b) $\Delta t_2 = \dfrac{2L}{\sqrt{c^2-v^2}} = \dfrac{2L/c}{\sqrt{1-v^2/c^2}}$;

(c) Sarah, who swims cross-stream, returns first

P4.84 (a) $v_i > \sqrt{gR}$; (b) $x - R = \left(\sqrt{2}-1\right)R$

P4.86 (a) See P4.86a for derivation; (b) $d_{max} = 45° + \dfrac{\phi}{2}$, $\theta_i = \dfrac{v_i^2\left(1-\sin\phi\right)}{g\cos^2\phi}$

P4.88 See P4.88 for complete derivation.

5

The Laws of Motion

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ5.1 Answer (d). The stopping distance will be the same if the mass of the truck is doubled. The normal force and the friction force both double, so the backward acceleration remains the same as without the load.

OQ5.2 Answer (b). Newton's 3rd law describes all objects, breaking or whole. The force that the locomotive exerted on the wall is the same as that exerted by the wall on the locomotive. The framing around the wall could not exert so strong a force on the section of the wall that broke out.

OQ5.3 Since they are on the order of a thousand times denser than the surrounding air, we assume the snowballs are in free fall. The net force

on each is the gravitational force exerted by the Earth, which does not depend on their speed or direction of motion but only on the snowball mass. Thus we can rank the missiles just by mass: d > a = e > b > c.

OQ5.4 Answer (e). The stopping distance will decrease by a factor of four if the initial speed is cut in half.

OQ5.5 Answer (b). An air track or air table is a wonderful thing. It exactly cancels out the force of the Earth's gravity on the gliding object, to display free motion and to imitate the effect of being far away in space.

OQ5.6 Answer (b). 200 N must be greater than the force of friction for the box's acceleration to be forward.

OQ5.7 Answer (a). Assuming that the cord connecting m_1 and m_2 has constant length, the two masses are a fixed distance (measured along the cord) apart. Thus, their speeds must always be the same, which means that their accelerations must have equal magnitudes. The magnitude of the downward acceleration of m_2 is given by Newton's second law as

$$a_2 = \frac{\sum F_y}{m_2} = \frac{m_2 g - T}{m_2} = g - \left(\frac{T}{m_2}\right) < g$$

where T is the tension in the cord, and downward has been chosen as the positive direction.

OQ5.8 Answer (d). Formulas a, b, and e have the wrong units for speed. Formulas a and c would give an imaginary answer.

OQ5.9 Answer (b). As the trailer leaks sand at a constant rate, the total mass of the vehicle (truck, trailer, and remaining sand) decreases at a steady rate. Then, with a constant net force present, Newton's second law states that the magnitude of the vehicle's acceleration ($a = F_{net}/m$) will *steadily increase*.

OQ5.10 Answer (c). When the truck accelerates forward, the crate has the natural tendency to remain at rest, so the truck tends to slip under the crate, leaving it behind. However, friction between the crate and the bed of the truck acts in such a manner as to oppose this relative motion between truck and crate. Thus, the friction force acting on the crate will be in the forward horizontal direction and tend to accelerate the crate forward. The crate will slide only when the coefficient of static friction is inadequate to prevent slipping.

OQ5.11 Both answers (d) and (e) are *not true*: (d) is not true because the value of the velocity's constant magnitude need not be zero, and (e) is not true because there may be *no force* acting on the object. An object in equilibrium has zero acceleration $(\vec{\mathbf{a}} = 0)$, so both the magnitude and direction of the object's velocity must be *constant*. Also, Newton's second law states that the *net force* acting on an object in equilibrium is zero.

OQ5.12 Answer (d). All the other possibilities would make the total force on the crate be different from zero.

OQ5.13 Answers (a), (c), and (d). A free-body diagram shows the forces exerted on the object by other objects, and the net force is the sum of those forces.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ5.1 A portion of each leaf of grass extends above the metal bar. This portion must accelerate in order for the leaf to bend out of the way. If the bar moves fast enough, the grass will not have time to increase its speed to match the speed of the bar. The leaf's mass is small, but when its acceleration is very large, the force exerted by the bar on the leaf puts the leaf under tension large enough to shear it off.

CQ5.2 When the hands are shaken, there is a large acceleration of the surfaces of the hands. If the water drops were to stay on the hands, they must accelerate along with the hands. The only force that can provide this acceleration is the friction force between the water and the hands. (There are adhesive forces also, but let's not worry about those.) The static friction force is not large enough to keep the water stationary with respect to the skin at this large acceleration. Therefore, the water breaks free and slides along the skin surface. Eventually, the water reaches the end of a finger and then slides off into the air. This is an example of Newton's first law in action in that the drops continue in motion while the hand is stopped.

CQ5.3 When the bus starts moving, the mass of Claudette is accelerated by the force of the back of the seat on her body. Clark is standing, however, and the only force on him is the friction between his shoes and the floor of the bus. Thus, when the bus starts moving, his feet start accelerating forward, but the rest of his body experiences almost no accelerating force (only that due to his being attached to his accelerating feet!). As a consequence, his body tends to stay almost at rest, according to Newton's first law, relative to the ground. Relative to Claudette, however, he is moving toward her and falls into her lap.

CQ5.4 The resultant force is zero, as the acceleration is zero.

CQ5.5 First ask, "Was the bus moving forward or backing up?" If it was moving forward, the passenger is lying. A fast stop would make the suitcase fly toward the front of the bus, not toward the rear. If the bus was backing up at any reasonable speed, a sudden stop could not make a suitcase fly far. Fine her for malicious litigiousness.

CQ5.6 Many individuals have a misconception that throwing a ball in the air gives the ball some kind of a "force of motion" that the ball carries after it leaves the hand. This is the "force of the throw" that is mentioned in the problem. The upward motion of the ball is explained by saying that the "force of the throw" exceeds the gravitational force—of course, this explanation confuses upward velocity with downward acceleration—the hand applies a force on the ball *only* while they are in contact; once the ball leaves the hand, the hand no longer has any influence on the ball's motion. The *only* property of the ball that it carries from its interaction with the hand is the *initial* upward velocity imparted to it by the thrower. Once the ball leaves the hand, the only force on the ball is the gravitational force. (a) If there were a "force of the throw" felt by the ball after it leaves the hand and the force exceeded the gravitational force, the ball would accelerate upward, not downward! (b) If the "force of the throw" equaled the gravitational force, the ball would move upward with a constant velocity, rather than slowing down and coming back down! (c) The magnitude is zero because there is no "force of the throw." (d) The ball moves away from the hand because the hand imparts a velocity to the ball and then the hand stops moving.

CQ5.7 (a) force: The *Earth* attracts the *ball* <u>downward</u> with the force of gravity—reaction force: the *ball* attracts the *Earth* <u>upward</u> with the force of gravity; force: the *hand* pushes <u>up</u> on the *ball*—reaction force: the *ball* pushes <u>down</u> on the *hand*.

(b) force: The *Earth* attracts the *ball* <u>downward</u> with the force of gravity—reaction force: the *ball* attracts the *Earth* <u>upward</u> with the force of gravity.

CQ5.8 (a) The air inside pushes outward on each patch of rubber, exerting a force perpendicular to that section of area. The air outside pushes perpendicularly inward, but not quite so strongly. (b) As the balloon takes off, all of the sections of rubber feel essentially the same outward forces as before, but the now-open hole at the opening on the west side feels no force – except for a small amount of drag to the west from the escaping air. The vector sum of the forces on the rubber is to the east.

The small-mass balloon moves east with a large acceleration. (c) Hot combustion products in the combustion chamber push outward on all the walls of the chamber, but there is nothing for them to push on at the open rocket nozzle. The net force exerted by the gases on the chamber is up if the nozzle is pointing down. This force is larger than the gravitational force on the rocket body, and makes it accelerate upward.

CQ5.9 The molecules of the floor resist the ball on impact and push the ball back, upward. The actual force acting is due to the forces between molecules that allow the floor to keep its integrity and to prevent the ball from passing through. Notice that for a ball passing through a window, the molecular forces weren't strong enough.

CQ5.10 The tension in the rope when pulling the car is twice that in the tug-of-war. One could consider the car as behaving like another team of twenty more people.

CQ5.11 An object cannot exert a force on itself, so as to cause acceleration. If it could, then objects would be able to accelerate themselves, without interacting with the environment. You cannot lift yourself by tugging on your bootstraps.

CQ5.12 Yes. The table bends down more to exert a larger upward force. The deformation is easy to see for a block of foam plastic. The sag of a table can be displayed with, for example, an optical lever.

CQ5.13 As the barbell goes through the bottom of a cycle, the lifter exerts an upward force on it, and the scale reads the larger upward force that the floor exerts on them together. Around the top of the weight's motion, the scale reads less than average. If the weightlifter throws the barbell upward so that it loses contact with his hands, the reading on the scale will return to normal, reading just the weight of the weightlifter, until the barbell lands back in his hands, at which time the reading will jump upward.

CQ5.14 The sack of sand moves up with the athlete, regardless of how quickly the athlete climbs. Since the athlete and the sack of sand have the same weight, the acceleration of the system must be zero.

CQ5.15 If you slam on the brakes, your tires will skid on the road. The force of kinetic friction between the tires and the road is less than the maximum static friction force. Antilock brakes work by "pumping" the brakes (much more rapidly than you can) to minimize skidding of the tires on the road.

CQ5.16 (a) Larger: the tension in A must accelerate two blocks and not just one. (b) Equal. Whenever A moves by 1 cm, B moves by 1 cm. The two blocks have equal speeds at every instant and have equal accelerations. (c) Yes, backward, equal. The force of cord B on block 1 is the tension in the cord.

CQ5.17 As you pull away from a stoplight, friction exerted by the ground on the tires of the car accelerates the car forward. As you begin running forward from rest, friction exerted by the floor on your shoes causes your acceleration.

CQ5.18 It is impossible to string a horizontal cable without its sagging a bit. Since the cable has a mass, gravity pulls it downward. A vertical component of the tension must balance the weight for the cable to be in equilibrium. If the cable were completely horizontal, then there would be no vertical component of the tension to balance the weight. If a physicist would testify in court, the city employees would win.

CQ5.19 (a) Yes, as exerted by a vertical wall on a ladder leaning against it. (b) Yes, as exerted by a hammer driving a tent stake into the ground. (c) Yes, as the ball accelerates upward in bouncing from the floor. (d) No; the two forces describe the same interaction.

CQ5.20 The clever boy bends his knees to lower his body, then starts to straighten his knees to push his body up—that is when the branch breaks. In order to give himself an upward acceleration, he must push down on the branch with a force greater than his weight so that the branch pushes up on him with a force greater than his weight.

CQ5.21 (a) As a man takes a step, the action is the force his foot exerts on the Earth; the reaction is the force of the Earth on his foot. (b) The action is the force exerted on the girl's back by the snowball; the reaction is the force exerted on the snowball by the girl's back. (c) The action is the force of the glove on the ball; the reaction is the force of the ball on the glove. (d) The action is the force exerted on the window by the air molecules; the reaction is the force on the air molecules exerted by the window. We could in each case interchange the terms "action" and "reaction."

CQ5.22 (a) Both students slide toward each other. When student A pulls on the rope, the rope pulls back, causing her to slide toward Student B. The rope also pulls on the pulley, so Student B slides because he is gripping a rope attached to the pulley. (b) Both chairs slide because there is tension in the rope that pulls on both Student A and the pulley connected to Student B. (c) Both chairs slide because when Student B pulls on his rope, he pulls the pulley which puts tension into the rope

passing over the pulley to Student A. (d) Both chairs slide because when Student A pulls on the rope, it pulls on her and also pulls on the pulley.

CQ5.23 If you have ever seen a car stuck on an icy road, with its wheels spinning wildly, you know the car has great difficulty moving forward until it "catches" on a rough patch. (a) Friction exerted by the road is the force making the car accelerate forward. Burning gasoline can provide energy for the motion, but only external forces—forces exerted by objects outside—can accelerate the car. (b) If the car moves forward slowly as it speeds up, then its tires do not slip on the surface. The rubber contacting the road moves toward the rear of the car, and static friction opposes relative sliding motion by exerting a force on the rubber toward the front of the car. If the car is under control (and not skidding), the relative speed is zero along the lines where the rubber meets the road, and static friction acts rather than kinetic friction.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 5.1 The Concept of Force
Section 5.2 Newton's First Law and Inertial Frames
Section 5.3 Mass
Section 5.4 Newton's Second Law
Section 5.5 The Gravitational Force and Weight
Section 5.6 Newton's Third Law

***P5.1** (a) The woman's weight is the magnitude of the gravitational force acting on her, given by

$$F_g = mg = 120\text{ lb} = (4.448\text{ N/lb})(120\text{ lb}) = \boxed{534\text{ N}}$$

(b) Her mass is $m = \dfrac{F_g}{g} = \dfrac{534\text{ N}}{9.80\text{ m/s}^2} = \boxed{54.5\text{ kg}}$

***P5.2** We are given $F_g = mg = 900\text{ N}$, from which we can find the man's mass,

$$m = \frac{900\text{ N}}{9.80\text{ m/s}^2} = 91.8\text{ kg}$$

Then, his weight on Jupiter is given by

$$(F_g)_{\text{on Jupiter}} = 91.8\text{ kg}(25.9\text{ m/s}^2) = \boxed{2.38\text{ kN}}$$

P5.3 We use Newton's second law to find the force as a vector and then the Pythagorean theorem to find its magnitude. The givens are $m = 3.00$ kg and $\vec{\mathbf{a}} = \left(2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)$ m/s^2.

(a) The total vector force is

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} = (3.00 \text{ kg})(2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}) \text{ m/s}^2 = \boxed{(6.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}) \text{ N}}$$

(b) Its magnitude is

$$\left|\vec{\mathbf{F}}\right| = \sqrt{(F_x)^2 + \left(F_y\right)^2} = \sqrt{(6.00 \text{ N})^2 + (15.0 \text{ N})^2} = \boxed{16.2 \text{ N}}$$

P5.4 Using the reference axes shown in Figure P5.4, we see that

$$\sum F_x = T\cos 14.0° - T\cos 14.0° = 0$$

and

$$\sum F_y = -T\sin 14.0° - T\sin 14.0° = -2T\sin 14.0°$$

Thus, the magnitude of the resultant force exerted on the tooth by the wire brace is

$$R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} = \sqrt{0 + (-2T\sin 14.0°)^2} = 2T\sin 14.0°$$

or

$$R = 2(18.0 \text{ N})\sin 14.0° = \boxed{8.71 \text{ N}}$$

P5.5 We use the particle under constant acceleration and particle under a net force models. We first calculate the acceleration of the puck:

$$\vec{\mathbf{a}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\left(8.00\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}}\right)\text{m/s} - 3.00\hat{\mathbf{i}} \text{ m/s}}{8.00 \text{ s}}$$
$$= 0.625\hat{\mathbf{i}} \text{ m/s}^2 + 1.25\hat{\mathbf{j}} \text{ m/s}^2$$

In $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$, the only horizontal force is the thrust $\vec{\mathbf{F}}$ of the rocket:

(a) $\vec{\mathbf{F}} = (4.00 \text{ kg})\left(0.625\hat{\mathbf{i}} \text{ m/s}^2 + 1.25\hat{\mathbf{j}} \text{ m/s}^2\right) = \boxed{\left(2.50\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right) \text{ N}}$

(b) Its magnitude is $\left|\vec{\mathbf{F}}\right| = \sqrt{(2.50 \text{ N})^2 + (5.00 \text{ N})^2} = \boxed{5.59 \text{ N}}$

P5.6 (a) Let the x axis be in the original direction of the molecule's motion. Then, from $v_f = v_i + at$, we have

$$a = \frac{v_f - v_i}{t} = \frac{-670\text{ m/s} - 670\text{ m/s}}{3.00\times10^{-13}\text{ s}} = \boxed{-4.47\times10^{15}\text{ m/s}^2}$$

(b) For the molecule, $\sum\vec{F} = m\vec{a}$. Its weight is negligible.

$$\vec{F}_{\text{wall on molecule}} = \left(4.68\times10^{-26}\text{ kg}\right)\left(-4.47\times10^{15}\text{ m/s}^2\right)$$
$$= -2.09\times10^{-10}\text{ N}$$
$$\vec{F}_{\text{molecule on wall}} = \boxed{+2.09\times10^{-10}\text{ N}}$$

***P5.7** Imagine a quick trip by jet, on which you do not visit the rest room and your perspiration is just canceled out by a glass of tomato juice. By subtraction, $\left(F_g\right)_p = mg_p$ and $\left(F_g\right)_C = mg_C$ give

$$\Delta F_g = m\left(g_p - g_C\right)$$

For a person whose mass is 90.0 kg, the change in weight is

$$\Delta F_g = 90.0\text{ kg}(9.809\,5 - 9.780\,8) = \boxed{2.58\text{ N}}$$

A precise balance scale, as in a doctor's office, reads the same in different locations because it compares you with the standard masses on its beams. A typical bathroom scale is not precise enough to reveal this difference.

P5.8 The force on the car is given by $\sum\vec{F} = m\vec{a}$, or, in one dimension, $\sum F = ma$. Whether the car is moving to the left or the right, since it's moving at constant speed, $a = 0$ and therefore $\sum F = \boxed{0}$ for both parts (a) and (b).

P5.9 We find the mass of the baseball from its weight: $w = mg$, so $m = w/g = 2.21\text{ N}/9.80\text{ m/s}^2 = 0.226\text{ kg}$.

(a) We use $x_f = x_i + \frac{1}{2}(v_i + v_f)t$ and $x_f - x_i = \Delta x$, with $v_i = 0$, $v_f = 18.0$ m/s, and $\Delta t = t = 170\text{ ms} = 0.170$ s:

$$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$$
$$\Delta x = \frac{1}{2}(0 + 18.0\text{ m/s})(0.170\text{ s}) = \boxed{1.53\text{ m}}$$

(b) We solve for acceleration using $v_{xf} = v_{xi} + a_x t$, which gives

$$a_x = \frac{v_{xf} - v_{xi}}{t}$$

where *a* is in m/s^2, *v* is in m/s, and *t* in s. Substituting gives

$$a_x = \frac{18.0\text{ m/s} - 0}{0.170\text{ s}} = 106\text{ m/s}^2$$

Call $\vec{F}_1$ = force of pitcher on ball, and $\vec{F}_2$ = force of Earth on ball (weight). We know that

$$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 = m\vec{a}$$

Writing this equation in terms of its components gives

$$\sum F_x = F_{1x} + F_{2x} = ma_x \qquad \sum F_y = F_{1y} + F_{2y} = ma_y$$

$$\sum F_x = F_{1x} + 0 = ma_x \qquad \sum F_y = F_{1y} - 2.21\text{ N} = 0$$

Solving,

$$F_{1x} = (0.226\text{ kg})(106\text{ m/s}^2) = 23.9\text{ N} \text{ and } F_{1y} = 2.21\text{ N}$$

Then,

$$F_1 = \sqrt{(F_{1x})^2 + (F_{1y})^2}$$
$$= \sqrt{(23.9\text{ N})^2 + (2.21\text{ N})^2} = 24.0\text{ N}$$

and $\theta = \tan^{-1}\left(\dfrac{2.21\text{ N}}{23.9\text{ N}}\right) = 5.29°$

$\boxed{\text{The pitcher exerts a force of 24.0 N forward at 5.29° above the horizontal.}}$

P5.10 (a) Use $\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$, where $v_i = 0$, $v_f = v$, and $\Delta t = t$:

$$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t = \boxed{\frac{1}{2}vt}$$

(b) Use $v_{xf} = v_{xi} + a_x t$:

$$v_{xf} = v_{xi} + a_x t \rightarrow a_x = \frac{v_{xf} - v_{xi}}{t} \rightarrow a_x = \frac{v - 0}{t} = \frac{v}{t}$$

Call $\vec{\mathbf{F}}_1$ = force of pitcher on ball, and $\vec{\mathbf{F}}_2 = -F_g = -mg =$ gravitational force on ball. We know that

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}$$

writing this equation in terms of its components gives

$$\sum F_x = F_{1x} + F_{2x} = ma_x \qquad \sum F_y = F_{1y} + F_{2y} = ma_y$$

$$\sum F_x = F_{1x} + 0 = ma_x \qquad \sum F_y = F_{1y} - mg = 0$$

Solving and substituting from above,

$$F_{1x} = mv/t \qquad F_{1y} = mg$$

then the magnitude of F_1 is

$$F_1 = \sqrt{(F_{1x})^2 + (F_{1y})^2}$$
$$= \sqrt{(mv/t)^2 + (mg)^2} = \boxed{m\sqrt{(v/t)^2 + g^2}}$$

and its direction is

$$\theta = \tan^{-1}\left(\frac{mg}{mv/t}\right) = \boxed{\tan^{-1}\left(\frac{gt}{v}\right)}$$

P5.11 Since this is a linear acceleration problem, we can use Newton's second law to find the force as long as the electron does not approach relativistic speeds (as long as its speed is much less than 3×10^8 m/s), which is certainly the case for this problem. We know the initial and final velocities, and the distance involved, so from these we can find the acceleration needed to determine the force.

(a) From $v_f^2 = v_i^2 + 2ax$ and $\sum F = ma$, we can solve for the acceleration and then the force: $a = \dfrac{v_f^2 - v_i^2}{2x}$

Substituting to eliminate a, $\sum F = \dfrac{m\left(v_f^2 - v_i^2\right)}{2x}$

Substituting the given information,

$$\sum F = \frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left[\left(7.00\times10^{5}\ \text{m/s}\right)^2 - \left(3.00\times10^{5}\ \text{m/s}\right)^2\right]}{2(0.050\,0\ \text{m})}$$

$$\sum F = \boxed{3.64\times10^{-18}\ \text{N}}$$

(b) The Earth exerts on the electron the force called weight,

$$F_g = mg = (9.11\times10^{-31}\,\text{kg})(9.80\ \text{m/s}^2) = 8.93\times10^{-30}\,\text{N}$$

The accelerating force is

$\boxed{4.08\times10^{11}\ \text{times the weight of the electron.}}$

P5.12 We first find the acceleration of the object:

$$\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i = \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$4.20\ \text{m}\hat{\mathbf{i}} - 3.30\ \text{m}\hat{\mathbf{j}} = 0 + \frac{1}{2}\vec{\mathbf{a}}(1.20\ \text{s})^2 = \left(0.720\ \text{s}^2\right)\vec{\mathbf{a}}$$

$$\vec{\mathbf{a}} = \left(5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}}\right)\ \text{m/s}^2$$

Now $\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ becomes

$$\vec{\mathbf{F}}_g + \vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}$$

$$\vec{\mathbf{F}}_2 = 2.80\ \text{kg}\left(5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}}\right)\ \text{m/s}^2 + (2.80\ \text{kg})\left(9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}}_2 = \boxed{\left(16.3\hat{\mathbf{i}} + 14.6\hat{\mathbf{j}}\right)\ \text{N}}$$

P5.13 (a) $\boxed{\text{Force exerted by spring on hand, to the left; force exerted by spring on wall, to the right.}}$

(b) Force exerted by wagon on handle, downward to the left. Force exerted by wagon on planet, upward. Force exerted by wagon on ground, downward.

(c) Force exerted by football on player, downward to the right. Force exerted by football on planet, upward.

(d) Force exerted by small-mass object on large-mass object, to the left.

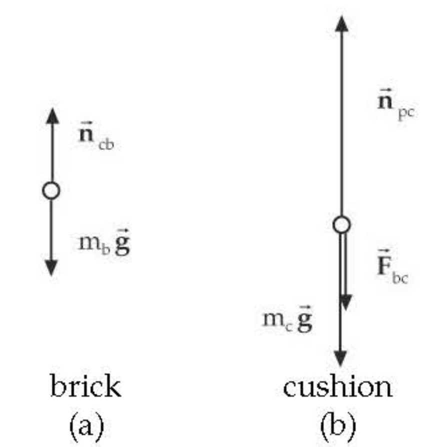
(e) Force exerted by negative charge on positive charge, to the left.

(f) Force exerted by iron on magnet, to the left.

P5.14 The free-body diagrams are shown in ANS. FIG. P5.14 below.

(a) $\vec{\mathbf{n}}_{cb}$ = normal force of cushion on brick
$m_b\vec{\mathbf{g}}$ = gravitational force on brick

(b) $\vec{\mathbf{n}}_{pc}$ = normal force of pavement on cushion
$m_b\vec{\mathbf{g}}$ = gravitational force on cushion
$\vec{\mathbf{F}}_{bc}$ = force of brick on cushion



ANS. FIG.P5.14

(c)

force: normal force of cushion on brick ($\vec{\mathbf{n}}_{cb}$) $\rightarrow$ reaction force: force of brick on cushion ($\vec{\mathbf{F}}_{bc}$)
force: gravitational force of Earth on brick ($m_b\vec{\mathbf{g}}$) $\rightarrow$ reaction force: gravitational force of brick on Earth
force: normal force of pavement on cushion ($\vec{\mathbf{n}}_{pc}$) $\rightarrow$ reaction force: force of cushion on pavement
force: gravitational force of Earth on cushion ($m_c\vec{\mathbf{g}}$) $\rightarrow$ reaction force: gravitational force of cushion on Earth

***P5.15** (a) We start from the sum of the two forces:

$$\sum\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \left(-6.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right) + \left(-3.00\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)$$
$$= \left(-9.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\text{ N}$$

The acceleration is then:

$$\vec{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} = \frac{\sum\vec{\mathbf{F}}}{m} = \frac{\left(-9.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\text{ N}}{2.00\text{ kg}}$$
$$= \left(-4.50\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right)\text{ m/s}^2$$

and the velocity is found from

$$\vec{\mathbf{v}}_f = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \vec{\mathbf{a}}t$$

$$\vec{\mathbf{v}}_f = \left[\left(-4.50\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right)\text{ m/s}^2\right](10.0\text{ s})$$
$$= \boxed{\left(-45.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) The direction of motion makes angle θ with the x direction.

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(-\frac{15.0\text{ m/s}}{45.0\text{ m/s}}\right)$$

$$\theta = -18.4° + 180° = \boxed{162°\text{ from the }+x\text{ axis}}$$

(c) Displacement:

$$x\text{-displacement} = x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$
$$= \frac{1}{2}(-4.50\ \text{m/s}^2)(10.0\ \text{s})^2 = -225\ \text{m}$$

$$y\text{-displacement} = y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2$$
$$= \frac{1}{2}(+1.50\ \text{m/s}^2)(10.0\ \text{s})^2 = +75.0\ \text{m}$$

$$\Delta\vec{\mathbf{r}} = \boxed{\left(-225\hat{\mathbf{i}} + 75.0\hat{\mathbf{j}}\right)\ \text{m}}$$

(d) Position: $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \Delta\vec{\mathbf{r}}$

$$\vec{\mathbf{r}}_f = \left(-2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}\right) + \left(-225\hat{\mathbf{i}} + 75.0\hat{\mathbf{j}}\right) = \boxed{\left(-227\hat{\mathbf{i}} + 79.0\hat{\mathbf{j}}\right)\ \text{m}}$$

***P5.16** Since the two forces are perpendicular to each other, their resultant is

$$F_R = \sqrt{(180\ \text{N})^2 + (390\ \text{N})^2} = 430\ \text{N}$$

at an angle of

$$\theta = \tan^{-1}\left(\frac{390\ \text{N}}{180\ \text{N}}\right) = 65.2°\ \text{N of E}$$

From Newton's second law,

$$a = \frac{F_R}{m} = \frac{430\ \text{N}}{270\ \text{kg}} = 1.59\ \text{m/s}^2$$

or

$$\vec{\mathbf{a}} = \boxed{1.59\ \text{m/s}^2\ \text{at}\ 65.2°\ \text{N of E}}$$

P5.17 (a) With the wind force being horizontal, the only vertical force acting on the object is its own weight, *mg*. This gives the object a downward acceleration of

$$a_y = \frac{\sum F_y}{m} = \frac{-mg}{m} = -g$$

The time required to undergo a vertical displacement $\Delta y = -h$, starting with initial vertical velocity $v_{0y} = 0$, is found from $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ as

$$-h = 0 - \frac{g}{2}t^2 \qquad \text{or} \qquad \boxed{t = \sqrt{\frac{2h}{g}}}$$

(b) The only horizontal force acting on the object is that due to the wind, so $\sum F_x = F$ and the horizontal acceleration will be

$$a_x = \frac{\sum F_x}{m} = \boxed{\frac{F}{m}}$$

(c) With $v_{0x} = 0$, the horizontal displacement the object undergoes while falling a vertical distance *h* is given by $\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$ as

$$\Delta x = 0 + \frac{1}{2}\left(\frac{F}{m}\right)\left(\sqrt{\frac{2h}{g}}\right)^2 = \boxed{\frac{Fh}{mg}}$$

(d) The total acceleration of this object while it is falling will be

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(F/m)^2 + (-g)^2} = \boxed{\sqrt{(F/m)^2 + g^2}}$$

P5.18 For the same force *F*, acting on different masses $F = m_1a_1$ and $F = m_2a_2$. Setting these expressions for *F* equal to one another gives:

(a) $\frac{m_1}{m_2} = \frac{a_2}{a_1} = \boxed{\frac{1}{3}}$

(b) The acceleration of the combined object is found from

$$F = (m_1 + m_2)a = 4m_1a$$

or $$a = \frac{F}{4m_1} = \frac{1}{4}(3.00 \text{ m/s}^2) = \boxed{0.750 \text{ m/s}^2}$$

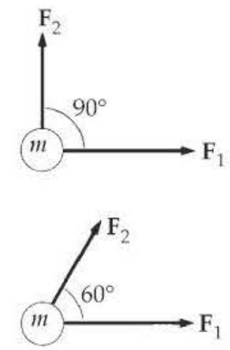
P5.19 We use the particle under a net force model and add the forces as vectors. Then Newton's second law tells us the acceleration.

(a) $\sum \vec{F} = \vec{F}_1 + \vec{F}_2 = (20.0\hat{i} + 15.0\hat{j})$ N

Newton's second law gives, with $m = 5.00$ kg,

$$\vec{a} = \frac{\sum \vec{F}}{m} = \left(4.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right) \text{ m/s}^2$$

or $\boxed{a = 5.00 \text{ m/s}^2 \text{ at } \theta = 36.9°}$



ANS. FIG. P5.19

(b) In this configuration,

$$F_{2x} = 15.0\cos 60.0° = 7.50 \text{ N}$$
$$F_{2y} = 15.0\sin 60.0° = 13.0 \text{ N}$$
$$\vec{F}_2 = \left(7.50\hat{i} + 13.0\hat{j}\right) \text{ N}$$

Then,

$$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 = \left[20.0\hat{i} + \left(7.50\hat{i} + 13.0\hat{j}\right)\right] \text{ N}$$
$$= (27.5\hat{i} + 13.0\hat{j}) \text{ N}$$

and $\vec{a} = \dfrac{\sum \vec{F}}{m} = \left(5.50\hat{\mathbf{i}} + 2.60\hat{\mathbf{j}}\right) \text{ m/s}^2 = \boxed{6.08 \text{ m/s}^2 \text{ at } 25.3°}$

P5.20 (a) You and the Earth exert equal forces on each other: $m_y g = M_E a_E$. If your mass is 70.0 kg,

$$a_E = \frac{(70.0 \text{ kg})(9.80 \text{ m/s}^2)}{5.98 \times 10^{24} \text{ kg}} = \boxed{\sim 10^{-22} \text{ m/s}^2} \quad \textbf{[1]}$$

(b) You and the planet move for equal time intervals Δt according to $\Delta x = \frac{1}{2}a(\Delta t)^2$. If the seat is 50.0 cm high,

$$\sqrt{\frac{2\Delta x_y}{a_y}} = \sqrt{\frac{2\Delta x_E}{a_E}}$$

$$\Delta x_E = \frac{a_E}{a_y}\Delta x_y$$

We substitute for $\frac{a_E}{a_y}$ from [1] to obtain

$$\Delta x_E = \frac{70.0\text{ kg}(0.500\text{ m})}{5.98\times 10^{24}\text{ kg}}$$

$$\boxed{\Delta x_E \sim 10^{-23}\text{ m}}$$

P5.21 (a) $\boxed{\text{15.0 lb up,}}$ to counterbalance the Earth's force on the block.

(b) $\boxed{\text{5.00 lb up,}}$ the forces on the block are now the Earth pulling down with 15.0 lb and the rope pulling up with 10.0 lb. The forces from the floor and rope together balance the weight.

(c) $\boxed{0,}$ the block now accelerates up away from the floor.

P5.22 $\sum \vec{F} = m\vec{a}$ reads

$$\left(-2.00\hat{i} + 2.00\hat{j} + 5.00\hat{i} - 3.00\hat{j} - 45.0\hat{i}\right)\text{ N} = m\left(3.75\text{ m/s}^2\right)\hat{a}$$

where $\hat{a}$ represents the direction of $\vec{a}$:

$$\left(-42.0\hat{i} - 1.00\hat{j}\right)\text{ N} = m\left(3.75\text{ m/s}^2\right)\hat{a}$$

$$\sum \vec{F} = \sqrt{(42.0)^2 + (1.00)^2}\text{ N at } \tan^{-1}\left(\frac{1.00}{42.0}\right)\text{ below the } -x\text{ axis}$$

$$\sum \vec{F} = 42.0\text{ N at } 181° = m\left(3.75\text{ m/s}^2\right)\hat{a}$$

For the vectors to be equal, their magnitudes and their directions must be equal.

(a) Therefore $\boxed{\hat{a}\text{ is at }181°}$ counter-clockwise from the x axis

(b) $m = \frac{42.0\text{ N}}{3.75\text{ m/s}^2} = \boxed{11.2\text{ kg}}$

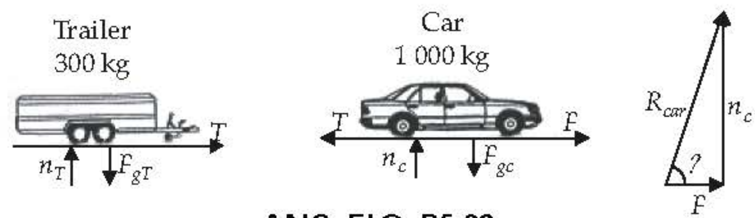
(c) $v = |\vec{v}| = 0 + |\vec{a}|t = (3.75\text{ m/s}^2)(10.00\text{ s}) = \boxed{37.5\text{ m/s}}$

(d) $\vec{\mathbf{v}} = \vec{\mathbf{v}}_i + |\vec{\mathbf{a}}|\,t = 0 + \dfrac{\vec{\mathbf{F}}}{m}t$

$$\vec{\mathbf{v}} = \frac{\left(-42.0\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)\text{ N}}{11.2\text{ kg}}(10.0\text{ s}) = \left(-37.5\hat{\mathbf{i}} - 0.893\hat{\mathbf{j}}\right)\text{ m/s}$$

So, $\vec{\mathbf{v}}_f = \boxed{\left(-37.5\hat{\mathbf{i}} - 0.893\hat{\mathbf{j}}\right)\text{ m/s}}$

* Choose the $+x$ direction to be horizontal and forward with the $+y$ vertical and upward. The common acceleration of the car and trailer then has components of $a_x = +2.15\text{ m/s}^2$ and $a_y = 0$.



ANS. FIG. P5.23

(a) The net force on the car is horizontal and given by

$$\left(\sum F_x\right)_{car} = F - T = m_{car}a_x = (1\,000\text{ kg})(2.15\text{ m/s}^2)$$
$$= \boxed{2.15 \times 10^3\text{ N forward}}$$

(b) The net force on the trailer is also horizontal and given by

$$\left(\sum F_x\right)_{trailer} = +T = m_{trailer}a_x = (300\text{ kg})(2.15\text{ m/s}^2)$$
$$= \boxed{645\text{ N forward}}$$

(c) Consider the free-body diagrams of the car and trailer. The only horizontal force acting on the trailer is $T = 645$ N forward, exerted on the trailer by the car. Newton's third law then states that the force the trailer exerts on the car is $\boxed{645\text{ N toward the rear}}$.

(d) The road exerts two forces on the car. These are F and n_c shown in the free-body diagram of the car. From part (a), $F = T + 2.15 \times 10^3\text{ N} = +2.80 \times 10^3\text{ N}$. Also, $\left(\sum F_y\right)_{car} = n_c - F_{gc} = m_{car}a_y = 0$, so $n_c = F_{gc} = m_{car}g = 9.80 \times 10^3$ N. The resultant force exerted on the car by the road is then

$$R_{car} = \sqrt{F^2 + n_c^2} = \sqrt{\left(2.80 \times 10^3\text{ N}\right)^2 + \left(9.80 \times 10^3\text{ N}\right)^2}$$
$$= 1.02 \times 10^4\text{ N}$$

at $\theta = \tan^{-1}\left(\frac{n_c}{F}\right) = \tan^{-1}(3.51) = 74.1°$ above the horizontal and forward. Newton's third law then states that the resultant force exerted on the road by the car is

$\boxed{1.02 \times 10^4 \text{ N at } 74.1° \text{ below the horizontal and rearward}}$.

P5.24 $v = v_i - kx$ implies the acceleration is given by

$$a = \frac{dv}{dt} = 0 - k\frac{dx}{dt} = -kv$$

Then the total force is

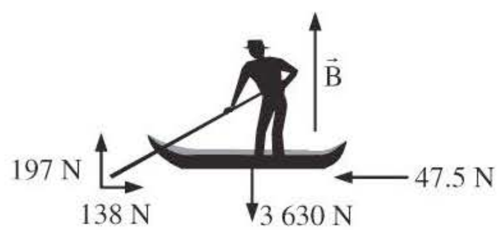
$$\sum F = ma = m(-kv)$$

The resistive force is opposite to the velocity:

$$\boxed{\sum \vec{F} = -km\vec{v}}$$

Section 5.7 Analysis Models Using Newton's Second Law

P5.25 As the worker through the pole exerts on the lake bottom a force of 240 N downward at 35° behind the vertical, the lake bottom through the pole exerts a force of 240 N upward at 35° ahead of the vertical. With the x axis horizontally forward, the pole force on the boat is



ANS. FIG. P5.25

$$\left(240\cos 35°\hat{j} + 240\sin 35°\hat{i}\right) \text{ N} = \left(138\hat{i} + 197\hat{j}\right) \text{ N}$$

The gravitational force of the whole Earth on boat and worker is $F_g = mg = 370 \text{ kg } (9.8 \text{ m/s}^2) = 3\,630 \text{ N}$ down. The acceleration of the boat is purely horizontal, so

$$\sum F_y = ma_y \text{ gives } +B + 197 \text{ N} - 3\,630 \text{ N} = 0$$

(a) The buoyant force is $B = \boxed{3.43 \times 10^3 \text{ N}}$.

(b) The acceleration is given by

$$\sum F_x = ma_x: \quad +138\text{ N} - 47.5\text{ N} = (370\text{ kg})a$$

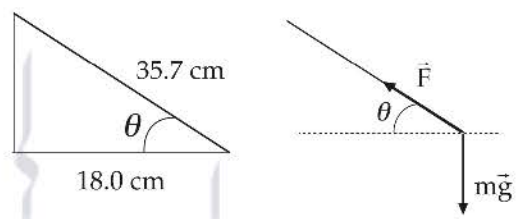
$$a = \frac{90.2\text{ N}}{370\text{ kg}} = 0.244\text{ m/s}^2$$

According to the constant-acceleration model,

$$\begin{aligned} v_{xf} &= v_{xi} + a_x t \\ &= 0.857\text{ m/s} + (0.244\text{ m/s}^2)(0.450\text{ s}) \\ &= 0.967\text{ m/s} \end{aligned}$$

$$\vec{\mathbf{v}}_f = \boxed{0.967\hat{\mathbf{i}}\text{ m/s}}$$

P5.26 (a) The left-hand diagram in ANS. FIG. P5.26(a) shows the geometry of the situation and lets us find the angle of the string with the horizontal:



ANS. FIG. P5.26(a)

$$\cos\theta = 28/35.7 = 0.784$$

or $\theta = 38.3°$

The right-hand diagram in ANS. FIG. P5.26(a) is the free-body diagram. The weight of the bolt is

$$w = mg = (0.065\text{ kg})(9.80\text{ m/s}^2) = 0.637\text{ N}$$

(b) To find the tension in the string, we apply Newton's second law in the x and y directions:

$$\sum F_x = ma_x: -T\cos 38.3° + F_{\text{magnetic}} = 0 \qquad \textbf{[1]}$$

$$\sum F_y = ma_y: +T\sin 38.3° - 0.637\text{ N} = 0 \qquad \textbf{[2]}$$

from equation [2],

$$T = \frac{0.637\text{ N}}{\sin 38.3°} = \boxed{1.03\text{ N}}$$

(c) Now, from equation [1],

$$F_{\text{magnetic}} = T\cos 38.3° = (1.03\text{ N})\cos 38.3° = \boxed{0.805\text{ N to the right}}$$

P5.27 (a) $\boxed{\begin{array}{l}P\cos 40.0° - n = 0 \text{ and } P\sin 40.0° - 220\text{ N} = 0\\ P = 342\text{ N and } n = 262\text{ N}\end{array}}$

(b) $\boxed{\begin{array}{l}P - n\cos 40.0° - 220\text{ N}\sin 40.0° = 0\\ \text{and } n\sin 40.0° - 220\text{ N}\cos 40.0° = 0\\ n = 262\text{ N and } P = 342\text{ N.}\end{array}}$

(c) The results agree. The methods are basically of the same level of difficulty. Each involves one equation in one unknown and one equation in two unknowns. If we are interested in n without finding P, method (b) is simpler.

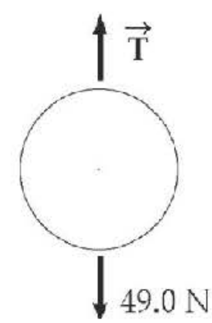
P5.28 (a) Isolate either mass:

$$T + mg = ma = 0$$

$$|T| = |mg|$$

The scale reads the tension T, so

$$T = mg = (5.00\text{ kg})(9.80\text{ m/s}^2) = \boxed{49.0\text{ N}}$$



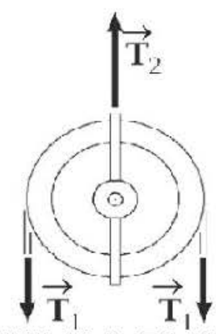
ANS. FIG. P5.28 (a) and (b)

(b) The solution to part (a) is also the solution to (b).

(c) Isolate the pulley:

$$\vec{T}_2 + 2\vec{T}_1 = 0$$

$$T_2 = 2|T_1| = 2mg = \boxed{98.0\text{ N}}$$



ANS. FIG. P5.28(c)

(d) $\sum \vec{F} = \vec{n} + \vec{T} + m\vec{g} = 0$

Take the component along the incline,

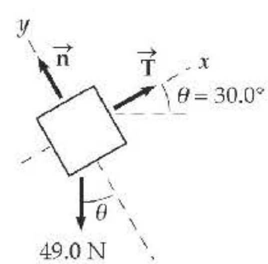
$$n_x + T_x + mg_x = 0$$

or $\quad 0 + T - mg\sin 30.0° = 0$

$$T = mg\sin 30.0° = \frac{mg}{2}$$

$$= \frac{(5.00\text{ kg})(9.80\text{ m/s}^2)}{2}$$

$$= \boxed{24.5\text{ N}}$$



ANS. FIG. P5.28(d)

***P5.29** (a) The resultant external force acting on this system, consisting of all three blocks having a total mass of 6.0 kg, is 42 N directed horizontally toward the right. Thus, the acceleration produced is

$$a = \frac{\sum F}{m} = \frac{42\text{ N}}{6.0\text{ kg}} = \boxed{7.0\text{ m/s}^2\text{ horizontally to the right}}$$

(b) Draw a free-body diagram of the 3.0-kg block and apply Newton's second law to the horizontal forces acting on this block:

$$\sum F_x = ma_x:$$

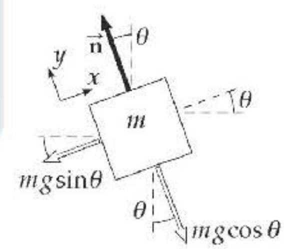
$$42\text{ N} - T = (3.0\text{ kg})(7.0\text{ m/s}^2) \quad \rightarrow \quad T = \boxed{21\text{ N}}$$

(c) The force accelerating the 2.0-kg block is the force exerted on it by the 1.0-kg block. Therefore, this force is given by

$$F = ma = (2.0\text{ kg})(7.0\text{ m/s}^2) = 14\text{ N}$$

or $\vec{F} = \boxed{14\text{ N horizontally to the right}}$

P5.30 (a) ANS. FIG. P5.30 shows the forces on the object. The two forces acting on the block are the normal force, n, and the weight, mg. If the block is considered to be a point mass and the x axis is chosen to be parallel to the plane, then the free-body diagram will be as shown in the figure to the right. The angle θ is the angle of inclination of the plane. Applying Newton's second law for the accelerating system (and taking the direction up the plane as the positive x direction), we have



ANS. FIG. P5.30(a)

$$\sum F_y = n - mg\cos\theta = 0: \; n = mg\cos\theta$$

$$\sum F_x = -mg\sin\theta = ma: \; a = -g\sin\theta$$

(b) When $\theta = 15.0°$,

$$a = \boxed{-2.54\text{ m/s}^2}$$

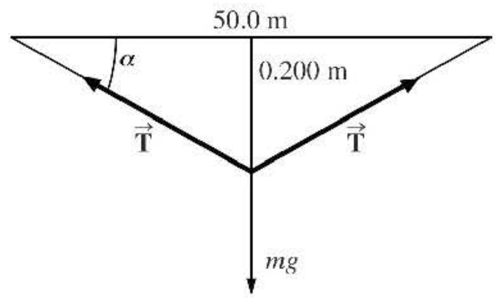
(c) Starting from rest,

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 2a\Delta x$$

$$\left|v_f\right| = \sqrt{2|a|\Delta x} = \sqrt{2\left|-2.54 \text{ m/s}^2\right|(2.00 \text{ m})} = \boxed{3.19 \text{ m/s}}$$

P5.31 We use Newton's second law with the forces in the x and y directions in equilibrium.

(a) At the point where the bird is perched, the wire's midpoint, the forces acting on the wire are the tension forces and the force of gravity acting on the bird. These forces are shown in ANS. FIG. P5.31(a) below.



ANS. FIG. P5.31(a)

(b) The mass of the bird is $m = 1.00$ kg, so the force of gravity on the bird, its weight, is $mg = (1.00 \text{ kg})(9.80 \text{ m/s}^2) = 9.80$ N. To calculate the angle α in the free-body diagram, we note that the base of the triangle is 25.0 m, so that

$$\tan\alpha = \frac{0.200 \text{ m}}{25.0 \text{ m}} \quad \rightarrow \quad \alpha = 0.458°$$

Each of the tension forces has x and y components given by

$$T_x = T\cos\alpha \quad \text{and} \quad T_y = T\sin\alpha$$

The x components of the two tension forces cancel out. In the y direction,

$$\sum F_y = 2T\sin\alpha - mg = 0$$

which gives

$$T = \frac{mg}{2\sin\alpha} = \frac{9.80 \text{ N}}{2\sin 0.458°} = \boxed{613 \text{ N}}$$

P5.32 To find the net force, we differentiate the equations for the position of the particle once with respect to time to obtain the velocity, and once again to obtain the acceleration:

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left(5t^2 - 1\right) = 10t, \quad v_y = \frac{dy}{dt} = \frac{d}{dt}\left(3t^3 + 2\right) = 9t^2$$

$$a_x = \frac{dv_x}{dt} = 10, \quad a_y = \frac{dv_y}{dt} = 18t$$

Then, at $t = 2.00$ s, $a_x = 10.0\ \text{m/s}^2$, $a_y = 36.0\ \text{m/s}^2$, and Newton's second law gives us

$$\sum F_x = ma_x: \ 3.00\ \text{kg}(10.0\ \text{m/s}^2) = 30.0\ \text{N}$$

$$\sum F_y = ma_y: \ 3.00\ \text{kg}(36.0\ \text{m/s}^2) = 108\ \text{N}$$

$$\sum F = \sqrt{F_x^2 + F_y^2} = \boxed{112\ \text{N}}$$

P5.33 From equilibrium of the sack:

$$T_3 = F_g \qquad [1]$$

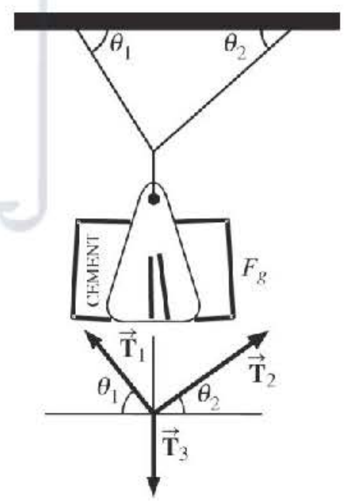
From $\sum F_y = 0$ for the knot:

$$T_1 \sin\theta_1 + T_2 \sin\theta_2 = F_g \qquad [2]$$

From $\sum F_x = 0$ for the knot:

$$T_1 \cos\theta_1 = T_2 \cos\theta_2 \qquad [3]$$

Eliminate T_2 by using $T_2 = T_1 \cos\theta_1 / \cos\theta_2$ and solve for T_1:



ANS. FIG. P5.33

$$T_1 = \frac{F_g \cos\theta_2}{\left(\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2\right)} = \frac{F_g \cos\theta_2}{\sin\left(\theta_1 + \theta_2\right)}$$

$$T_3 = F_g = \boxed{325\ \text{N}}$$

$$T_1 = F_g\left(\frac{\cos 40.0°}{\sin 100.0°}\right) = \boxed{253\ \text{N}}$$

$$T_2 = T_1\left(\frac{\cos\theta_1}{\cos\theta_2}\right) = (253\ \text{N})\left(\frac{\cos 60.0°}{\cos 40.0°}\right) = \boxed{165\ \text{N}}$$

P5.34 See the solution for T_1 in Problem 5.33. The equations indicate that the tension is directly proportional to F_g.

***P5.35** Let us call the forces exerted by each person F_1 and F_2. Thus, for pulling in the same direction, Newton's second law becomes

$$F_1 + F_2 = (200 \text{ kg})(1.52 \text{ m/s}^2)$$

or $$F_1 + F_2 = 304 \text{ N} \qquad \textbf{[1]}$$

When pulling in opposite directions,

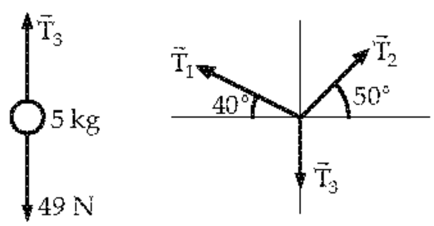
$$F_1 - F_2 = (200 \text{ kg})(-0.518 \text{ m/s}^2)$$

or $$F_1 - F_2 = -104 \text{ N} \qquad \textbf{[2]}$$

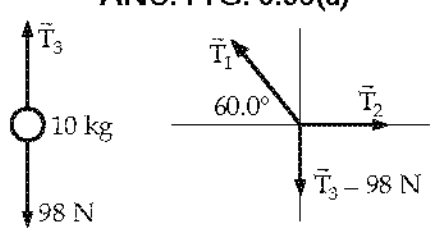
Solving [1] and [2] simultaneously, we find

$$F_1 = \boxed{100 \text{ N}} \text{ and } F_2 = \boxed{204 \text{ N}}$$

***P5.36** (a) First construct a free-body diagram for the 5.00-kg mass as shown in the Figure 5.36a. Since the mass is in equilibrium, we can require $T_3 - 49.0 \text{ N} = 0$ or $T_3 = 49.0$ N. Next, construct a free-body diagram for the knot as shown in ANS. FIG. P5.36(a).



ANS. FIG. 5.36(a)



ANS. FIG. 5.36(b)

Again, since the system is moving at constant velocity, $a = 0$, and applying Newton's second law in component form gives

$$\sum F_x = T_2 \cos 50.0° - T_1 \cos 40.0° = 0$$
$$\sum F_y = T_2 \sin 50.0° + T_1 \sin 40.0° - 49.0 \text{ N} = 0$$

Solving the above equations simultaneously for T_1 and T_2 gives $\boxed{T_1 = 31.5 \text{ N}}$ and $\boxed{T_2 = 37.5 \text{ N}}$ and above we found $\boxed{T_3 = 49.0 \text{ N}}$.

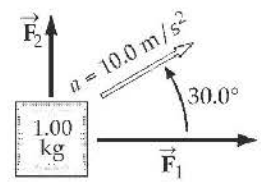
(b) Proceed as in part (a) and construct a free-body diagram for the mass and for the knot as shown in ANS. FIG. P5.36(b). Applying Newton's second law in each case (for a constant-velocity system), we find:

$$T_3 - 98.0\text{ N} = 0$$
$$T_2 - T_1\cos 60.0° = 0$$
$$T_1\sin 60.0° - T_3 = 0$$

Solving this set of equations we find:

$$\boxed{T_1 = 113\text{ N},}\ \boxed{T_2 = 56.6\text{ N},}\ \text{and}\ \boxed{T_3 = 98.0\text{ N}}$$

P5.37 Choose a coordinate system with $\hat{\mathbf{i}}$ East and $\hat{\mathbf{j}}$ North. The acceleration is



ANS. FIG. P5.37

$$\vec{\mathbf{a}} = [(10.0\cos 30.0°)\hat{\mathbf{i}} + (10.0\sin 30.0°)\hat{\mathbf{j}}]\text{ m/s}^2$$
$$= (8.66\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})\text{ m/s}^2$$

From Newton's second law,

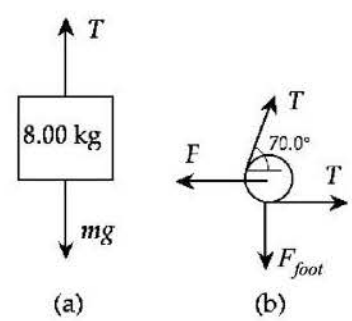
$$\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}} = (1.00\text{ kg})(8.66\hat{\mathbf{i}}\text{ m/s}^2 + 5.00\hat{\mathbf{j}}\text{ m/s}^2)$$
$$= (8.66\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})\text{ N}$$

and $\sum\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$

So the force we want is

$$\vec{\mathbf{F}}_1 = \sum\vec{\mathbf{F}} - \vec{\mathbf{F}}_2 = (8.66\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 5.00\hat{\mathbf{j}})\text{ N}$$
$$= 8.66\hat{\mathbf{i}}\text{ N} = \boxed{8.66\text{ N east}}$$

P5.38 (a) Assuming frictionless pulleys, the tension is uniform through the entire length of the rope. Thus, the tension at the point where the rope attaches to the leg is the same as that at the 8.00-kg block. ANS. FIG. P5.38(a) gives a free-body diagram of the suspended block. Recognizing that the block has zero acceleration, Newton's second law gives



ANS. FIG. P5.38

$$\sum F_y = T - mg = 0$$

or

$$T = mg = (8.00 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{78.4 \text{ N}}$$

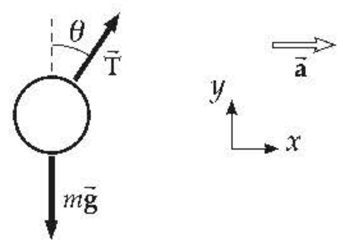
(b) ANS. FIG. P5.38(b) gives a free-body diagram of the pulley near the foot. Here, F is the magnitude of the force the foot exerts on the pulley. By Newton's third law, this is the same as the magnitude of the force the pulley exerts on the foot. Applying the second law gives

$$\sum F_x = T + T\cos 70.0° - F = ma_x = 0$$

or

$$F = T(1+\cos 70.0°) = (78.4 \text{ N})(1+\cos 70.0°) = \boxed{105 \text{ N}}$$

***P5.39** (a) Assume the car and mass accelerate horizontally. We consider the forces on the suspended object.



ANS. FIG. P5.39

$$\sum F_y = ma_y: \ +T\cos\theta - mg = 0$$

$$\sum F_x = ma_x: \ +T\sin\theta = ma$$

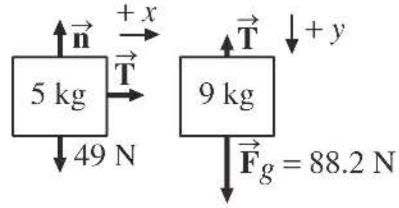
Substitute $T = \dfrac{mg}{\cos\theta}$ from the first equation into the second,

$$\frac{mg\sin\theta}{\cos\theta} = mg\tan\theta = ma$$

$$\boxed{a = g\tan\theta}$$

(b) $a = (9.80 \text{ m/s}^2)\tan 23.0° = \boxed{4.16 \text{ m/s}^2}$

P5.40 (a) The forces on the objects are shown in ANS. FIG. P5.40.



ANS. FIG. P5.40

(b) and (c) First, consider m_1, the block moving along the horizontal. The only force in the direction of movement is T. Thus,

$$\sum F_x = ma$$

or $\quad T = (5.00\text{ kg})a$ **[1]**

Next consider m_2, the block that moves vertically. The forces on it are the tension T and its weight, 88.2 N.

We have $\sum F_y = ma$:

$$88.2\text{ N} - T = (9.00\text{ kg})a \qquad \textbf{[2]}$$

Note that both blocks must have the same magnitude of acceleration. Equations [1] and [2] can be added to give 88.2 N = (14.0 kg)a. Then

$$\boxed{a = 6.30\text{ m/s}^2 \text{ and } T = 31.5\text{ N}}$$

P5.41 (a) and (b) The slope of the graph of upward velocity versus time is the acceleration of the person's body. At both time 0 and time 0.5 s, this slope is (18 cm/s)/0.6 s = 30 cm/s^2.

For the person's body,

$$\sum F_y = ma_y:$$
$$+F_{bar} - (64.0\text{ kg})(9.80\text{ m/s}^2) = (64.0\text{ kg})(0.3\text{ m/s}^2)$$

Note that there is no floor touching the person to exert a normal force, and that he does not exert any extra force "on himself." Solving, $F_{bar} = \boxed{646\text{ N up}}$.

(c) a_y = slope of v_y versus t graph = 0 at t = 1.1 s. The person is moving with maximum speed and is momentarily in equilibrium:

$$\sum F_y = ma_y:$$
$$+F_{bar} - (64.0\text{ kg})(9.80\text{ m/s}^2) = 0$$

$$F_{bar} = \boxed{627\text{ N up}}$$

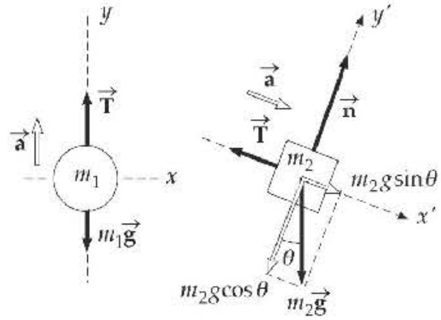
(d) a_y = slope of v_y versus t graph = (0 – 24 cm/s)/(1.7 s – 1.3 s) = –60 cm/s^2

$$\sum F_y = ma_y:$$
$$+F_{bar} - (64.0\text{ kg})(9.80\text{ m/s}^2) = (64.0\text{ kg})(-0.6\text{ m/s}^2)$$

$$F_{bar} = \boxed{589\text{ N up}}$$

P5.42 $m_1 = 2.00 \text{ kg}, m_2 = 6.00 \text{ kg}, \theta = 55.0°$

(a) The forces on the objects are shown in ANS. FIG. P5.42.



ANS. FIG. P5.42

(b) $\sum F_x = m_2 g \sin\theta - T = m_2 a$ and

$$T - m_1 g = m_1 a$$

$$a = \frac{m_2 g \sin\theta - m_1 g}{m_1 + m_2}$$

$$= \frac{(6.00 \text{ kg})(9.80 \text{ m/s}^2)\sin 55.0° - (2.00 \text{ kg})(9.80 \text{ m/s}^2)}{2.00 \text{ kg} + 6.00 \text{ kg}}$$

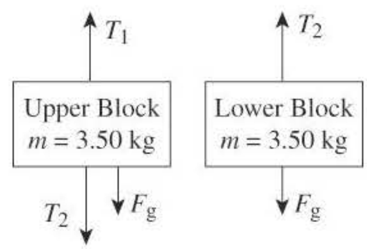
$$= \boxed{3.57 \text{ m/s}^2}$$

(c) $T = m_1(a + g) = (2.00 \text{ kg})(3.57 \text{ m/s}^2 + 9.80 \text{ m/s}^2) = \boxed{26.7 \text{ N}}$

(d) Since $v_i = 0$, $v_f = at = (3.57 \text{ m/s}^2)(2.00 \text{ s}) = \boxed{7.14 \text{ m/s}}$.

P5.43 (a) Free-body diagrams of the two blocks are shown in ANS. FIG. P5.43. Note that each block experiences a downward gravitational force

$$F_g = (3.50 \text{ kg})(9.80 \text{ m/s}^2) = 34.3 \text{ N}$$



ANS. FIG. P5.43

Also, each has the same upward acceleration as the elevator, in this case $a_y = +1.60 \text{ m/s}^2$.

Applying Newton's second law to the lower block:

$$\sum F_y = ma_y \quad \Rightarrow \quad T_2 - F_g = ma_y$$

or

$$T_2 = F_g + ma_y = 34.3 \text{ N} + (3.50 \text{ kg})(1.60 \text{ m/s}^2) = \boxed{39.9 \text{ N}}$$

Next, applying Newton's second law to the upper block:

$$\sum F_y = ma_y \quad \Rightarrow \quad T_1 - T_2 - F_g = ma_y$$

or

$$T_1 = T_2 + F_g + ma_y = 39.9 \text{ N} + 34.3 \text{ N} + (3.50 \text{ kg})(1.60 \text{ m/s}^2)$$

$$= \boxed{79.8 \text{ N}}$$

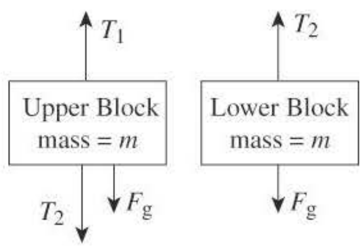
(b) Note that the tension is greater in the upper string, and this string will break first as the acceleration of the system increases. Thus, we wish to find the value of a_y when $T_1 = 85.0$. Making use of the general relationships derived in (a) above gives:

$$T_1 = T_2 + F_g + ma_y = \left(F_g + ma_y\right) + F_g + ma_y = 2F_g + 2ma_y$$

or

$$a_y = \frac{T_1 - 2F_g}{2m} = \frac{85.0\text{ N} - 2(34.3\text{ N})}{2(3.50\text{ kg})} = \boxed{2.34\text{ m/s}^2}$$

P5.44 (a) Free-body diagrams of the two blocks are shown in ANS. FIG. P5.44. Note that each block experiences a downward gravitational force $F_g = mg$.



ANS. FIG. P5.44

Also, each has the same upward acceleration as the elevator, $a_y = +a$.

Applying Newton's second law to the lower block:

$$\sum F_y = ma_y \quad \Rightarrow \quad T_2 - F_g = ma_y$$

or $T_2 = mg + ma = \boxed{m(g + a)}$

Next, applying Newton's second law to the upper block:

$$\sum F_y = ma_y \quad \Rightarrow \quad T_1 - T_2 - F_g = ma_y$$

or

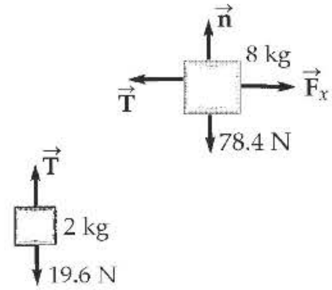
$$T_1 = T_2 + F_g + ma_y = (mg + ma) + mg + ma = 2(mg + ma)$$
$$= \boxed{2m(g + a)} = 2T_2$$

(b) Note that $\boxed{T_1 = 2T_2\text{, so the upper string breaks first}}$ as the acceleration of the system increases.

(c) When the upper string breaks, both blocks will be in free fall with $a = -g$. Then, using the results of part (a), $T_2 = m(g + a) = m(g - g) = \boxed{0}$ and $T_1 = 2T_2 = \boxed{0}$.

P5.45 Forces acting on $m_1 = 2.00$-kg block:

$$T - m_1 g = m_1 a \qquad [1]$$



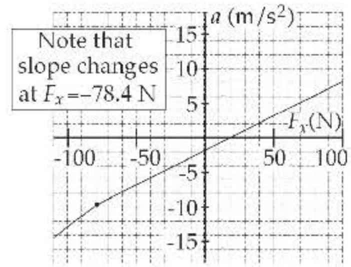
Forces acting on $m_2 = 8.00$-kg block:

$$F_x - T = m_2 a \qquad [2]$$

(a) Eliminate T and solve for a:

$$a = \frac{F_x - m_1 g}{m_1 + m_2}$$

$$\boxed{a > 0 \text{ for } F_x > m_1 g = 19.6 \text{ N}}$$



ANS. FIG. P5.45

(b) Eliminate a and solve for T:

$$T = \frac{m_1}{m_1 + m_2}(F_x + m_2 g)$$

$$\boxed{T = 0 \text{ for } F_x \le -m_2 g = -78.4 \text{ N}}$$

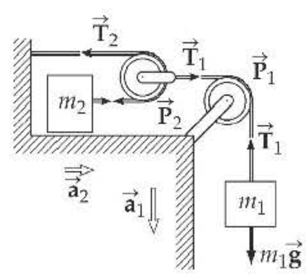
Note that if $F_x < -m_2 g$, the cord is loose, so mass m_2 is in free fall and mass m_1 accelerates under the action of F_x only.

(c) See ANS. FIG. P5.45.

F_x, N	−100	−78.4	−50.0	0	50.0	100
a_x, m/s^2	−12.5	−9.80	−6.96	−1.96	3.04	8.04

P5.46 (a) Pulley P_2 has acceleration a_1.

Since m_2 moves *twice* the distance P_2 moves in the same time, m_2 has twice the acceleration of P_2, i.e., $\boxed{a_2 = 2a_1}$.



ANS. FIG. P5.46

(b) From the figure, and using

$$\sum F = ma: \quad m_1 g - T_1 = m_1 a_1 \qquad [1]$$

$$T_2 = m_2 a_2 = 2 m_2 a_1 \qquad [2]$$

$$T_1 - 2T_2 = 0 \qquad [3]$$

Equation [1] becomes $m_1g - 2T_2 = m_1a_1$. This equation combined with equation [2] yields

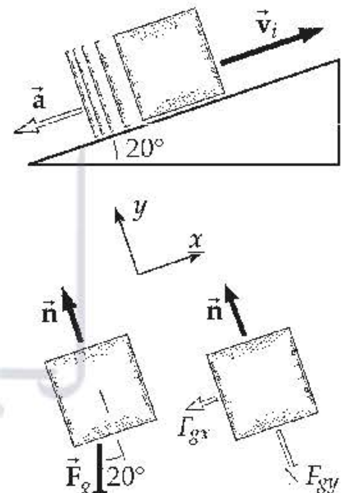
$$\frac{T_2}{m_2}\left(2m_2 + \frac{m_2}{2}\right) = m_1 g$$

$$\boxed{T_2 = \frac{m_1 m_2}{2m_2 + \frac{1}{2}m_1} g} \quad \text{and} \quad \boxed{T_2 = \frac{m_1 m_2}{m_2 + \frac{1}{4}m_1} g}$$

(c) From the values of T_2 and T_1, we find that

$$a_2 = \frac{T_2}{m_2} = \boxed{\frac{m_1 g}{2m_2 + \frac{1}{2}m_1}} \quad \text{and } a_1 = \frac{1}{2}a_2 = \boxed{\frac{m_1 g}{4m_2 + m_1}}$$

***P5.47** We use the particle under constant acceleration and particle under a net force models. Newton's law applies for each axis. After it leaves your hand, the block's speed changes only because of one component of its weight:



ANS. FIG. P5.47

$$\sum F_x = ma_x$$
$$-mg\sin 20.0° = ma$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

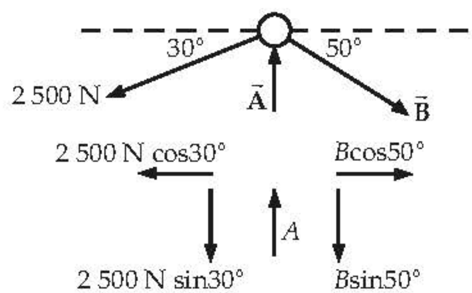
Taking $v_f = 0$, $v_i = 5.00$ m/s, and $a = -g\sin(20.0°)$ gives, suppressing units,

$$0 = (5.00)^2 - 2(9.80)\sin(20.0°)(x_f - 0)$$

or

$$x_f = \frac{25.0}{2(9.80)\sin(20.0°)} = \boxed{3.73\text{ m}}$$

***P5.48** We assume the vertical bar is in compression, pushing up on the pin with force *A*, and the tilted bar is in tension, exerting force *B* on the pin at –50.0°.



ANS. FIG. P5.48

$\sum F_x = 0$:

$$-2\,500\text{ N}\cos 30° + B\cos 50° = 0$$

$$\boxed{B = 3.37 \times 10^3\text{ N}}$$

$\sum F_y = 0$:

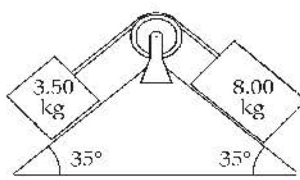
$$-2\,500\text{ N}\sin 30° + A - \left(3.37 \times 10^3\text{ N}\right)\sin 50° = 0$$

$$\boxed{A = 3.83 \times 10^3\text{ N}}$$

Positive answers confirm that

$$\boxed{\text{B is in tension and A is in compression.}}$$

P5.49 Since it has a larger mass, we expect the 8.00-kg block to move down the plane. The acceleration for both blocks should have the same magnitude since they are joined together by a non-stretching string. Define up the left-hand plane as positive for the 3.50-kg object and down the right-hand plane as positive for the 8.00-kg object.



ANS. FIG. P5.49

$$\sum F_1 = m_1 a_1: \quad -m_1 g\sin 35.0° + T = m_1 a$$
$$\sum F_2 = m_2 a_2: \quad m_2 g\sin 35.0° - T = m_2 a$$

and, suppressing units,

$$-(3.50)(9.80)\sin 35.0° + T = 3.50a$$
$$(8.00)(9.80)\sin 35.0° - T = 8.00a.$$

Adding, we obtain $+45.0\text{ N} - 19.7\text{ N} = (11.5\text{ kg})a$.

(a) Thus the acceleration is $\boxed{a = 2.20\text{ m/s}^2}$. By substitution,

$$-19.7\text{ N} + T = (3.50\text{ kg})(2.20\text{ m/s}^2) = 7.70\text{ N}$$

(b) The tension is $\boxed{T = 27.4\text{ N}}$

P5.50 Both blocks move with acceleration $a=\left(\dfrac{m_2-m_1}{m_2+m_1}\right)g$:

$$a=\left(\frac{7\text{ kg}-2\text{ kg}}{7\text{ kg}+2\text{ kg}}\right)(9.80\text{ m/s}^2)=5.44\text{ m/s}^2$$

(a) Take the upward direction as positive for m_1.

$$v_{yf}^2=v_{yi}^2+2a_x(y_f-y_i)$$

$$0=(-2.4\text{ m/s})^2+2(5.44\text{ m/s}^2)(y_f-0)$$

$$y_f=-\frac{5.76\text{ m}^2/\text{s}^2}{2(5.44\text{ m/s}^2)}=-0.529\text{ m}$$

$$y_f=\boxed{0.529\text{ m below its initial level}}$$

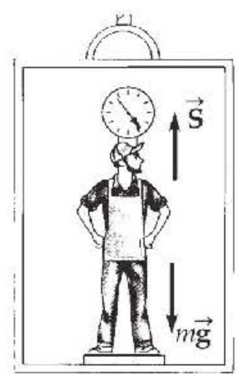
(b) $v_{yf}=v_{yi}+a_yt$: $v_{yf}=-2.40\text{ m/s}+(5.44\text{ m/s}^2)(1.80\text{ s})$

$$v_{yf}=\boxed{7.40\text{ m/s upward}}$$

P5.51 We draw a force diagram and apply Newton's second law for each part of the elevator trip to find the scale force. The acceleration can be found from the change in speed divided by the elapsed time.

Consider the force diagram of the man shown as two arrows. The force F is the upward force exerted on the man by the scale, and his weight is

$$F_g=mg=(72.0\text{ kg})(9.80\text{ m/s}^2)=706\text{ N}$$



ANS. FIG. P5.51

With $+y$ defined to be upwards, Newton's second law gives

$$\sum F_y=+F_s-F_g=ma$$

Thus, we calculate the upward scale force to be

$$F_s=706\text{ N}+(72.0\text{ kg})a \qquad [1]$$

where a is the acceleration the man experiences as the elevator changes speed.

(a) Before the elevator starts moving, the elevator's acceleration is zero ($a = 0$). Therefore, equation [1] gives the force exerted by the scale on the man as 706 N upward, and the man exerts a downward force of $\boxed{706\text{ N}}$ on the scale.

(b) During the first 0.800 s of motion, the man accelerates at a rate of

$$a_x = \frac{\Delta v}{\Delta t} = \frac{1.20\text{ m/s} - 0}{0.800\text{ s}} = 1.50\text{ m/s}^2$$

Substituting a into equation [1] then gives

$$F = 706\text{ N} + (72.0\text{ kg})(1.50\text{ m/s}^2) = \boxed{814\text{ N}}$$

(c) While the elevator is traveling upward at constant speed, the acceleration is zero and equation [1] again gives a scale force $F = \boxed{706\text{ N}}$.

(d) During the last 1.50 s, the elevator first has an upward velocity of 1.20 m/s, and then comes to rest with an acceleration of

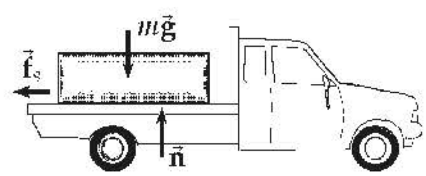
$$a = \frac{\Delta v}{\Delta t} = \frac{0 - 1.20\text{ m/s}}{1.50\text{ s}} = -0.800\text{ m/s}^2$$

Thus, the force of the man on the scale is

$$F = 706\text{ N} + (72.0\text{ kg})(-0.800\text{ m/s}^2) = \boxed{648\text{ N}}$$

Section 5.8 Forces of Friction

***P5.52** If the load is on the point of sliding forward on the bed of the slowing truck, static friction acts backward on the load with its maximum value, to give it the same acceleration as the truck:



ANS. FIG. P5.52

$$\Sigma F_x = ma_x: \quad -f_s = m_{load} a_x$$

$$\Sigma F_y = ma_y: \quad n - m_{load} g = 0$$

Solving for the normal force and substituting into the x equation gives:

$$-\mu_s m_{load} g = m_{load} a_x \qquad \text{or} \qquad a_x = -\mu_s g$$

We can then use

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Which becomes

$$0 = v_{xi}^2 + 2(-\mu_s g)\left(x_f - 0\right)$$

(a) $$x_f = \frac{v_{xi}^2}{2\mu_s g} = \frac{(12.0\ \text{m/s})^2}{2(0.500)(9.80\ \text{m/s}^2)} = \boxed{14.7\ \text{m}}$$

(b) From the expression $x_f = \dfrac{v_{xi}^2}{2\mu_s g}$,

$\boxed{\text{neither mass affects the answer}}$

P5.53 Using $m = 12.0 \times 10^{-3}$ kg, $v_i = 260$ m/s, $v_f = 0$, $\Delta x = (x_f - x_i) = 0.230$ m, and $v_f^2 = v_i^2 + 2a(x_f - x_i)$, we find the acceleration of the bullet: $a = -1.47 \times 10^5\ \text{m/s}^2$. Newton's second law then gives

$$\sum F_x = ma_x$$

$$f_k = ma = -1.76 \times 10^5\ \text{N}$$

The (kinetic) friction force is $\boxed{1.76 \times 10^5\ \text{N in the negative } x \text{ direction}}$.

P5.54 We apply Newton's second law to the car to determine the maximum static friction force acting on the car:

$$\sum F_y = ma_y: \quad +n - mg = 0$$

$$f_s \le \mu_s n = \mu_s mg$$

This maximum magnitude of static friction acts so long as the tires roll without skidding.

$$\sum F_x = ma_x \rightarrow -f_s = ma$$

The maximum acceleration is

$$a = -\mu_s g$$

The initial and final conditions are: $x_i = 0$, $v_i = 50.0$ mi/h $= 22.4$ m/s, and $v_f = 0$. Then,

$$v_f^2 = v_i^2 + 2a(x_f - x_i) \rightarrow v_i^2 = 2\mu_s g x_f$$

(a) $$x_f = \frac{v_i^2}{2\mu_s g}$$

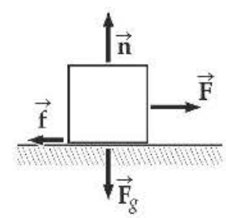
$$x_f = \frac{(22.4 \text{ m/s})^2}{2(0.100)(9.80 \text{ m/s}^2)} = \boxed{256 \text{ m}}$$

(b) $$x_f = \frac{v_i^2}{2\mu_s g}$$

$$x_f = \frac{(22.4 \text{ m/s})^2}{2(0.600)(9.80 \text{ m/s}^2)} = \boxed{42.7 \text{ m}}$$

P5.55 For equilibrium: $f = F$ and $n = F_g$. Also, $f = \mu n$, i.e.,

$$\mu = \frac{f}{n} = \frac{F}{F_g}$$



ANS. FIG. P5.55

In parts (a) and (b), we replace F with the magnitude of the applied force and μ with the appropriate coefficient of friction.

(a) The coefficient of static friction is found from

$$\mu_s = \frac{F}{F_g} = \frac{75.0 \text{ N}}{(25.0 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{0.306}$$

(b) The coefficient of kinetic friction is found from

$$\mu_k = \frac{F}{F_g} = \frac{60.0 \text{ N}}{(25.0 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{0.245}$$

P5.56 Find the acceleration of the car, which is the same as the acceleration of the book because the book does not slide.

For the car: $v_i = 72.0$ km/h $= 20.0$ m/s, $v_f = 0$, $\Delta x = (x_f - x_i) = 30.0$ m. Using $v_f^2 = v_i^2 + 2a(x_f - x_i)$, we find the acceleration of the car:

$$a = -6.67 \text{ m/s}^2$$

Now, find the maximum acceleration that friction can provide. Because the book does not slide, static friction provides the force that slows down the book. We have the coefficient of static friction, $\mu_s = 0.550$, and we know $f_s \le \mu_s n$. The book is on a horizontal seat, so friction acts in the horizontal direction, and the vertical normal force that the seat exerts on the book is equal in magnitude to the force of gravity on the book: $n = F_g = mg$. For maximum acceleration, the static friction force will be a maximum, so $f_s = \mu_s n = \mu_s mg$. Applying Newton's second law, we find the acceleration that friction can provide for the book:

$$\sum F_x = ma_x:$$

$$-f_s = ma$$

$$-\mu_s mg = ma$$

which gives $a = -\mu_s g = -(0.550)(9.80 \text{ m/s}^2) = -5.39 \text{ m/s}^2$, which is too small for the stated conditions.

The situation is impossible because maximum static friction cannot provide the acceleration necessary to keep the book stationary on the seat.

P5.57 The x and y components of Newton's second law as the eraser begins to slip are

$$-f + mg\sin\theta = 0 \quad \text{and} \quad +n - mg\cos\theta = 0$$

with $f = \mu_s n$ or $\mu_k n$, these equations yield

$$\mu_s = \tan\theta_c = \tan 36.0° = \boxed{0.727}$$

$$\mu_k = \tan\theta_c = \tan 30.0° = \boxed{0.577}$$

P5.58 We assume that all the weight is on the rear wheels of the car.

(a) We find the record time from

$$F = ma: \quad \mu_s mg = ma \quad \text{or} \quad a = \mu_s g$$

But

$$\Delta x = \frac{at^2}{2} = \frac{\mu_s gt^2}{2}$$

so $\mu_s = \dfrac{2\Delta x}{gt^2}$

$$\mu_s = \frac{2(0.250 \text{ mi})(1\,609 \text{ m/mi})}{(9.80 \text{ m/s}^2)(4.43 \text{ s})^2} = \boxed{4.18}$$

(b) $\boxed{\text{Time would increase, as the wheels would skid and only kinetic friction would act; or perhaps the car would flip over.}}$

P5.59 Maximum static friction provides the force that produces maximum acceleration, resulting in a minimum time interval to accelerate through $\Delta x = 3.00$ m. We know that the maximum force of static friction is $f_s = \mu_s n$. If the shoe is on a horizontal surface, friction acts in the horizontal direction. Assuming that the vertical normal force is maximal, equal in magnitude to the force of gravity on the person, we have $n = F_g = mg$; therefore, the maximum static friction force is

$$f_s = \mu_s n = \mu_s mg$$

Applying Newton's second law:

$$\sum F_x = ma_x:$$

$$f_s = ma$$

$$\mu_s mg = ma \quad \rightarrow \quad a = \mu_s g$$

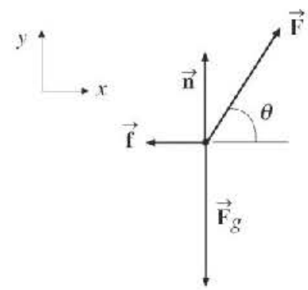
We find the time interval $\Delta t = t$ to accelerate from rest through $\Delta x =$ 3.00 m using $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$:

$$\Delta x = \frac{1}{2}a_x(\Delta t)^2 \rightarrow \Delta t = \sqrt{\frac{2\Delta x}{a_x}} = \sqrt{\frac{2\Delta x}{\mu_s g}}$$

(a) For $\mu_s = 0.500$, $\Delta t = \boxed{1.11\ \text{s}}$

(b) For $\mu_s = 0.800$, $\Delta t = \boxed{0.875\ \text{s}}$

P5.60 (a) See the free-body diagram of the suitcase in ANS. FIG. P5.60(a).



ANS. FIG. P5.60(a)

(b) $m_{\text{suitcase}} = 20.0$ kg, $F = 35.0$ N

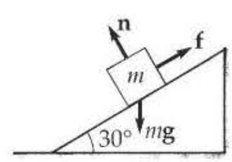
$$\sum F_x = ma_x: \quad -20.0\ \text{N} + F\cos\theta = 0$$
$$\sum F_y = ma_y: \quad +n + F\sin\theta - F_g = 0$$

$$F\cos\theta = 20.0\ \text{N}$$
$$\cos\theta = \frac{20.0\ \text{N}}{35.0\ \text{N}} = 0.571$$
$$\boxed{\theta = 55.2°}$$

(c) With $F_g = (20.0\ \text{kg})(9.80\ \text{m/s}^2)$,

$$n = F_g - F\sin\theta = [196\ \text{N} - (35.0\ \text{N})(0.821)]$$
$$\boxed{n = 167\ \text{N}}$$

P5.61 We are given: $m = 3.00$ kg, $\theta = 30.0°$, $x = 2.00$ m, $t = 1.50$ s



ANS. FIG. P5.61

(a) At constant acceleration,

$$x_f = v_i t + \tfrac{1}{2}at^2$$

Solving,

$$a = \frac{2(x_f - v_i t)}{t^2} = \frac{2(2.00\ \text{m} - 0)}{(1.50\ \text{s})^2} = \boxed{1.78\ \text{m/s}^2}$$

From the acceleration, we can calculate the friction force, answer (c), next.

(c) Take the positive x axis down parallel to the incline, in the direction of the acceleration. We apply Newton's second law:

$$\sum F_x = mg\sin\theta - f = ma$$

Solving, $f = m(g \sin\theta - a)$

Substituting,

$$f = (3.00\text{ kg})[(9.80\text{ m/s}^2)\sin 30.0° - 1.78\text{ m/s}^2] = \boxed{9.37\text{ N}}$$

(b) Applying Newton's law in the y direction (perpendicular to the incline), we have no burrowing-in or taking-off motion. Then the y component of acceleration is zero:

$$\sum F_y = n - mg\cos\theta = 0$$

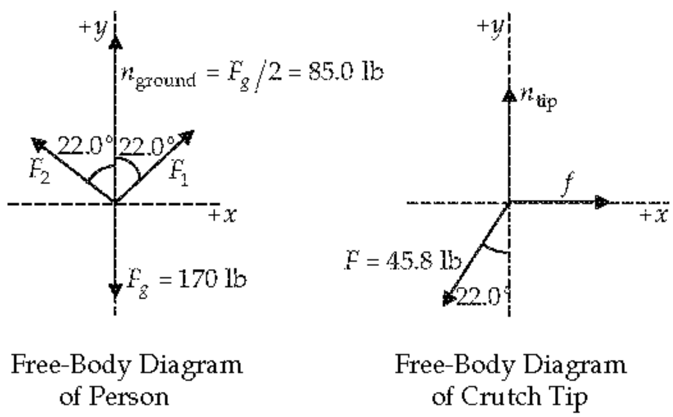
Thus $n = mg\cos\theta$

Because $f = \mu_k n$

we have $\mu_k = \dfrac{f}{mg\cos\theta} = \dfrac{9.37\text{ N}}{(3.00\text{ kg})(9.80\text{ m/s}^2)\cos 30.0°} = \boxed{0.368}$

(d) $v_f = v_i + at$ so $v_f = 0 + (1.78\text{ m/s}^2)(1.50\text{ s}) = \boxed{2.67\text{ m/s}}$

***P5.62** The free-body diagrams for this problem are shown in ANS. FIG. P5.62.



ANS. FIG. P5.62

From the free-body diagram for the person,

$$\sum F_x = F_1 \sin(22.0°) - F_2 \sin(22.0°) = 0$$

which gives $F_1 = F_2 = F$. Then, $\sum F_y = 2F\cos 22.0° + 85.0\text{ lbs} - 170\text{ lbs} = 0$ yields $F = 45.8$ lb.

(a) Now consider the free-body diagram of a crutch tip.

$$\sum F_x = f - (45.8 \text{ lb})\sin 22.0° = 0$$

or $f = 17.2$ lb.

$$\sum F_y = n_{tip} - (45.8 \text{ lb})\cos 22.0° = 0$$

which gives n_{tip} = 42.5 lb.

For minimum coefficient of friction, the crutch tip will be on the verge of slipping, so $f = (f_s)_{max} = \mu_s n_{tip}$ and

$$\mu_s = \frac{f}{n_{tip}} = \frac{17.2 \text{ lb}}{42.5 \text{ lb}} = \boxed{0.404}$$

(b) As found above, the compression force in each crutch is

$$F_1 = F_2 = F = \boxed{45.8 \text{ lb}}$$

P5.63 Newton's second law for the 5.00-kg mass gives

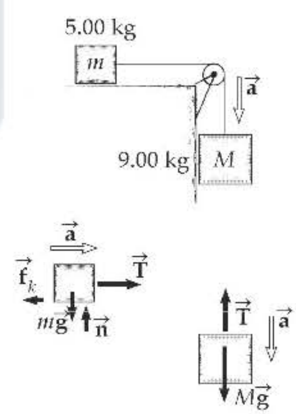
$$T - f_k = (5.00 \text{ kg})a$$

Similarly, for the 9.00-kg mass,

$$(9.00 \text{ kg})g - T = (9.00 \text{ kg})a$$

Adding these two equations gives:

$$(9.00 \text{ kg})(9.80 \text{ m/s}^2) - 0.200(5.00 \text{ kg})(9.80 \text{ m/s}^2) = (14.0 \text{ kg})a$$

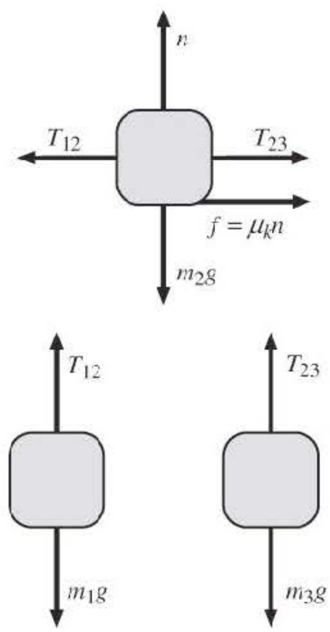


ANS. FIG. P5.63

Which yields $a = 5.60$ m/s^2. Plugging this into the first equation above gives

$$T = (5.00 \text{ kg})(5.60 \text{ m/s}^2) + 0.200(5.00 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{37.8 \text{ N}}$$

P5.64 (a) The free-body diagrams for each object appear on the right.



ANS. FIG. P5.64(a)

(b) Let a represent the positive magnitude of the acceleration $-a\hat{\mathbf{j}}$ of m_1, of the acceleration $-a\hat{\mathbf{i}}$ of m_2, and of the acceleration $+a\hat{\mathbf{j}}$ of m_3. Call T_{12} the tension in the left cord and T_{23} the tension in the cord on the right.

For m_1, $\sum F_y = ma_y$:

$$+T_{12} - m_1 g = -m_1 a$$

For m_2, $\sum F_x = ma_x$:

$$-T_{12} + \mu_k n + T_{23} = -m_2 a$$

and $\sum F_y = ma_y$, giving $n - m_2 g = 0$.

For m_3, $\sum F_y = ma_y$, giving $T_{23} - m_3 g = +m_3 a$.

We have three simultaneous equations:

$$-T_{12} + 39.2\text{ N} = (4.00\text{ kg})a$$
$$+T_{12} - 0.350(9.80\text{ N}) - T_{23} = (1.00\text{ kg})a$$
$$+T_{23} - 19.6\text{ N} = (2.00\text{ kg})a$$

Add them up (this cancels out the tensions):

$$+39.2\text{ N} - 3.43\text{ N} - 19.6\text{ N} = (7.00\text{ kg})a$$

$$a = \boxed{2.31\text{ m/s}^2\text{, down for } m_1\text{, left for } m_2\text{, and up for } m_3}$$

(c) Now $-T_{12} + 39.2\text{ N} = (4.00\text{ kg})(2.31\text{ m/s}^2)$

$$\boxed{T_{12} = 30.0\text{ N}}$$

and $T_{23} - 19.6\text{ N} = (2.00\text{ kg})(2.31\text{ m/s}^2)$

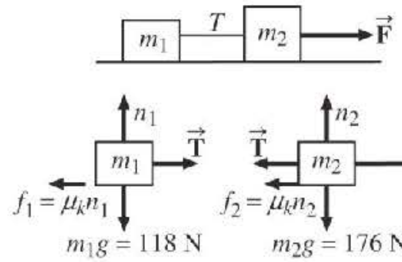
$$\boxed{T_{23} = 24.2\text{ N}}$$

(d) If the tabletop were smooth, friction disappears ($\mu_k = 0$), and so the acceleration would become larger. For a larger acceleration, according to the equations above, the tensions change:

$$T_{12} = m_1 g - m_1 a \rightarrow \boxed{T_{12} \text{ decreases}}$$

$$T_{23} = m_3 g + m_3 a \rightarrow \boxed{T_{23} \text{ increases}}$$

P5.65 Because the cord has constant length, both blocks move the same number of centimeters in each second and so move with the same acceleration. To find just this acceleration, we could model the 30-kg system as a particle under a net force. That method would not help to finding the tension, so we treat the two blocks as separate accelerating particles.



ANS. FIG. P5.65

(a) ANS. FIG. P5.65 shows the free-body diagrams for the two blocks. The tension force exerted by block 1 on block 2 is the same size as the tension force exerted by object 2 on object 1. The tension in a light string is a constant along its length, and tells how strongly the string pulls on objects at both ends.

(b) We use the free-body diagrams to apply Newton's second law.

For m_1: $\sum F_x = T - f_1 = m_1 a$ or $T = m_1 a + f_1$ **[1]**

And also $\sum F_y = n_1 - m_1 g = 0$ or $n_1 = m_1 g$

Also, the definition of the coefficient of friction gives

$$f_1 = \mu n_1 = (0.100)(12.0 \text{ kg})(9.80 \text{ m/s}^2) = 11.8 \text{ N}$$

For m_2: $\sum F_x = F - T - f_2 = ma$ **[2]**

Also from the y component, $n_2 - m_2 g = 0$ or $n_2 = m_2 g$

And again $f_2 = \mu n_2 = (0.100)(18.0 \text{ kg})(9.80 \text{ m/s}^2) = 17.6 \text{ N}$

Substituting T from equation [1] into [2], we get

$$F - m_1 a - f_1 - f_2 = m_2 a \quad \text{or} \quad F - f_1 - f_2 = m_2 a + m_1 a$$

Solving for a,

$$a = \frac{F - f_1 - f_2}{m_1 + m_2} = \frac{(68.0\text{ N} - 11.8\text{ N} - 17.6\text{ N})}{(12.0\text{ kg} + 18.0\text{ kg})} = \boxed{1.29\text{ m/s}^2}$$

(c) From equation [1],

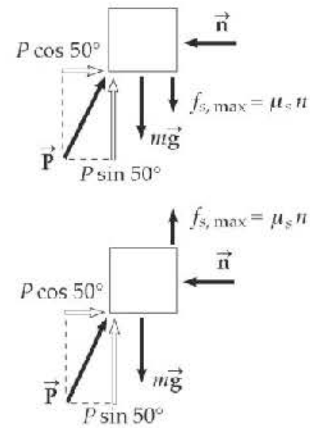
$$T = m_1 a + f_1 = (12.0\text{ kg})(1.29\text{ m/s}^2) + 11.8\text{ N} = \boxed{27.2\text{ N}}$$

P5.66 (a) To find the maximum possible value of P, imagine impending upward motion as case 1. Setting $\sum F_x = 0$:

$$P\cos 50.0° - n = 0$$

with $f_{s,\,max} = \mu_s n$:

$$f_{s,\,max} = \mu_s P\cos 50.0°$$
$$= 0.250(0.643)P = 0.161P$$



ANS. FIG. P5.66

Setting $\sum F_y = 0$:

$$P\sin 50.0° - 0.161P - (3.00\text{ kg})(9.80\text{ m/s}^2) = 0$$

$$P_{max} = \boxed{48.6\text{ N}}$$

To find the minimum possible value of P, consider impending downward motion. As in case 1,

$$f_{s,\,max} = 0.161P$$

Setting $\sum F_y = 0$:

$$P\sin 50.0° + 0.161P - (3.00\text{ kg})(9.80\text{ m/s}^2) = 0$$

$$P_{min} = \boxed{31.7\text{ N}}$$

(b) $\boxed{\text{If } P > 48.6\text{ N, the block slides up the wall. If } P < 31.7\text{ N, the block slides down the wall.}}$

(c) We repeat the calculation as in part (a) with the new angle.

Consider impending upward motion as case 1. Setting

$$\sum F_x = 0: \qquad P\cos 13° - n = 0$$
$$f_{s,\,max} = \mu_s n: \quad f_{s,\,max} = \mu_s P\cos 13°$$
$$= 0.250(0.974)P = 0.244P$$

Setting

$$\sum F_y = 0: \quad P\sin 13° - 0.244P - (3.00\text{ kg})(9.80\text{ m/s}^2) = 0$$
$$P_{max} = -1\ 580\text{ N}$$

The push cannot really be negative. However large or small it is, it cannot produce upward motion. To find the minimum possible value of *P*, consider impending downward motion. As in case 1,

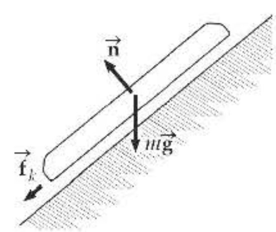
$$f_{s,\,max} = 0.244P$$

Setting

$$\sum F_y = 0: \quad P\sin 13° + 0.244P - (3.00\text{ kg})(9.80\text{ m/s}^2) = 0$$
$$P_{min} = \boxed{62.7\text{ N}}$$

$\boxed{P \geq 62.7\text{ N. The block cannot slide up the wall. If } P < 62.7\text{ N, the block slides down the wall.}}$

P5.67 We must consider separately the rock when it is in contact with the roof and when it has gone over the top into free fall. In the first case, we take *x* and *y* as parallel and perpendicular to the surface of the roof:



ANS. FIG. P5.67

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$n = mg\cos\theta$$

then friction is $f_k = \mu_k n = \mu_k mg\cos\theta$

$$\sum F_x = ma_x: \quad -f_k - mg\sin\theta = ma_x$$

$$\begin{aligned} a_x &= -\mu_k g\cos\theta - g\sin\theta = (-0.400\cos 37.0° - \sin 37.0°)(9.80\ \text{m/s}^2) \\ &= -9.03\ \text{m/s}^2 \end{aligned}$$

The rock goes ballistic with speed given by

$$\begin{aligned} v_{xf}^2 &= v_{xi}^2 + 2a_x\left(x_f - x_i\right) = (15\ \text{m/s})^2 + 2\left(-9.03\ \text{m/s}^2\right)(10\ \text{m} - 0) \\ &= 44.4\ \text{m}^2/\text{s}^2 \\ v_{xf} &= 6.67\ \text{m/s} \end{aligned}$$

For the free fall, we take x and y horizontal and vertical:

$$\begin{aligned} v_{yf}^2 &= v_{yi}^2 + 2a_y\left(y_f - y_i\right) \\ y_f &= \frac{v_{yf}^2 - v_{yi}^2}{2a_y} \\ &= \frac{0 - (6.67\ \text{m/s}\ \ \sin 37°)^2}{2\left(-9.8\ \text{m/s}^2\right)} = 0.822\ \text{m above the top of the roof} \end{aligned}$$

Then $y_{tot} = 10.0\ \text{m}\ \sin 37.0° + 0.822\ \text{m} = \boxed{6.84\ \text{m}}$.

P5.68 The motion of the salmon as it breaks the surface of the water and eventually leaves must be modeled in two steps. The first is over a distance of 0.750 m, until half of the salmon is above the surface, while a constant force, P, is applied upward. In this motion, the initial velocity of the salmon as it nears the surface is 3.58 m/s and ends with the salmon having a velocity, $v_{1/2}$, when it is half out of the water. This is then the initial velocity for the second motion, where gravity is a second force to be considered acting on the fish. This motion is again over a distance of 0.750 m, and results with the salmon having a final velocity of 6.26 m/s.

The vertical motion equations, in each case, would be

$$a_{1y} = \frac{v_{1yf}^2 - v_{1yi}^2}{2\ \Delta y} = \frac{v_{1/2}^2 - (3.58\ \text{m/s})^2}{2\ (0.750\ \text{m})} = \frac{v_{1/2}^2 - \left(12.8\ \text{m}^2/\text{s}^2\right)}{1.50\ \text{m}}$$

and

$$a_{2y} = \frac{v_{2yf}^2 - v_{2yi}^2}{2\ \Delta y} = \frac{(6.26\ \text{m/s})^2 - v_{1/2}^2}{2\ (0.750\ \text{m})} = \frac{\left(39.2\ \text{m}^2/\text{s}^2\right) - v_{1/2}^2}{1.50\ \text{m}}$$

Solving for the square of the velocity in each case and equating the expressions, we find

$$v_{1/2}^2 = (1.50\ \text{m})a_{1y} + \left(12.8\ \text{m}^2/\text{s}^2\right)$$

$$v_{1/2}^2 = \left(39.2\ \text{m}^2/\text{s}^2\right) - (1.50\ \text{m})a_{2y}$$

$$(1.50\ \text{m})a_{1y} + \left(12.8\ \text{m}^2/\text{s}^2\right) = \left(39.2\ \text{m}^2/\text{s}^2\right) - (1.50\ \text{m})a_{2y}$$
$$a_{1y} = \left(17.6\ \text{m/s}^2\right) - a_{2y}$$

In the first motion, the relationship between the net acceleration and the net force can be written as

$$\sum F_y = P = ma_{1y}$$
$$P = (61.0\ \text{kg})a_{1y}$$

Substituting from above,

$$P = (61.0\ \text{kg})\left[\left(17.6\ \text{m/s}^2\right) - a_{2y}\right]$$
$$P = 1\ 070\ \text{N} - (61.0\ \text{kg})a_{2y}$$

In the second motion, the relationship between the net acceleration and the net force can be written as

$$\sum F_y = P - mg = ma_{2y}$$
$$P = mg + ma_{2y} = (61.0\ \text{kg})\left(9.80\ \text{m/s}^2\right) + (61.0\ \text{kg})a_{2y}$$
$$P = 598\ \text{N} + (61.0\ \text{kg})a_{2y}$$

Equating these two equations for, *P*,

$$1\ 070\ \text{N} - (61.0\ \text{kg})a_{2y} = 598\ \text{N} + (61.0\ \text{kg})a_{2y}$$
$$-(122.0\ \text{kg})a_{2y} = -472\ \text{N}$$
$$a_{2y} = 3.87\ \text{m/s}^2$$

Plugging into either of the above,

$$P = 598\ \text{N} + (61.0\ \text{kg})\left(3.87\ \text{m/s}^2\right)$$
$$P = \boxed{834\ \text{N}}$$

P5.69 Take $+x$ in the direction of motion of the tablecloth. For the mug:

$$\sum F_x = ma_x: \quad 0.1 \text{ N} = 0.2 \text{ kg } a_x$$
$$a_x = 0.5 \text{ m/s}^2$$

Relative to the tablecloth, the acceleration of the mug is $0.500 \text{ m/s}^2 - 3.00 \text{ m/s}^2 = -2.50 \text{ m/s}^2$. The mug reaches the edge of the tablecloth after time given by

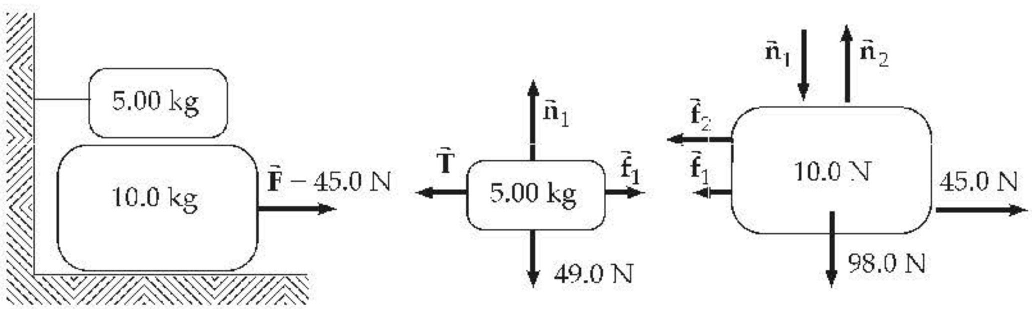
$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$
$$-0.300 \text{ m} = 0 + \frac{1}{2}\left(-2.50 \text{ m/s}^2\right)t^2$$
$$t = 0.490 \text{ s}$$

The motion of the mug relative to tabletop is over distance

$$\frac{1}{2}a_x t^2 = \frac{1}{2}\left(0.500 \text{ m/s}^2\right)\left(0.490 \text{ s}\right)^2 = \boxed{0.060\,0 \text{ m}}$$

The tablecloth slides 36 cm over the table in this process.

***P5.70** (a) The free-body diagrams are shown in the figure below.



ANS. FIG. P5.70(a)

f_1 and n_1 appear in both diagrams as action-reaction pairs.

(b) For the 5.00-kg mass, Newton's second law in the y direction gives:

$$n_1 = m_1 g = (5.00 \text{ kg})\left(9.80 \text{ m/s}^2\right) = 49.0 \text{ N}$$

In the x direction,

$$f_1 - T = 0$$

$$T = f_1 = \mu mg = 0.200(5.00 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{9.80 \text{ N}}$$

For the 10.0-kg mass, Newton's second law in the *x* direction gives:

$$45.0 \text{ N} - f_1 - f_2 = (10.0 \text{ kg})a$$

In the *y* direction,

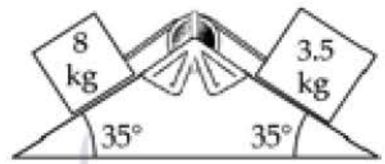
$$n_2 - n_1 - 98.0 \text{ N} = 0$$

$$f_2 = \mu n_2 = \mu(n_1 + 98.0 \text{ N}) = 0.20(49.0 \text{ N} + 98.0 \text{ N}) = 29.4 \text{ N}$$

$$45.0 \text{ N} - 9.80 \text{ N} - 29.4 \text{ N} = (10.0 \text{ kg})a$$

$$a = \boxed{0.580 \text{ m/s}^2}$$

***P5.71** For the right-hand block (m_1), $\sum F_1 = m_1 a$ gives



ANS. FIG. P5.71

$$-m_1 g \sin 35.0° - f_{k,1} + T = m_1 a$$

or

$$-(3.50 \text{ kg})(9.80 \text{ m/s}^2)\sin 35.0° - \mu_s(3.50 \text{ kg})(9.80 \text{ m/s}^2)\cos 35.0° + T = (3.50 \text{ kg})(1.50 \text{ m/s}^2) \quad \textbf{[1]}$$

For the left-hand block (m_2), $\sum F_2 = m_2 a$ gives

$$+m_2 g \sin 35.0° - f_{k,2} - T = m_2 a$$

$$+(8.00 \text{ kg})(9.80 \text{ m/s}^2)\sin 35.0° - \mu_s(8.00 \text{ kg})(9.80 \text{ m/s}^2)\cos 35.0° - T = (8.00 \text{ kg})(1.50 \text{ m/s}^2) \quad \textbf{[2]}$$

Solving equations [1] and [2] simultaneously gives

(a) $\boxed{\mu_k = 0.087\ 1}$

(b) $\boxed{T = 27.4 \text{ N}}$

Additional Problems

P5.72 (a) Choose the black glider plus magnet as the system.

$$\sum F_x = ma_x \rightarrow +0.823 \text{ N} = (0.24 \text{ kg})a$$

$$a = \boxed{3.43 \text{ m/s}^2 \text{ toward the scrap iron}}$$

(b) The force of attraction the magnet exerts on the scrap iron is the same as in (a):

$$a_{\text{black}} = \boxed{3.43 \text{ m/s}^2 \text{ toward the scrap iron}}$$

By Newton's third law, the force the black glider exerts on the magnet is equal and opposite to the force exerted on the scrap iron:

$$\sum F_x = ma_x \rightarrow -0.823 \text{ N} = -(0.12 \text{ kg})\, a$$

$$a = \boxed{-6.86 \text{ m/s}^2 \text{ toward the magnet}}$$

P5.73 Let situation 1 be the original situation, with $\sum F_1 = m_1 a_1 = m_1(8.40 \text{ mi/h} \cdot \text{s})$. Let situation 2 be the case with larger force $\sum F_2 = (1 + 0.24)\sum F_1 = m_1 a_2 = 1.24 m_1 a_1$, so $a_2 = 1.24 a_1$. Let situation 3 be the case with the original force but with smaller mass:

$$\sum F_3 = \sum F_1 = m_3 a_3 = (1 - 0.24) m_1 a_3$$

$$a_3 = \frac{\sum F_1}{0.76 m_1} = 1.32 a_1$$

(a) With 1.32*a* greater than $1.24a_1$, $\boxed{\text{reducing the mass}}$ gives a larger increase in acceleration.

(b) Now with both changes,

$$\sum F_4 = m_4 a_4$$

$$1.24 \sum F_1 = 0.76 m_1 a_4$$

$$a_4 = \frac{1.24}{0.76}\frac{\sum F_1}{m_1} = \frac{1.24}{0.76}(8.40 \text{ mi/h} \cdot \text{s}) = \boxed{13.7 \text{ mi/h} \cdot \text{s}}$$

P5.74 Find the acceleration of the block according to the kinematic equations. The book travels through a displacement of 1.00 m in a time interval of 0.483 s. Use the equation $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$, where $\Delta x = x_f - x_i$ = 1.00 m, $\Delta t = t$ = 0.483 s, and $v_i = 0$:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \rightarrow a = \frac{2\Delta x}{t^2} = 8.57 \text{ m/s}^2$$

Now, find the acceleration of the block caused by the forces. See the free-body diagram below. We take the positive *y* axis is perpendicular to the incline; the positive *x* axis is parallel and down the incline.

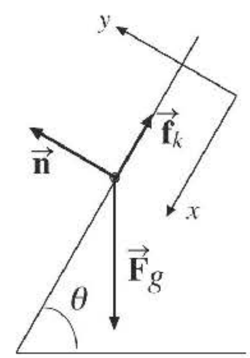
$$\sum F_y = ma_y:$$

$$n - mg\cos\theta = 0 \rightarrow n = mg\cos\theta$$

$$\sum F_x = ma_x:$$

$$mg\sin\theta - f_k = ma$$

where $f_k = \mu_k n = \mu_k mg\cos\theta$



ANS. FIG. P5.74

Substituting the express for kinetic friction into the *x*-component equation gives

$$mg\sin\theta - \mu_k mg\cos\theta = ma \rightarrow a = g(\sin\theta - \mu_k\cos\theta)$$

For $\mu_k = 0.300$, and $\theta = 60.0°$, $a = 7.02\text{ m/s}^2$.

The situation is impossible because these forces on the book cannot produce the acceleration described.

P5.75 (a) Since the puck is on a horizontal surface, the normal force is vertical. With $a_y = 0$, we see that

$$\sum F_y = ma_y \rightarrow n - mg = 0 \quad \text{or} \quad n = mg$$

Once the puck leaves the stick, the only horizontal force is a friction force in the negative *x* direction (to oppose the motion of the puck). The acceleration of the puck is

$$a_x = \frac{\Sigma F_x}{m} = \frac{-f_k}{m} = \frac{-\mu_k n}{m} = \frac{-\mu_k(mg)}{m} = \boxed{-\mu_k g}$$

(b) Then $v_{xf}^2 = v_{xi}^2 + 2a\Delta x$ gives the horizontal displacement of the puck before coming to rest as

$$\Delta x = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{0 - v_i^2}{2(-\mu_k g)} = \boxed{\frac{v_i^2}{2\mu_k g}}$$

***P5.76** (a) Let *x* represent the position of the glider along the air track. Then $z^2 = x^2 + h_0^2$, $x = (z^2 - h_0^2)^{1/2}$, and $v_x = \frac{dx}{dt} = \frac{1}{2}(z^2 - h_0^2)^{-1/2}(2z)\frac{dz}{dt}$.

Now $\frac{dz}{dt}$ is the rate at which the string passes over the pulley, so it is equal to v_y of the counterweight.

$$v_x = z(z^2 - h_0^2)^{-1/2} v_y = uv_y$$

(b) $a_x = \frac{dv_x}{dt} = \frac{d}{dt}uv_y = u\frac{dv_y}{dt} + v_y\frac{du}{dt}$

At release from rest, $v_y = 0$ and $a_x = ua_y$.

(c) $\sin 30.0° = \frac{80.0\text{ cm}}{z}$, $z = 1.60$ m,

$$u = (z^2 - h_0^2)^{-1/2} z = \left[(1.6\text{ m})^2 - (0.8\text{ m})^2\right]^{-1/2}(1.6\text{ m}) = 1.15\text{ m}$$

For the counterweight, $\sum F_y = ma_y$:

$$T - (0.5\text{ kg})(9.80\text{ m/s}^2) = -(0.5\text{ kg})a_y$$

$$a_y = (-2\text{ kg}^{-1})T + (9.80\text{ m/s}^2)$$

For the glider, $\sum F_x = ma_x$:

$$\begin{aligned} T\cos 30° &= (1.00\text{ kg})\ a_x = (1.15\text{ kg})a_y \\ &= (1.15\text{ kg})\left[(-2\text{ kg}^{-1})T + 9.80\text{ m/s}^2\right] \\ &= -2.31T + 11.3\text{ N} \end{aligned}$$

$$3.18T = 11.3\text{ N}$$

$$T = \boxed{3.56\text{ N}}$$

***P5.77** When an object of mass m is on this frictionless incline, the only force acting parallel to the incline is the parallel component of weight, $mg\sin\theta$, directed down the incline. The acceleration is then

$$a = \frac{mg\sin\theta}{m} = g\sin\theta = \left(9.80 \text{ m/s}^2\right)\sin 35.0° = 5.62 \text{ m/s}^2$$

directed down the incline.

(a) Taking up the incline as positive, the time for the sled projected up the incline to come to rest is given by

$$t = \frac{v_f - v_i}{a} = \frac{0 - 5.00 \text{ m/s}}{-5.62 \text{ m/s}^2} = 0.890 \text{ s}$$

The distance the sled travels up the incline in this time is

$$\Delta x = v_{avg}t = \left(\frac{v_f + v_i}{2}\right) = \left(\frac{0 + 5.00 \text{ m/s}}{2}\right)(0.890 \text{ s}) = \boxed{2.23 \text{ m}}$$

(b) The time required for the first sled to return to the bottom of the incline is the same as the time needed to go up, that is, t = 0.890 s. In this time, the second sled must travel down the entire 10.0-m length of the incline. The needed initial velocity is found from

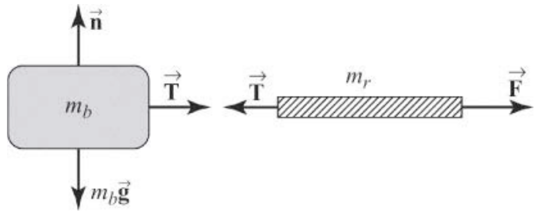
$$\Delta x = v_i t + \frac{1}{2}at^2$$

which gives

$$v_i = \frac{\Delta x}{t} - \frac{at}{2} = \frac{-10.0 \text{ m}}{0.890 \text{ s}} - \frac{\left(-5.62 \text{ m/s}^2\right)(0.890 \text{ s})}{2} = -8.74 \text{ m/s}$$

or $\boxed{8.74 \text{ m/s down the incline}}$

P5.78 (a) free-body diagrams of block and rope are shown in ANS. FIG. P5.78(a):



ANS. FIG. P5.78(a)

(b) Applying Newton's second law to the rope yields

$$\sum F_x = ma_x \quad \Rightarrow \quad F - T = m_r a \quad \text{or} \quad T = F - m_r a \qquad \textbf{[1]}$$

Then, applying Newton's second law to the block, we find

$$\sum F_x = ma_x \quad \Rightarrow T = m_b a \quad \text{or} \quad F - m_r a = m_b a$$

which gives

$$\boxed{a = \frac{F}{m_b + m_r}}$$

(c) Substituting the acceleration found above back into equation [1] gives the tension at the left end of the rope as

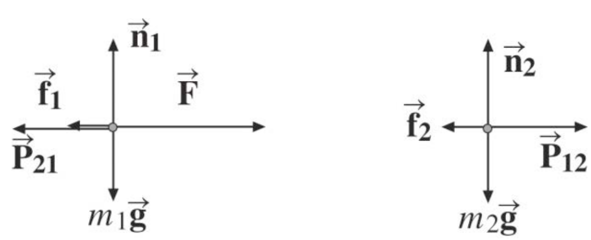
$$T = F - m_r a = F - m_r\left(\frac{F}{m_b + m_r}\right) = F\left(\frac{m_b + \cancel{m_r} - \cancel{m_r}}{m_b + m_r}\right)$$

or $$\boxed{T = \left(\frac{m_b}{m_b + m_r}\right)F}$$

(d) From the result of (c) above, we see that as m_r approaches zero, T approaches F. Thus,

$\boxed{\text{the tension in a cord of negligible mass is constant along its length.}}$

P5.79 (a) The free-body diagrams of the two blocks shown in ANS. FIG. P5.79(a):



ANS. FIG. P5.79(a)

Vertical forces sum to zero because the blocks move on a horizontal surface; therefore, $a_y = 0$ for each block.

$\sum F_{1y} = m_1 a_y$: $\quad$ $\sum F_{2y} = m_2 a_y$:

$-m_1 g + n_1 = 0 \rightarrow n_1 = m_1 g$ $\quad$ $-m_2 g + n_2 = 0 \rightarrow n_2 = m_2 g$

Kinetic friction is: $\quad$ Kinetic friction is:

$f_1 = \mu_1 n_1 = \mu_1 m_1 g$ $\quad$ $f_2 = \mu_2 n_2 = \mu_2 m_2 g$

(b) The net force on the system of the blocks would be equal to the magnitude of the force, F, minus the friction force on each block. The blocks will have the same acceleration.

(c) The net force on the mass, m_1, would be equal to the force, F, minus the friction force on m_1 and the force P_{21}, as identified in the free-body diagram.

(d) The net force on the mass, m_2, would be equal to the force, P_{12}, minus the friction force on m_2, as identified in the free-body diagram.

(e) The blocks are pushed to the right by force $\vec{\mathbf{F}}$, so kinetic friction $\vec{\mathbf{f}}$ acts on each block to the left. Each block has the same horizontal acceleration, $a_x = a$. Each block exerts an equal and opposite force on the other, so those forces have the same magnitude: $P_{12} = P_{21} = P$.

$\sum F_{1x} = m_1 a_x$: $\quad$ $\sum F_{2x} = m_2 a_x$:

$F - P - f_1 = m_1 a$ $\quad$ $P - f_2 = m_2 a$

$\boxed{F - P - \mu_1 m_1 g = m_1 a}$ $\quad$ $\boxed{P - \mu_2 m_2 g = m_2 a}$

(f) Adding the above two equations of x components, we find

$$F - P - \mu_1 m_1 g + P - \mu_2 m_2 g = m_1 a + m_2 a$$

$$F - \mu_1 m_1 g - \mu_2 m_2 g = (m_1 + m_2)a \rightarrow$$

$$\boxed{a = \frac{F - \mu_1 m_1 g - \mu_2 m_2 g}{m_1 + m_2}}$$

(g) From the x component equation for block 2, we have

$$P - \mu_2 m_2 g = m_2 a \rightarrow P = \mu_2 m_2 g + m_2 a$$

$$\boxed{P = \left(\frac{m_2}{m_1 + m_2}\right)\left[F + (\mu_2 - \mu_1) m_1 g\right]}$$

We see that when the coefficients of friction are equal, $\mu_1 = \mu_2$, the magnitude P is independent of friction.

P5.80 (a) The cable does not stretch: Whenever one car moves 1 cm, the other moves 1 cm.

$\boxed{\text{At any instant they have the same velocity and at all instants they have the same acceleration.}}$

(b) Consider the BMW as the object.

$$\sum F_y = ma_y:$$

$$+T - mg = ma$$

$$+T - (1\ 461\ \text{kg})(9.80\ \text{m/s}^2) = (1\ 461\ \text{kg})(1.25\ \text{m/s}^2)$$

$$T = \boxed{1.61 \times 10^4\ \text{N}}$$

(c) Consider both cars as the object.

$$\sum F_y = ma_y:$$

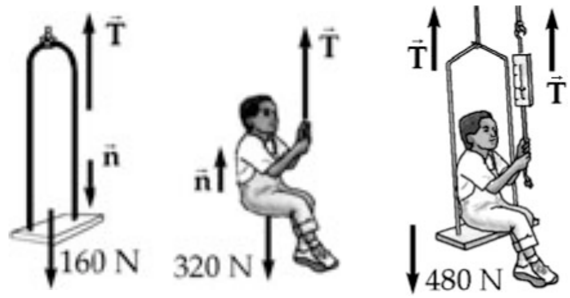
$$+T - (m + M)g = (m + M)a$$

$$+T - (1\ 461\ \text{kg} + 1\ 207\ \text{kg})(9.80\ \text{m/s}^2)$$
$$= (1\ 461\ \text{kg} + 1\ 207\ \text{kg})(1.25\ \text{m/s}^2)$$

$$T_{\text{above}} = \boxed{2.95 \times 10^4\ \text{N}}$$

P5.81 (a) ANS. FIG. P5.81(a) shows the free-body diagrams for this problem.

Note that the same-size force n acts up on Nick and down on chair, and cancels out in the diagram. The same-size force T = 250 N acts up on Nick and up on chair, and appears twice in the diagram.



ANS. FIG. P5.81(a)

(b) First consider Nick and the chair together as the system. Note that **two** ropes support the system, and $T = 250$ N in each rope.

Applying $\sum F = ma$, $\quad 2T - (160\text{ N} + 320\text{ N}) = ma$

where $\quad m = \dfrac{480\text{ N}}{9.80\text{ m/s}^2} = 49.0\text{ kg}$

Solving for a gives $\quad a = \dfrac{(500 - 480)\text{ N}}{49.0\text{ kg}} = \boxed{0.408\text{ m/s}^2}$

(c) On Nick, we apply

$\sum F = ma$: $\quad n + T - 320\text{ N} = ma$

where $\quad m = \dfrac{320\text{ N}}{9.80\text{ m/s}^2} = 32.7\text{ kg}$

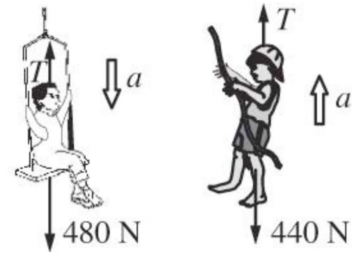
The normal force is the one remaining unknown:

$$n = ma + 320\text{ N} - T$$

Substituting, $\quad n = (32.7\text{ kg})\left(0.408\text{ m/s}^2\right) + 320\text{ N} - 250\text{ N}$

gives $\quad n = \boxed{83.3\text{ N}}$

P5.82 See ANS. FIG. P5.82 showing the free-body diagrams. The rope has tension T.



ANS. FIG. P5.82

(a) As soon as Nick passes the rope to the other child,

Nick and the seat, with total weight 480 N, will accelerate down and the child, with smaller weight 440 N, will accelerate up.

On Nick and the seat,

$$\sum F_y = +480\text{ N} - T = \frac{480\text{ N}}{9.80\text{ m/s}^2}a$$

On the child,

$$\sum F_y = +T - 440\text{ N} = \frac{440\text{ N}}{9.80\text{ m/s}^2}a$$

Adding,

$$+480\text{ N} - T + T - 440\text{ N} = (49.0\text{ kg} + 44.9\text{ kg})a$$

$$a = \frac{40\text{ N}}{93.9\text{ kg}} = \boxed{0.426\text{ m/s}^2 = a}$$

The rope tension is $T = 440\text{ N} + (44.9\text{ kg})(0.426\text{ m/s}^2) = 459\text{ N}$.

(b) The rope must support Nick and the seat, so the rope tension is 480 N.

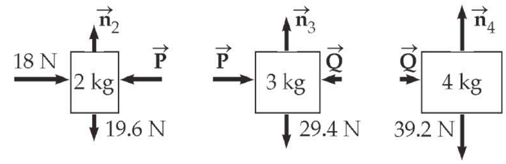
In problem 81, a rope tension of 250 N does not make the rope break. In part (a), the rope is strong enough to support tension 459 N. But now the tension everywhere in the rope is 480 N, so it can exceed the breaking strength of the rope.

The tension in the chain supporting the pulley is 480 N + 480 N = 960 N, so the chain may break first.

P5.83 (a) See free-body diagrams in ANS. FIG. P5.83.

(b) We write $\sum F_x = ma_x$ for each object.

$$18\text{ N} - P = (2\text{ kg})a$$
$$P - Q = (3\text{ kg})a$$
$$Q = (4\text{ kg})a$$



ANS. FIG. P5.83

Adding gives

$$18\text{ N} = (9\text{ kg})a \rightarrow a = \boxed{2.00\text{ m/s}^2}$$

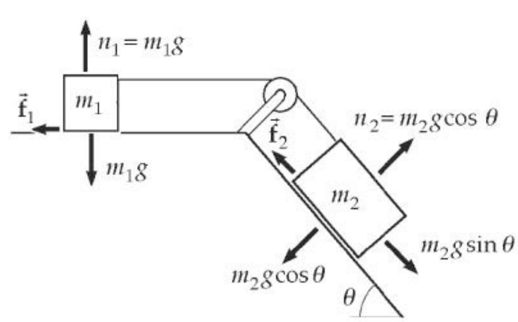
(c) The resultant force on any object is $\sum \vec{F} = m\vec{a}$: All have the same acceleration:

$$\sum \vec{F} = (4\text{ kg})(2\text{ m/s}^2) = \boxed{8.00\text{ N on the 4-kg object}}$$
$$\sum \vec{F} = (3\text{ kg})(2\text{ m/s}^2) = \boxed{6.00\text{ N on the 3-kg object}}$$
$$\sum \vec{F} = (2\text{ kg})(2\text{ m/s}^2) = \boxed{2.00\text{ N on the 2-kg object}}$$

(d) From above, $P = 18\text{ N} - (2\text{ kg})a \rightarrow \boxed{P = 14.0\text{ N}}$, and $Q = (4\text{ kg})a \rightarrow \boxed{Q = 8.00\text{ N}}$.

(e) Introducing the heavy block reduces the acceleration because the mass of the system (plasterboard-heavy block-you) is greater. The 3-kg block models the heavy block of wood. The contact force on your back is represented by Q, which is much less than the force F. The difference between F and Q is the net force causing acceleration of the 5-kg pair of objects.

P5.84 (a) For the system to start to move when released, the force tending to move m_2 down the incline, $m_2 g \sin\theta$, must exceed the maximum friction force which can retard the motion:



ANS. FIG. P5.84

$$f_{max} = f_{1,max} + f_{2,max} = \mu_{s,1}n_1 + \mu_{s,2}n_2$$
$$f_{max} = \mu_{s,1}m_1 g + \mu_{s,2}m_2 g\cos\theta$$

From the table of coefficients of friction in the text, we take $\mu_{s,1} = 0.610$ (aluminum on steel) and $\mu_{s,2} = 0.530$ (copper on steel). With

$$m_1 = 2.00 \text{ kg}, m_2 = 6.00 \text{ kg}, \theta = 30.0°$$

the maximum friction force is found to be $f_{\max} = 38.9$ N. This exceeds the force tending to cause the system to move,

$m_2 g \sin\theta = 6.00 \text{ kg}\left(9.80 \text{ m/s}^2\right)\sin 30° = 29.4 \text{ N}$. Hence,

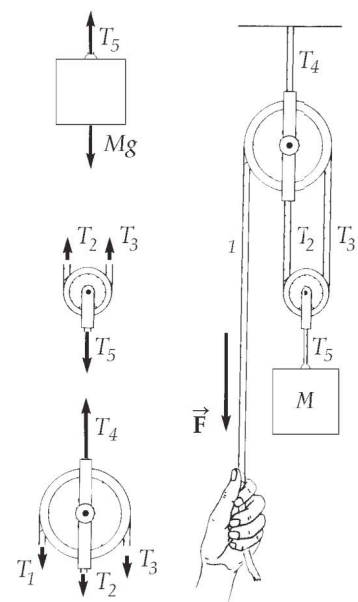
$\boxed{\text{the system will not start to move when released}}$

(b) and (c) $\boxed{\text{No answer because the blocks do not move.}}$

(d) The friction forces increase in magnitude until the total friction force retarding the motion, $f = f_1 + f_2$, equals the force tending to set the system in motion. That is, until

$$\boxed{f = m_2 g \sin\theta = 29.4 \text{ N}}$$

P5.85 (a) See ANS. FIG. P5.85 showing the forces. All forces are in the vertical direction. The lifting can be done at constant speed, with zero acceleration and total force zero on each object.



ANS. FIG. 5.85

(b) For M, $\sum F = 0 = T_5 - Mg$

so $T_5 = Mg$

Assume frictionless pulleys. The tension is constant throughout a light, continuous rope. Therefore, $T_1 = T_2 = T_3$.

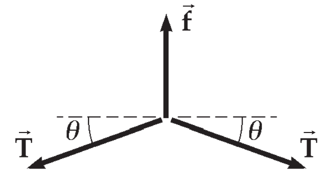
For the bottom pulley,

$$\sum F = 0 = T_2 + T_3 - T_5$$

so $2T_2 = T_5$. Then $\boxed{T_1 = T_2 = T_3 = \dfrac{Mg}{2}}$, $\boxed{T_4 = \dfrac{3Mg}{2}}$, and $\boxed{T_5 = Mg}$.

(c) Since $F = T_1$, we have $\boxed{F = \frac{Mg}{2}}$.

***P5.86** (a) Consider forces on the midpoint of the rope. It is nearly in equilibrium just before the car begins to move. Take the y axis in the direction of the force you exert:



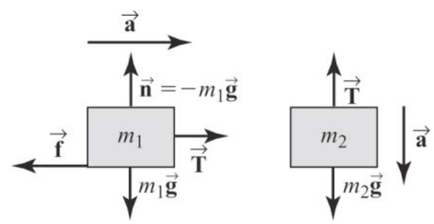
ANS. FIG. P5.86

$$\sum F_y = ma_y: \quad -T\sin\theta + f - T\sin\theta = 0$$

$$\boxed{T = \frac{f}{2\sin\theta}}$$

(b) $T = \dfrac{100\text{ N}}{2\sin 7°} = \boxed{410\text{ N}}$

***P5.87** The acceleration of the system is found from



ANS. FIG. 5.87

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$$

since $v_{yi} = 0$, we obtain

$$a = \frac{2\Delta y}{t^2} = \frac{2(1.00\text{ m})}{(1.20\text{ s})^2} = 1.39\text{ m/s}^2$$

Using the free-body diagram for m_2, Newton's second law gives

$$\begin{aligned}
&\sum F_{y2} = m_2 a: \\
&\quad m_2 g - T = m_2 a \\
&\quad T = m_2(g - a) \\
&\quad\ = (5.00\text{ kg})(9.80\text{ m/s}^2 - 1.39\text{ m/s}^2) \\
&\quad\ = 42.1\text{ N}
\end{aligned}$$

Then, applying Newton's second law to the horizontal motion of m_1,

$$\begin{aligned}
&\sum F_{x1} = m_1 a: \\
&\quad T - f = m_1 a \\
&\quad f = T - m_1 a \\
&\quad\ = 42.1\text{ N} - (10.0\text{ kg})(1.39\text{ m/s}^2) = 28.2\text{ N}
\end{aligned}$$

Since $n = m_1g = 98.0$ N, we have

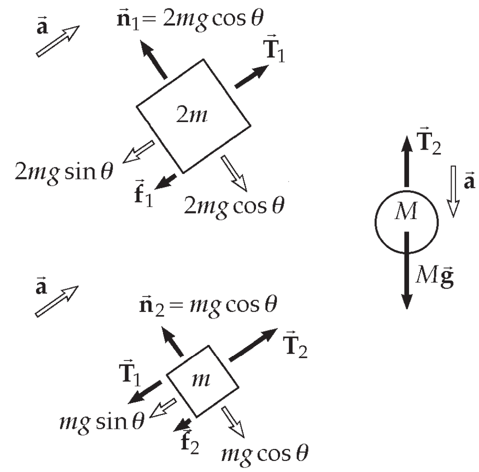
$$\mu_k = \frac{f}{n} = \frac{28.2\text{ N}}{98.0\text{ N}} = \boxed{0.288}$$

***P5.88** Applying Newton's second law to each object gives:

$$T_1 = f_1 + 2m(g\sin\theta + a) \qquad \textbf{[1]}$$

$$T_2 - T_1 = f_2 + m(g\sin\theta + a) \qquad \textbf{[2]}$$

$$T_2 = M(g - a) \qquad \textbf{[3]}$$



ANS. FIG. P5.88

(a), (b) Assuming that the system is in equilibrium ($a = 0$) and that the incline is frictionless, ($f_1 = f_2 = 0$), the equations reduce to

$$\boxed{T_1 = 2mg\sin\theta} \qquad \textbf{[1']}$$

$$T_2 - T_1 = mg\sin\theta \qquad \textbf{[2']}$$

$$T_2 = Mg \qquad \textbf{[3']}$$

Substituting [1′] and [3′] into equation [2′] then gives

$$\boxed{M = 3m\sin\theta}$$

so equation [3′] becomes $\boxed{T_2 = 3mg\sin\theta}$

(c), (d) $M = 6m\sin\theta$ (double the value found above), and $f_1 = f_2 = 0$. With these conditions present, the equations become $T_1 = 2m(g\sin\theta + a)$, $T_2 - T_1 = m(g\sin\theta + a)$ and $T_2 = 6m\sin\theta(g - a)$. Solved simultaneously, these yield

$$\boxed{a = \frac{g\sin\theta}{1 + 2\sin\theta}}, \quad \boxed{T_1 = 4mg\sin\theta\left(\frac{1+\sin\theta}{1+2\sin\theta}\right)} \text{ and}$$

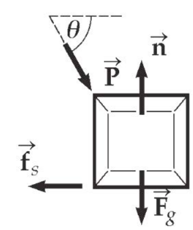
$$\boxed{T_2 = 6mg\sin\theta\left(\frac{1+\sin\theta}{1+2\sin\theta}\right)}$$

(e) Equilibrium ($a = 0$) and impending motion **up** the incline, so $M = M_{max}$ while $f_1 = 2\mu_s mg\cos\theta$ and $f_2 = \mu_s mg\cos\theta$, both directed **down** the incline. Under these conditions, the equations become $T_1 = 2mg(\sin\theta + \mu_s\cos\theta)$, $T_2 - T_1 = mg(\sin\theta + \mu_s\cos\theta)$, and $T_2 = M_{max}g$, which yield $\boxed{M_{max} = 3m(\sin\theta + \mu_s\cos\theta)}$.

(f) Equilibrium ($a = 0$) and impending motion **down** the incline, so $M = M_{min}$, while $f_1 = 2\mu_s mg\cos\theta$ and $f_2 = \mu_s mg\cos\theta$, both directed **up** the incline. Under these conditions, the equations are $T_1 = 2mg(\sin\theta - \mu_s\cos\theta)$, $T_2 - T_1 = mg(\sin\theta - \mu_s\cos\theta)$, and $T_2 = M_{min}g$, which yield $\boxed{M_{min} = 3m(\sin\theta - \mu_s\cos\theta)}$. When this expression gives a negative value, it corresponds physically to a mass *M* hanging from a cord over a pulley at the bottom end of the incline.

(g) $\boxed{T_{2,max} - T_{2,min} = M_{max}g - M_{min}g = 6\mu_s mg\cos\theta}$

P5.89 (a) The crate is in equilibrium, just before it starts to move. Let the normal force acting on it be n and the friction force, f_s.



ANS. FIG. P5.89

Resolving vertically: $\sum F_y = ma_y$ gives

$$n = F_g + P\sin\theta$$

Horizontally, $\sum F_x = ma_x$ gives

$$P\cos\theta = f$$

But,

$$f_s \le \mu_s n$$

i.e.,

$$P\cos\theta \le \mu_s\left(F_g + P\sin\theta\right)$$

or

$$P(\cos\theta - \mu_s\sin\theta) \le \mu_s F_g$$

Divide by $\cos\theta$:

$$P(1 - \mu_s\tan\theta) \le \mu_s F_g\sec\theta$$

Then

$$\boxed{P_{\text{minimum}} = \frac{\mu_s F_g \sec\theta}{1 - \mu_s \tan\theta}}$$

(b) To set the crate into motion, the x component ($P\cos\theta$) must overcome friction $f_s = \mu_s n$:

$$P\cos\theta \geq \mu_s n = \mu_s\left(F_g + P\sin\theta\right)$$

$$P\left(\cos\theta - \mu_s \sin\theta\right) \geq \mu_s F_g$$

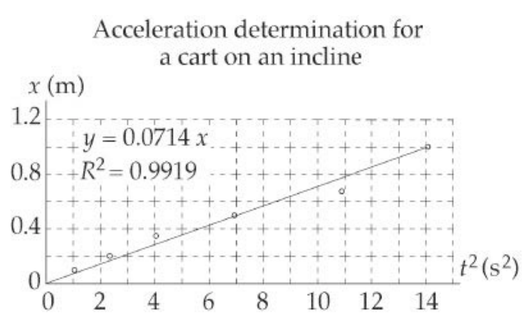
For this condition to be satisfied, it must be true that

$$\left(\cos\theta - \mu_s \sin\theta\right) > 0 \rightarrow \mu_s \tan\theta < 1 \rightarrow \tan\theta < \frac{1}{\mu_s}$$

If this condition is not met, no value of P can move the crate.

P5.90 (a) See table below and graph in ANS. FIG. P5.90(a).

t(s)	t^2(s^2)	x(m)
0	0	0
1.02	1.04 0	0.100
1.53	2.34 1	0.200
2.01	4.04 0	0.350
2.64	6.97 0	0.500
3.30	10.89	0.750
3.75	14.06	1.00



ANS. FIG. P5.90(a)

(b) From $x = \frac{1}{2}at^2$ the slope of a graph of x versus t^2 is $\frac{1}{2}a$, and

$$a = 2 \times \text{slope} = 2\left(0.071\,4\ \text{m/s}^2\right) = \boxed{0.143\ \text{m/s}^2}$$

(c) From $a' = g\sin\theta$,

$$a' = 9.80\ \text{m/s}^2\left(\frac{1.77\,4}{127.1}\right) = 0.137\ \text{m/s}^2\text{, different by 4\%.}$$

The difference is accounted for by the uncertainty in the data, which we may estimate from the third point as

$$\frac{0.350 - (0.071\,4)(4.04)}{0.350} = 18\%$$

$\boxed{\text{Thus the acceleration values agree.}}$

P5.91 (a) The net force on the cushion is in a fixed direction, downward and forward making angle $\theta = \tan^{-1}(F/mg)$ with the vertical.

Because the cushion starts from rest, the direction of its line of motion will be the same as that of the net force.

We show the path is a straight line another way. In terms of a standard coordinate system, the x and y coordinates of the cushion are

$$y = h - \frac{1}{2}gt^2$$

$$x = \frac{1}{2}(F/m)t^2 \to t^2 = (2m/F)x$$

Substitution of t^2 into the equation for y gives

$$y = h - (mg/f)x$$

which is an equation for a straight line.

(b) Because the cushion starts from rest, it will move in the direction of the net force which is the direction of its acceleration; therefore, it will move with increasing speed and its velocity changes in magnitude.

(c) Since the line of motion is in the direction of the net force, they both make the same angle with the vertical. Refer to Figure P5.91 in the textbook: in terms of a right triangle with angle θ, height h, and base x,

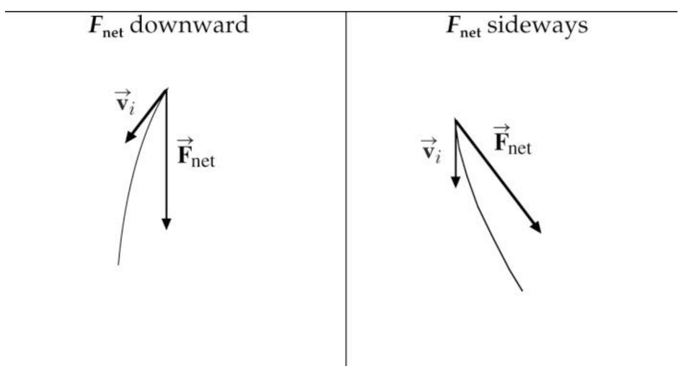
$$\tan\theta = x/h = F/mg \rightarrow x = hF/mg$$

$$x = \frac{(8.00\text{ m})(2.40\text{ N})}{(1.20\text{ kg})(9.80\text{ m/s}^2)}$$

and the cushion will land a distance

$x = \boxed{\text{1.63 m from the base of the building}}$.

(d) The cushion will move along a tilted parabola. If the cushion were experiencing a constant net force directed vertically downward (as is normal with gravity), and if its initial velocity were down and somewhat to the left, the trajectory would have the shape of a parabola that we would expect for projectile motion. Because the constant net force is "sideways"—at an angle θ counterclockwise from the vertical—the cushion would travel a similar trajectory as described above, but rotated counterclockwise by the angle θ so that the initial velocity is directed downward. See the figures.



ANS. FIG. P5.91(d)

P5.92 (a) When block 2 moves down 1 cm, block 1 moves 2 cm forward, so block 1 always has twice the speed of block 2, and $\boxed{a_1 = 2a_2}$ relates the magnitudes of the accelerations.

(b) Let T represent the uniform tension in the cord.

For block 1 as object,

$$\sum F_x = m_1a_1: \quad T = m_1a_1 = m_1(2a_2)$$

$$T = 2m_1a_2 \qquad \textbf{[1]}$$

For block 2 as object,

$$\sum F_y = m_2a_2: \quad T + T - m_2g = m_2(-a_2)$$
$$2T - m_2g = -m_2a_2 \qquad \textbf{[2]}$$

To solve simultaneously we substitute equation [1] into equation [2]:

$$2(2m_1a_2) - m_2g = -m_2a_2 \rightarrow 4m_1a_2 + m_2a_2 = m_2g$$
$$a_2 = \frac{m_2g}{4m_1 + m_2}$$

for $m_2 = 1.30$ kg: $\boxed{a_2 = 12.7\text{ N }(1.30\text{ kg} + 4\,m_1)^{-1}\text{ down}}$

(c) If m_1 is very much less than 1.30 kg, a_2 approaches

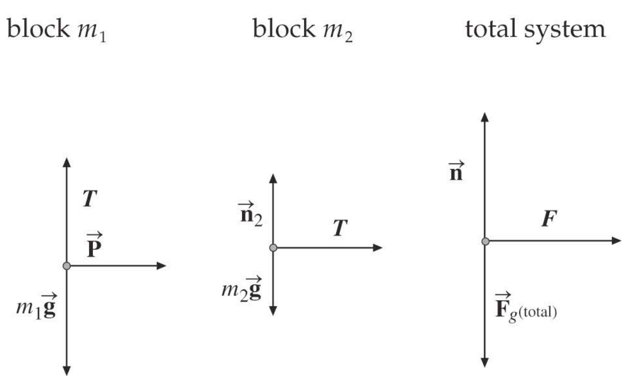
$$12.7\text{ N}/1.30\text{ kg} = \boxed{9.80\text{ m/s}^2\text{ down}}$$

(d) If m_1 approaches infinity, $\boxed{a_2\text{ approaches zero}}$.

(e) From equation (2) above, $2T = m_2g + m_2a_2 = 12.74\text{ N} + 0$, $\boxed{T = 6.37\text{ N}}$

(f) $\boxed{\text{Yes. As } m_1 \text{ approaches zero, block 2 is essentially in free fall. As } m_2 \text{ becomes negligible compared to } m_1,\ m_2 \text{ has very little weight, so the system is nearly in equilibrium.}}$

P5.93 We will use $\sum F = ma$ on each object, so we draw force diagrams for the $M + m_1 + m_2$ system, and also for blocks m_1 and m_2. Remembering that normal forces are always perpendicular to the contacting surface, and always **push** on a body, draw n_1 and n_2 as shown.



ANS. FIG. P5.93

Note that m_1 is in contact with the cart, and therefore feels a normal force exerted by the cart. Remembering that ropes always **pull** on

bodies toward the center of the rope, draw the tension force $\vec{\mathbf{T}}$. Finally, draw the gravitational force on each block, which always points downwards.

Applying $\sum F = ma$,

For m_1: $\quad T - m_1 g = 0$

For m_2: $\quad T = m_2 a$

Eliminating T,

$$a = \frac{m_1 g}{m_2}$$

For all three blocks:

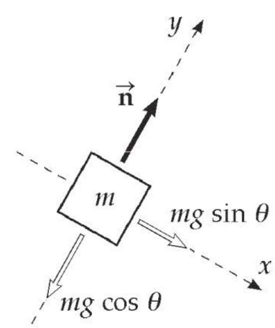
$$\boxed{F = \left(M + m_1 + m_2\right)\frac{m_1 g}{m_2}}$$

P5.94 (a) $\sum F_y = ma_y$:

$n - mg\cos\theta = 0$

or $n = (8.40 \text{ kg})(9.80 \text{ m/s}^2)\cos\theta$

$$\boxed{n = (82.3 \text{ N})\cos\theta}$$



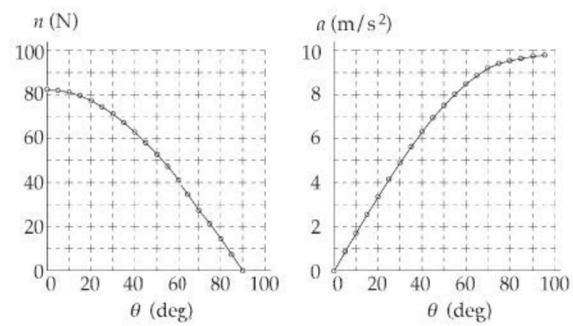
(b) $\sum F_x = ma_x$:

$mg\sin\theta = ma$

or

$a = g\sin\theta$

$$\boxed{a = (9.80 \text{ m/s}^2)\sin\theta}$$



ANS. FIG. P5.94

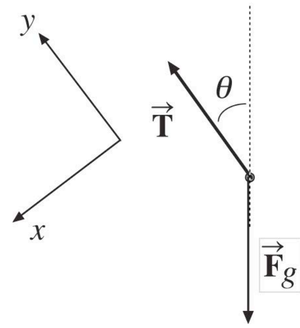
(c)

θ, deg	n, N	a, m/s^2
0.00	82.3	0.00
5.00	82.0	0.854
10.0	81.1	1.70
15.0	79.5	2.54
20.0	77.4	3.35
25.0	74.6	4.14
30.0	71.3	4.90
35.0	67.4	5.62
40.0	63.1	6.30
45.0	58.2	6.93
50.0	52.9	7.51
55.0	47.2	8.03
60.0	41.2	8.49
65.0	34.8	8.88
70.0	28.2	9.21
75.0	21.3	9.47
80.0	14.3	9.65
85.0	7.17	9.76
90.0	0.00	9.80

(d) At 0°, the normal force is the full weight and the acceleration is zero. At 90°, the mass is in free fall next to the vertical incline.

P5.95 Refer to the free-body diagram in ANS. FIG. P5.95. Choose the x axis pointing down the slope so that the string makes the angle θ with the vertical. The acceleration is obtained from $v_f = v_i + at$:

$$a = (v_f - v_i)/t = (30.0\ \text{m/s}^2 - 0)/6.00\ \text{s}$$
$$a = 5.00\ \text{m/s}^2$$



ANS. FIG. P5.95

Because the string stays perpendicular to the ceiling, we know that the toy moves with the same acceleration as the van, 5.00 m/s^2 parallel to the hill. We take the x axis in this direction, so

$$a_x = 5.00\ \text{m/s}^2 \quad \text{and} \quad a_y = 0$$

The only forces on the toy are the string tension in the y direction and the planet's gravitational force, as shown in the force diagram. The size of the latter is $mg = (0.100\ \text{kg})(9.80\ \text{m/s}^2) = 0.980\ \text{N}$

(a) Using $\sum F_x = ma_x$ gives $(0.980\ \text{N})\sin\theta = (0.100\ \text{kg})(5.00\ \text{m/s}^2)$

Then $\sin\theta = 0.510$ and $\theta = \boxed{30.7°}$

(b) Using $\sum F_y = ma_y$ gives $+T - (0.980\ \text{N})\cos\theta = 0$

$T = (0.980\ \text{N})\cos 30.7° = \boxed{0.843\ \text{N}}$

Challenge Problems

P5.96 $\sum \vec{F} = m\vec{a}$ gives the object's acceleration:

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \frac{\left(8.00\hat{\mathbf{i}} - 4.00t\hat{\mathbf{j}}\right)\ \text{N}}{2.00\ \text{kg}}$$

$$\vec{\mathbf{a}} = \left(4.00\ \text{m/s}^2\right)\hat{\mathbf{i}} - \left(2.00\ \text{m/s}^3\right)t\hat{\mathbf{j}} = \frac{d\vec{\mathbf{v}}}{dt}$$

(a) To arrive at an equation for the instantaneous velocity of object, we must integrate the above equation.

$$d\vec{\mathbf{v}} = \left(4.00\ \text{m/s}^2\right)dt\hat{\mathbf{i}} - \left(2.00\ \text{m/s}^3\right)tdt\hat{\mathbf{j}}$$
$$\int d\vec{\mathbf{v}} = \int\left(4.00\ \text{m/s}^2\right)dt\hat{\mathbf{i}} - \int\left(2.00\ \text{m/s}^3\right)t\,dt\hat{\mathbf{j}}$$
$$\vec{\mathbf{v}} = \left[\left(4.00\ \text{m/s}^2\right)t + c_1\right]\hat{\mathbf{i}} - \left[\left(1.00\ \text{m/s}^3\right)t^2 + c_2\right]\hat{\mathbf{j}}$$

In order to evaluate the constants of integration, we observe that the object is at rest when $t = 0$ s.

$$\vec{\mathbf{v}}(t=0) = 0 = \left[\left(4.00\ \text{m/s}^2\right)0 + c_1\right]\hat{\mathbf{i}} - \left[\left(1.00\ \text{m/s}^3\right)0^2 + c_2\right]\hat{\mathbf{j}}$$

or $c_1 = c_2 = 0$

and

$$\vec{\mathbf{v}} = \left[\left(4.00\ \text{m/s}^2\right)t\right]\hat{\mathbf{i}} - \left[\left(1.00\ \text{m/s}^3\right)t^2\right]\hat{\mathbf{j}}$$

Thus, when $v = 15.0$ m/s,

$$|\vec{\mathbf{v}}| = 15.0\ \text{m/s} = \sqrt{\left[\left(4.00\ \text{m/s}^2\right)t\right]^2 + \left[\left(1.00\ \text{m/s}^3\right)t^2\right]^2}$$
$$15.0\ \text{m/s} = \sqrt{\left[\left(16.0\ \text{m}^2/\text{s}^4\right)t^2\right] + \left[\left(1.00\ \text{m}^2/\text{s}^6\right)t^4\right]}$$
$$225\ \text{m}^2/\text{s}^2 = \left[\left(16.0\ \text{m}^2/\text{s}^4\right)t^2\right] + \left[\left(1.00\ \text{m}^2/\text{s}^6\right)t^4\right]$$
$$0 = \left(1.00\ \text{m}^2/\text{s}^6\right)t^4 + \left(16.0\ \text{m}^2/\text{s}^4\right)t^2 - 225\ \text{m}^2/\text{s}^2$$

We now need a solution to the above equation, in order to find t. The equation can be factored as,

$$0 = \left(t^2 - 9\right)\left(t^2 + 25\right)$$

The solution for t, here, comes from the first factor:

$$t^2 - 9.00 = 0$$
$$t = \pm 3.00\ \text{s} = \boxed{3.00\ \text{s}}$$

(b) In order to find the object's position at this time, we need to integrate the velocity equation, using the assumption that the objects starts at the origin (the constants of integration will again be equal to 0, as before).

$$d\vec{\mathbf{r}} = \left(4.00\ \text{m/s}^2\right)t\,dt\,\hat{\mathbf{i}} - \left(1.00\ \text{m/s}^3\right)t^2\,dt\,\hat{\mathbf{j}}$$
$$\int d\vec{\mathbf{r}} = \int\left(4.00\ \text{m/s}^2\right)t\,dt\,\hat{\mathbf{i}} - \int\left(1.00\ \text{m/s}^3\right)t^2\,dt\,\hat{\mathbf{j}}$$
$$\vec{\mathbf{r}} = \left[\left(2.00\ \text{m/s}^2\right)t^2\right]\hat{\mathbf{i}} - \left[\left(0.333\ \text{m/s}^3\right)t^3\right]\hat{\mathbf{j}}$$

Now, using the time above and finding the magnitude of this displacement vector,

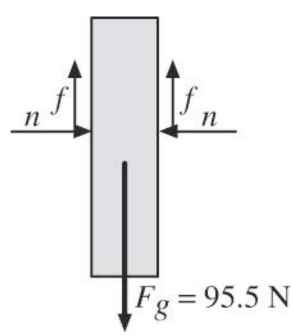
$$|\vec{\mathbf{r}}| = \sqrt{\left[\left(2.00\ \text{m/s}^2\right)\left(3.00\ \text{s}\right)^2\right]^2 + \left[\left(0.333\ \text{m/s}^3\right)\left(3.00\ \text{s}\right)^3\right]^2}$$
$$|\vec{\mathbf{r}}| = \boxed{20.1\ \text{m}}$$

(c) Using the displacement vector found in part (b),

$$\vec{\mathbf{r}} = \left[\left(2.00\ \text{m/s}^2\right)t^2\right]\hat{\mathbf{i}} - \left[\left(0.333\ \text{m/s}^3\right)t^3\right]\hat{\mathbf{j}}$$
$$\vec{\mathbf{r}} = \left[\left(2.00\ \text{m/s}^2\right)\left(3.00\ \text{s}\right)^2\right]\hat{\mathbf{i}} - \left[\left(0.333\ \text{m/s}^3\right)\left(3.00\ \text{s}\right)^3\right]\hat{\mathbf{j}}$$
$$\vec{\mathbf{r}} = \boxed{\left(18.0\ \text{m}\right)\hat{\mathbf{i}} - \left(9.00\ \text{m}\right)\hat{\mathbf{j}}}$$

P5.97 Since the board is in equilibrium, $\sum F_x = 0$ and we see that the normal forces must be the same on both sides of the board. Also, if the minimum normal forces (compression forces) are being applied, the board is on the verge of slipping and the friction force on each side is

$$f = \left(f_s\right)_{\max} = \mu_s n$$



ANS. FIG. P5.97

The board is also in equilibrium in the vertical direction, so

$$\sum F_y = 2f - F_g = 0\text{, or } f = \frac{F_g}{2}$$

The minimum compression force needed is then

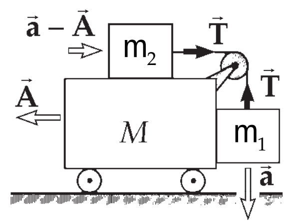
$$n = \frac{f}{\mu_s} = \frac{F_g}{2\mu_s} = \frac{95.5\text{ N}}{2(0.663)} = \boxed{72.0\text{ N}}$$

***P5.98** We apply Newton's second law to each of the three masses, reading the forces from ANS. FIG. P5.98:

$$m_2(a - A) = T \Rightarrow \quad a = \frac{T}{m_2} + A \qquad \textbf{[1]}$$

$$MA = R_x = T \Rightarrow \quad A = \frac{T}{M} \qquad \textbf{[2]}$$

$$m_1 a = m_1 g - T \Rightarrow \quad T = m_1(g - a) \qquad \textbf{[3]}$$



ANS. FIG. P5.98

(a) Substitute the value for *a* from [1] into [3] and solve for *T*:

$$T = m_1\left[g - \left(\frac{T}{m_2} + A\right)\right]$$

Substitute for *A* from [2]:

$$T = m_1\left[g - \left(\frac{T}{m_2} + \frac{T}{M}\right)\right] \Rightarrow T = \boxed{m_2 g\left[\frac{m_1 M}{m_2 M + m_1(m_2 + M)}\right]}$$

(b) Solve [3] for *a* and substitute value of *T*:

$$a = g - \frac{T}{m_1} = g - m_2 g\left[\frac{M}{m_2 M + m_1(m_2 + M)}\right]$$

$$= g\left[1 - \frac{m_2 M}{m_2 M + m_1(m_2 + M)}\right]$$

$$= \boxed{\left[\frac{g m_1(m_2 + M)}{m_2 M + m_1(m_2 + M)}\right]}$$

(c) From [2], $A = \frac{T}{M}$. Substitute the value of T:

$$A = \frac{T}{M} = \boxed{\left[\frac{m_1 m_2 g}{m_2 M + m_1(m_2 + M)}\right]}$$

(d) The acceleration of m_1 is given by

$$a - A = \boxed{\left[\frac{m_1 M g}{m_2 M + m_1(m_2 + M)}\right]}$$

P5.99 (a) The cord makes angle θ with the horizontal where

$$\theta = \tan^{-1}\left(\frac{0.100 \text{ m}}{0.400 \text{ m}}\right) = 14.0°$$

Applying Newton's second law in the y direction gives

$$\sum F_y = ma_y:$$
$$T\sin\theta - mg + n = 0$$
$$(+10 \text{ N})\sin 14.0° - (2.20 \text{ kg})(9.80 \text{ m/s}^2) + n = 0$$

which gives n = 19.1 N. Applying Newton's second law in the x direction then gives

$$\sum F_x = ma_x:$$
$$T\cos\theta - f_k = ma$$
$$T\cos\theta - \mu_k n = ma$$
$$(+10 \text{ N})\cos 14.0° - 0.400(19.1 \text{ N}) = (2.20 \text{ kg})\, a$$

which gives

$$a = \boxed{0.931 \text{ m/s}^2}$$

(b) When x is large we have $n = 21.6$ N, $f_k = 8.62$ N, and $a = (10\text{ N} - 8.62\text{ N})/2.2\text{ kg} = 0.625\text{ m/s}^2$.

As x decreases, the acceleration increases gradually, passes through a maximum, and then drops more rapidly, becoming negative. At $x = 0$ it reaches the value $a = [0 - 0.4(21.6\text{ N} - 10\text{ N})]/2.2\text{ kg} = -2.10\text{ m/s}^2$.

(c) We carry through the same calculations as in part (a) for a variable angle, for which $\cos\theta = x[x^2 + (0.100\text{ m})^2]^{-1/2}$ and $\sin\theta = (0.100\text{ m})[x^2 + (0.100\text{ m})^2]^{-1/2}$. We find

$$a = \left(\frac{1}{2.20\text{ kg}}\right)(10\text{ N})x\left[x^2 + 0.100^2\right]^{-1/2}$$
$$- 0.400\left(21.6\text{ N} - (10\text{ N})(0.100)\left[x^2 + 0.100^2\right]^{-1/2}\right)$$
$$a = 4.55x\left[x^2 + 0.100^2\right]^{-1/2} - 3.92 + 0.182\left[x^2 + 0.100^2\right]^{-1/2}$$

Now to maximize a we take its derivative with respect to x and set it equal to zero:

$$\frac{da}{dx} = 4.55\left(x^2 + 0.100^2\right)^{-1/2} + 4.55x\left(-\frac{1}{2}\right)2x\left(x^2 + 0.100^2\right)^{-3/2}$$
$$+ 0.182\left(-\frac{1}{2}\right)2x\left(x^2 + 0.100^2\right)^{-3/2} = 0$$

Solving,

$$4.55\left(x^2 + 0.1^2\right) - 4.55x^2 - 0.182x = 0$$

or $x = \boxed{0.250\text{ m}}$

At this point, suppressing units,

$$a = (4.55)(0.250)\left[0.250^2 + 0.100^2\right]^{-1/2} - 3.92$$
$$+ 0.182\left[0.250^2 + 0.100^2\right]^{-1/2}$$
$$= \boxed{0.976\text{ m/s}^2}$$

(d) We solve, suppressing units,

$$0 = 4.55x\left[x^2 + 0.100^2\right]^{-1/2} - 3.92 + 0.182\left[x^2 + 0.100^2\right]^{-1/2}$$
$$3.92\left[x^2 + 0.100^2\right]^{1/2} = 4.55x + 0.182$$
$$15.4\left[x^2 + 0.100^2\right] = 20.7x^2 + 1.65x + 0.033\ 1$$

which gives the quadratic equation

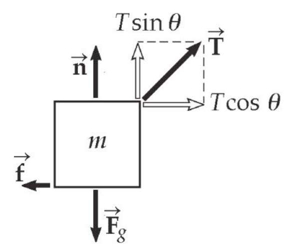
$$5.29x^2 + 1.65x - 0.121 = 0$$

Only the positive root is directly meaningful, so

$$x = \boxed{0.0610\text{ m}}$$

P5.100 The force diagram is shown on the right. With motion impending,

$$n + T\sin\theta - mg = 0$$
$$f = \mu_s\left(mg - T\sin\theta\right)$$



ANS. FIG. P5.100

and

$$T\cos\theta - \mu_s mg + \mu_s T\sin\theta = 0$$

so $$T = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}$$

To minimize T, we maximize $\cos\theta + \mu_s\sin\theta$:

$$\frac{d}{d\theta}\left(\cos\theta + \mu_s\sin\theta\right) = 0 = -\sin\theta + \mu_s\cos\theta$$

Therefore, the angle where tension T is a minimum is

$$\theta = \tan^{-1}\left(\mu_s\right) = \tan^{-1}\left(0.350\right) = 19.3°$$

What is the tension at this angle? From above,

$$T = \frac{0.350\left(1.30\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{\cos 19.3° + 0.350\sin 19.3°} = 4.21\text{ N}$$

$\boxed{\text{The situation is impossible because at the angle of minimum tension, the tension exceeds 4.00 N.}}$

P5.101 (a) Following the in-chapter example about a block on a frictionless incline, we have

$$a = g\sin\theta = (9.80\ \text{m/s}^2)\sin 30.0°$$

$$\boxed{a = 4.90\ \text{m/s}^2}$$

(b) The block slides distance *x* on the incline, with $\sin 30.0° = \dfrac{0.500\ \text{m}}{x}$.

$$x = 1.00\ \text{m: } v_f^2 = v_i^2 + 2a(x_f - x_i) = 0 + 2(4.90\ \text{m/s}^2)(1.00\ \text{m})$$

$$v_f = \boxed{3.13\ \text{m/s}} \text{ after time } t_s = \frac{2x_f}{v_f} = \frac{2(1.00\ \text{m})}{3.13\ \text{m/s}} = 0.639\ \text{s}$$

(c) To calculate the horizontal range of the block, we need to first determine the time interval during which it is in free fall. We use $y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2$, and substitute, noting that $v_{yi} = (-3.13\ \text{m/s}) \sin 30.0°$.

$$-2.00 = (-3.13\ \text{m/s})\sin 30.0° t - \frac{1}{2}(9.80\ \text{m/s}^2)t^2$$

$$(4.90\ \text{m/s}^2)t^2 + (1.56\ \text{m/s})t - 2.00\ \text{m} = 0$$

Solving for *t* gives

$$t = \frac{-1.56\ \text{m/s} \pm \sqrt{(1.56\ \text{m/s})^2 - 4(4.90\ \text{m/s}^2)(-2.00\ \text{m})}}{9.80\ \text{m/s}^2}$$

Only the positive root is physical, with $t = 0.499$ s. The horizontal range of the block is then

$$x_f = v_x t = [(3.13\ \text{m/s})\cos 30.0°](0.499\ \text{s}) = \boxed{1.35\ \text{m}}$$

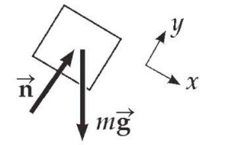
(d) The total time from release to impact is then

$$\text{total time } = t_s + t = 0.639\ \text{s} + 0.499\ \text{s} = \boxed{1.14\ \text{s}}$$

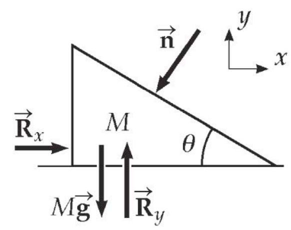
(e) $\boxed{\text{The mass of the block makes no difference}}$, as acceleration due to gravity, whether an object is in free fall or on a frictionless incline, is independent of the mass of the object.

P5.102 Throughout its up and down motion after release the block has

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$n = mg\cos\theta$$



Let $\vec{R} = R_x\hat{i} + R_y\hat{j}$ represent the force of table on incline. We have



ANS. FIG. P5.102

$$\sum F_x = ma_x: \quad +R_x - n\sin\theta = 0$$
$$R_x = mg\cos\theta\sin\theta$$
$$\sum F_y = ma_y: \quad -Mg - n\cos\theta + R_y = 0$$
$$R_y = Mg + mg\cos^2\theta$$

$$\boxed{\vec{R} = mg\cos\theta\sin\theta \text{ to the right} + \left(M + m\cos^2\theta\right)g \text{ upward}}$$

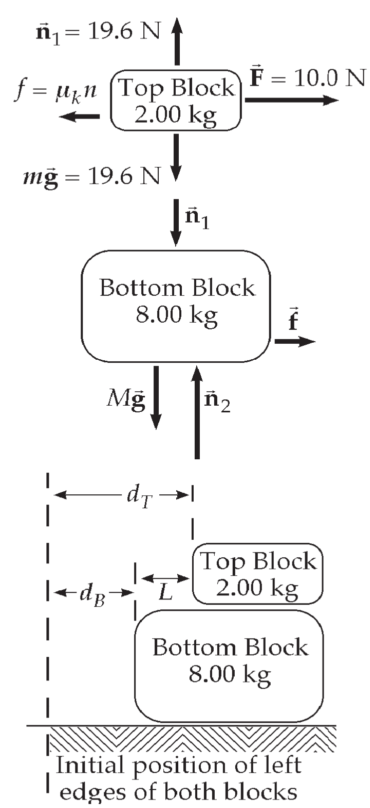
***P5.103** (a) First, draw a free-body diagram of the top block (top panel in ANS. FIG. P5.103). Since $a_y = 0$, $n_1 = 19.6$ N, and

$$f_k = \mu_k n_1 = 0.300(19.6\text{ N})$$
$$= 5.88\text{ N}$$

From $\sum F_x = ma_T$,

$$10.0\text{ N} - 5.88\text{ N} = (2.00\text{ kg})a_T$$

or $a_T = 2.06\text{ m/s}^2$ (for top block). Now draw a free-body diagram (middle figure) of the bottom block and observe that $\sum F_x = Ma_B$ gives $f = 5.88\text{ N} = (8.00\text{ kg})a_B$ or $a_B = 0.735\text{ m/s}^2$ (for the bottom block). In time t, the distance each block moves (starting from rest) is



ANS. FIG. P5.103

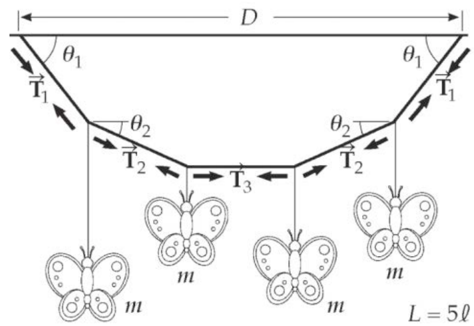
$d_T = \frac{1}{2}a_T t^2$ and $d_B = \frac{1}{2}a_B t^2$. For the top block to reach the right edge of the bottom block (see bottom figure), it is necessary that $d_T = d_B + L$ or

$$\frac{1}{2}\left(2.06 \text{ m/s}^2\right)t^2 = \frac{1}{2}\left(0.735 \text{ m/s}^2\right)t^2 + 3.00 \text{ m}$$

which gives $t = \boxed{2.13 \text{ s}}$.

(b) From above, $d_B = \frac{1}{2}\left(0.735 \text{ m/s}^2\right)\left(2.13 \text{ s}\right)^2 = \boxed{1.67 \text{ m}}$.

P5.104 (a) Apply Newton's second law to two points where butterflies are attached on either half of mobile (the other half is the same, by symmetry).



ANS. FIG. P5.104

$$T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0 \qquad \textbf{[1]}$$

$$T_1 \sin\theta_1 - T_2 \sin\theta_2 - mg = 0 \qquad \textbf{[2]}$$

$$T_2 \cos\theta_2 - T_3 = 0 \qquad \textbf{[3]}$$

$$T_2 \sin\theta_2 - mg = 0 \qquad \textbf{[4]}$$

Substituting [4] into [2] for $T_2 \sin\theta_2$,

$$T_1 \sin\theta_1 - mg - mg = 0$$

Then

$$\boxed{T_1 = \frac{2mg}{\sin\theta_1}}$$

Substitute [3] into [1] for $T_2 \cos\theta_2$:

$$T_3 - T_1 \cos\theta_1 = 0,\ T_3 = T_1 \cos\theta_1$$

Substitute value of T_1:

$$T_3 = 2mg\frac{\cos\theta_1}{\sin\theta_1} = \boxed{\frac{2mg}{\tan\theta_1} = T_3}$$

From equation [4],

$$T_2 = \frac{mg}{\sin\theta_2}$$

(b) Divide [4] by [3]:

$$\frac{T_2 \sin\theta_2}{T_2 \cos\theta_2} = \frac{mg}{T_3}$$

Substitute value of T_3:

$$\tan\theta_2 = \frac{mg\tan\theta_1}{2mg}, \boxed{\theta_2 = \tan^{-1}\left(\frac{\tan\theta_1}{2}\right)}$$

Then we can finish answering part (a):

$$\boxed{T_2 = \frac{mg}{\sin\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right]}}$$

(c) D is the horizontal distance between the points at which the two ends of the string are attached to the ceiling.

$$\boxed{\begin{array}{l} D = 2\ell\cos\theta_1 + 2\ell\cos\theta_2 + \ell \text{ and } L = 5\ell \\ D = \dfrac{L}{5}\left\{2\cos\theta_1 + 2\cos\left[\tan^{-1}\left(\dfrac{1}{2}\tan\theta_1\right)\right] + 1\right\} \end{array}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P5.2 2.38 kN

P5.4 8.71 N

P5.6 (a) -4.47×10^{15} m/s^2; (b) $+2.09 \times 10^{-10}$ N

P5.8 (a) zero; (b) zero

P5.10 (a) $\frac{1}{2}vt$; (b) magnitude: $m\sqrt{(v/t)^2 + g^2}$, direction: $\tan^{-1}\left(\frac{gt}{v}\right)$

P5.12 $\left(16.3\hat{\mathbf{i}} + 14.6\hat{\mathbf{j}}\right)$ N

P5.14 (a–c) See free-body diagrams and corresponding forces in P5.14.

P5.16 1.59 m/s^2 at 65.2° N of E

P5.18 (a) $\frac{1}{3}$; (b) 0.750 m/s^2

P5.20 (a) $\sim 10^{-22}$ m/s^2; (b) $\Delta x \sim 10^{-23}$ m

P5.22 (a) $\hat{\mathbf{a}}$ is at 181°; (b) 11.2 kg; (c) 37.5 m/s; (d) $\left(-37.5\hat{\mathbf{i}} - 0.893\hat{\mathbf{j}}\right)$ m/s

P5.24 $\sum \vec{\mathbf{F}} = -km\vec{\mathbf{v}}$

P5.26 (a) See ANS. FIG. P5.26; (b) 1.03 N; (c) 0.805 N to the right

P5.28 (a) 49.0 N; (b) 49.0 N; (c) 98.0 N; (d) 24.5 N

P5.30 (a) See ANS. FIG. P5.30(a); (b) -2.54 m/s^2; (c) 3.19 m/s

P5.32 112 N

P5.34 See P5.33 for complete derivation.

P5.36 (a) $T_1 = 31.5$ N, $T_2 = 37.5$ N, $T_3 = 49.0$ N; (b) $T_1 = 113$ N, $T_2 = 56.6$ N, $T_3 = 98.0$ N

P5.38 (a) 78.4 N; (b) 105 N

P5.40 $a = 6.30$ m/s^2 and $T = 31.5$ N

P5.42 (a) See ANS FIG P5.42; (b) 3.57 m/s^2; (c) 26.7 N; (d) 7.14 m/s

P5.44 (a) $2m(g + a)$; (b) $T_1 = 2T_2$, so the upper string breaks first; (c) 0, 0

P5.46 (a) $a_2 = 2a_1$; (b) $T_2 = \dfrac{m_1 m_2}{2m_2 + \frac{1}{2}m_1} g$ and $T_2 = \dfrac{m_1 m_2}{m_2 + \frac{1}{4}m_1} g$; (c) $\dfrac{m_1 g}{2m_2 + \frac{1}{2}m_1}$ and $\dfrac{m_1 g}{4m_2 + m_1}$

P5.48 $B = 3.37 \times 10^3$ N, $A = 3.83 \times 10^3$ N, B is in tension and A is in compression.

P5.50 (a) 0.529 m below its initial level; (b) 7.40 m/s upward

P5.52 (a) 14.7 m; (b) neither mass is necessary

P5.54 (a) 256 m; (b) 42.7 m

P5.56 The situation is impossible because maximum static friction cannot provide the acceleration necessary to keep the book stationary on the seat.

P5.58 (a) 4.18; (b) Time would increase, as the wheels would skid and only kinetic friction would act; or perhaps the car would flip over.

P5.60 (a) See ANS. FIG. P5.60; (b) $\theta = 55.2°$; (c) $n = 167$ N

P5.62 (a) 0.404; (b) 45.8 lb

P5.64 (a) See ANS. FIG. P5.64; (b) 2.31 m/s^2, down for m_1, left for m_2, and up for m_3; (c) $T_{12} = 30.0$ N and $T_{23} = 24.2$ N; (d) T_{12} decreases and T_{23} increases

P5.66 (a) 48.6 N, 31. 7 N; (b) If $P > 48.6$ N, the block slides up the wall. If $P < 31.7$ N, the block slides down the wall; (c) 62.7 N, $P \geq 62.7$ N, the block cannot slide up the wall. If $P < 62.7$ N, the block slides down the wall

P5.68 834 N

P5.70 (a) See P5.70 for complete solution; (b) 9.80 N, 0.580 m/s^2

P5.72 (a) 3.43 m/s^2 toward the scrap iron; (b) 3.43 m/s^2 toward the scrap iron; (c) –6.86 m/s^2 toward the magnet

P5.74 The situation is impossible because these forces on the book cannot produce the acceleration described.

P4.76 (a) and (b) See P5.76 for complete derivation; (c) 3.56 N

P5.78 (a) See ANS. FIG. P5.78(a); (b) $a = \dfrac{F}{m_b + m_r}$; (c) $T = \left(\dfrac{m_b}{m_b + m_r}\right)F$; (d) the tension in a cord of negligible mass is constant along its length

P5.80 (a) At any instant they have the same velocity and at all instants they have the same acceleration; (b) 1.61×10^4 N; (c) 2.95×10^4 N

P5.82 (a) Nick and the seat, with total weight 480 N, will accelerate down and the child, with smaller weight 440 N, will accelerate up; (b) In P5.81, a rope tension of 250 N does not make the rope break. In part (a), the rope is strong enough to support tension 459 N. But now the tension everywhere in the rope is 480 N, so it can exceed the breaking strength of the rope.

P5.84 (a) The system will not start to move when released; (b and c) no answer; (d) $f = m_2 g \sin\theta = 29.4$ N

P5.86 (a) $T = \dfrac{f}{2\sin\theta}$; (b) 410 N

P5.88 (a) $M = 3m\sin\theta$; (b) $T_1 = 2mg\sin\theta,\ T_2 = 3mg\sin\theta$; (c) $a = \dfrac{g\sin\theta}{1+2\sin\theta}$;

(d) $T_1 = 4mg\sin\theta\left(\dfrac{1+\sin\theta}{1+2\sin\theta}\right),\ T_2 = 6mg\sin\theta\left(\dfrac{1+\sin\theta}{1+2\sin\theta}\right)$;

(e) $M_{\max} = 3m(\sin\theta + \mu_s\cos\theta)$; (f) $M_{\min} = 3m(\sin\theta - \mu_s\cos\theta)$;

(g) $T_{2,\max} - T_{2,\min} = M_{\max}g - M_{\min}g = 6\mu_s mg\cos\theta$

P5.90 See table in P5.90 and ANS. FIG P5.90; (b) 0.143 m/s^2; (c) The acceleration values agree.

P5.92 (a) $a_1 = 2a_2$; (b) $a_2 = 12.7$ N $(1.30\text{ kg} + 4m_1)^{-1}$ down; (c) 9.80 m/s^2 down; (d) a_2 approaches zero; (e) $T = 6.37$ N; (f) yes

P5.94 (a) $n = (8.23\text{ N})\cos\theta$; (b) $a = (9.80\text{ m/s}^2)\sin\theta$; (c) See ANS. FIG P5.94; (d) At 0°, the normal force is the full weight, and the acceleration is zero. At 90° the mass is in free fall next to the vertical incline.

P5.96 (a) 3.00 s; (b) 20.1 m; (c) $(18.0\text{m})\hat{\mathbf{i}} - (9.00\text{m})\hat{\mathbf{j}}$

P5.98 (a) $m_2 g\left[\dfrac{m_1 M}{m_2 M + m_1(m_2 + M)}\right]$; (b) $\left[\dfrac{g m_1(m_2 + M)}{m_2 M + m_1(m_2 + M)}\right]$;
(c) $\left[\dfrac{m_1 m_2 g}{m_2 M + m_1(m_2 + M)}\right]$; (d) $\left[\dfrac{m_1 M g}{m_2 M + m_1(m_2 + M)}\right]$

P5.100 The situation is impossible because at the angle of minimum tension, the tension exceeds 4.00 N

P5.102 $\vec{\mathbf{R}} = mg\cos\theta\sin\theta$ to the right $+\left(M + m\cos^2\theta\right)g$ upward

P5.104 (a) $T_1 = \dfrac{2mg}{\sin\theta_1}, \dfrac{2mg}{\tan\theta_1} = T_3$; (b) $\theta_2 = \tan^{-1}\left(\dfrac{\tan\theta_1}{2}\right)$,
$T_2 = -\dfrac{mg}{\sin\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right]}$; (c) See P5.104 for complete explanation.

6

Circular Motion and Other Applications of Newton's Laws

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ6.1 (a) A > C = D > B = E = 0. At constant speed, centripetal acceleration is largest when radius is smallest. A straight path has infinite radius of curvature. (b) Velocity is north at *A*, west at *B*, and south at *C*. (c) Acceleration is west at *A*, nonexistent at *B*, east at *C*, to be radially inward.

OQ6.2 Answer (a). Her speed increases, until she reaches terminal speed.

OQ6.3 (a) Yes. Its path is an arc of a circle; the direction of its velocity is changing. (b) No. Its speed is not changing.

OQ6.4 (a) Yes, point *C*. Total acceleration here is centripetal acceleration, straight up. (b) Yes, point *A*. The speed at *A* is zero where the bob is reversing direction. Total acceleration here is tangential acceleration, to the right and downward perpendicular to the cord. (c) No. (d) Yes, point *B*. Total acceleration here is to the right and either downwards or upwards depending on whether the magnitude of the centripetal acceleration is smaller or larger than the magnitude of the tangential acceleration.

OQ6.5 Answer (b). The magnitude of acceleration decreases as the speed increases because the air resistance force increases, counterbalancing more and more of the gravitational force.

OQ6.6 (a) No. When $v = 0$, $v^2/r = 0$.

(b) Yes. Its speed is changing because it is reversing direction.

OQ6.7 (i) Answer (c). The iPod shifts backward relative to the student's hand. The cord then pulls the iPod upward and forward, to make it gain speed horizontally forward along with the airplane. (ii) Answer (b). The angle stays constant while the plane has constant acceleration. This experiment is described in the book *Science from your Airplane Window* by Elizabeth Wood.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ6.1 (a) Friction, either static or kinetic, exerted by the roadway where it meets the rubber tires accelerates the car forward and then maintains its speed by counterbalancing resistance forces. Most of the time static friction is at work. But even kinetic friction (racers starting) will still move the car forward, although not as efficiently. (b) The air around the propeller pushes forward on its blades. Evidence is that the propeller blade pushes the air toward the back of the plane. (c) The water pushes the blade of the oar toward the bow. Evidence is that the blade of the oar pushes the water toward the stern.

CQ6.2 The drag force is proportional to the speed squared and to the effective area of the falling object. At terminal velocity, the drag and gravity forces are in balance. When the parachute opens, its effective area increases greatly, causing the drag force to increase greatly. Because the drag and gravity forces are no longer in balance, the greater drag force causes the speed to decrease, causing the drag force to decrease until it and the force of gravity are in balance again.

CQ6.3 The speed changes. The tangential force component causes tangential acceleration.

CQ6.4 (a) The object will move in a circle at a constant speed.

(b) The object will move in a straight line at a changing speed.

CQ6.5 The person in the elevator is in an accelerating reference frame. The apparent acceleration due to gravity, "g," is changed inside the elevator. "g" $= g \pm a$

CQ6.6 I would not accept that statement for two reasons. First, to be "beyond the pull of gravity," one would have to be infinitely far away from all

other matter. Second, astronauts in orbit are moving in a circular path. It is the gravitational pull of Earth on the astronauts that keeps them in orbit. In the space shuttle, just above the atmosphere, gravity is only slightly weaker than at the Earth's surface. Gravity does its job most clearly on an orbiting spacecraft, because the craft feels no other forces and is in free fall.

CQ6.7 This is the same principle as the centrifuge. All the material inside the cylinder tends to move along a straight-line path, but the walls of the cylinder exert an inward force to keep everything moving around in a circular path.

CQ6.8 (a) The larger drop has higher terminal speed. In the case of spheres, the text demonstrates that terminal speed is proportional to the square root of radius. (b) When moving with terminal speed, an object is in equilibrium and has zero acceleration.

CQ6.9 Blood pressure cannot supply the force necessary both to balance the gravitational force and to provide the centripetal acceleration to keep blood flowing up to the pilot's brain.

CQ6.10 The water has inertia. The water tends to move along a straight line, but the bucket pulls it in and around in a circle.

CQ6.11 The current consensus is that the laws of physics are probabilistic in nature on the fundamental level. For example, the Uncertainty Principle (to be discussed later) states that the position and velocity (actually, momentum) of any particle cannot both be known exactly, so the resulting predictions cannot be exact. For another example, the moment of the decay of any given radioactive atomic nucleus cannot be predicted, only the average rate of decay of a large number of nuclei can be predicted—in this sense, quantum mechanics implies that the future is indeterminate. How the laws of physics are related to our sense of free will is open to debate.

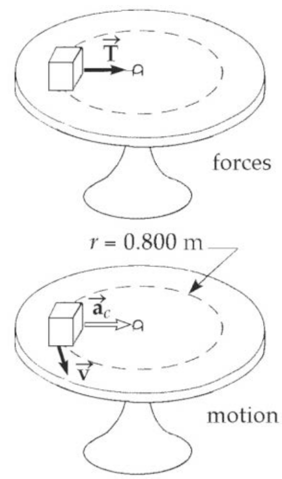
SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 6.1 Extending the Particle in Uniform Circular Motion Model

P6.1 We are given m = 3.00 kg, r = 0.800 m. The string will break if the tension exceeds the weight corresponding to 25.0 kg, so

$$T_{max} = Mg = (25.0 \text{ kg})(9.80 \text{ m/s}^2) = 245 \text{ N}$$

When the 3.00-kg mass rotates in a horizontal circle, the tension causes the centripetal acceleration,



ANS. FIG. P6.1

so $\quad T = \dfrac{mv^2}{r}$

Then

$$v^2 = \frac{rT}{m} = \frac{(0.800 \text{ m})T}{3.00 \text{ kg}} \le \frac{(0.800 \text{ m})T_{max}}{3.00 \text{ kg}}$$
$$= \frac{(0.800 \text{ m})(245 \text{ N})}{3.00 \text{ kg}} = 65.3 \text{ m}^2/\text{s}^2$$

This represents the maximum value of v^2, or

$$0 \le v \le \sqrt{65.3} \text{ m/s}$$

which gives

$$\boxed{0 \le v \le 8.08 \text{ m/s}}$$

P6.2 (a) The astronaut's orbital speed is found from Newton's second law, with

$$\sum F_y = ma_y: \ mg_{moon} \text{ down} = \frac{mv^2}{r} \text{ down}$$

solving for the velocity gives

$$v = \sqrt{g_{moon} r} = \sqrt{(1.52 \text{ m/s}^2)(1.7 \times 10^6 \text{ m} + 100 \times 10^3 \text{ m})}$$
$$v = \boxed{1.65 \times 10^3 \text{ m/s}}$$

(b) To find the period, we use $v = \dfrac{2\pi r}{T}$ and solve for T:

$$T = \frac{2\pi(1.8 \times 10^6 \text{ m})}{1.65 \times 10^3 \text{ m/s}} = \boxed{6.84 \times 10^3 \text{ s}} = 1.90 \text{ h}$$

P6.3 (a) The force acting on the electron in the Bohr model of the hydrogen atom is directed radially inward and is equal to

$$F = \frac{mv^2}{r} = \frac{(9.11\times10^{-31}\text{ kg})(2.20\times10^{6}\text{ m/s})^2}{0.529\times10^{-10}\text{ m}}$$
$$= \boxed{8.33\times10^{-8}\text{ N inward}}$$

(b) $a = \frac{v^2}{r} = \frac{(2.20\times10^{6}\text{ m/s})^2}{0.529\times10^{-10}\text{ m}} = \boxed{9.15\times10^{22}\text{ m/s}^2\text{ inward}}$

P6.4 In $\sum F = m\frac{v^2}{r}$, both *m* and *r* are unknown but remain constant. Symbolically, write

$$\sum F_{\text{slow}} = \left(\frac{m}{r}\right)(14.0\text{ m/s})^2 \text{ and } \sum F_{\text{fast}} = \left(\frac{m}{r}\right)(18.0\text{ m/s})^2$$

Therefore, $\sum F$ is proportional to v^2 and increases by a factor of $\left(\frac{18.0}{14.0}\right)^2$ as v increases from 14.0 m/s to 18.0 m/s. The total force at the higher speed is then

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 \sum F_{\text{slow}} = \left(\frac{18.0}{14.0}\right)^2 (130\text{ N}) = \boxed{215\text{ N}}$$

This force must be $\boxed{\text{horizontally inward}}$ to produce the driver's centripetal acceleration.

P6.5 We neglecting relativistic effects. With 1 u = 1.661 x 10^{-27} kg, and from Newton's second law, we obtain

$$F = ma_c = \frac{mv^2}{r}$$
$$= (2\times1.661\times10^{-27}\text{ kg})\frac{(2.998\times10^{7}\text{ m/s})^2}{(0.480\text{ m})}$$
$$= \boxed{6.22\times10^{-12}\text{ N}}$$

P6.6 (a) The car's speed around the curve is found from

$$v = \frac{235\text{ m}}{36.0\text{ s}} = 6.53\text{ m/s}$$

This is the answer to part (b) of this problem. We calculate the radius of the curve from $\frac{1}{4}(2\pi r) = 235\text{ m}$, which gives $r = 150$ m.

The car's acceleration at point *B* is then

$$\begin{aligned}\vec{\mathbf{a}}_r &= \left(\frac{v^2}{r}\right) \text{ toward the center} \\ &= \frac{(6.53\ \text{m/s})^2}{150\ \text{m}} \text{ at } 35.0° \text{ north of west} \\ &= (0.285\ \text{m/s}^2)\left(\cos 35.0°(-\hat{\mathbf{i}}) + \sin 35.0°\hat{\mathbf{j}}\right) \\ &= \boxed{(-0.233\hat{\mathbf{i}} + 0.163\hat{\mathbf{j}})\ \text{m/s}^2}\end{aligned}$$

(b) From part (a), $v = \boxed{6.53\ \text{m/s}}$

(c) We find the average acceleration from

$$\begin{aligned}\vec{\mathbf{a}}_{\text{avg}} &= \frac{(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i)}{\Delta t} \\ &= \frac{(6.53\hat{\mathbf{j}} - 6.53\hat{\mathbf{i}})\ \text{m/s}}{36.0\ \text{s}} \\ &= \boxed{(-0.181\hat{\mathbf{i}} + 0.181\hat{\mathbf{j}})\ \text{m/s}^2}\end{aligned}$$

P6.7 Standing on the inner surface of the rim, and moving with it, each person will feel a normal force exerted by the rim. This inward force causes the $3.00\ \text{m/s}^2$ centripetal acceleration:

$$a_c = v^2/r \quad \text{so} \quad v = \sqrt{a_c r} = \sqrt{(3.00\ \text{m/s}^2)(60.0\ \text{m})} = 13.4\ \text{m/s}$$

The period of rotation comes from $v = \dfrac{2\pi r}{T}$:

$$T = \frac{2\pi r}{v} = \frac{2\pi(60.0\ \text{m})}{13.4\ \text{m/s}} = 28.1\ \text{s}$$

so the frequency of rotation is

$$f = \frac{1}{T} = \frac{1}{28.1\ \text{s}} = \left(\frac{1}{28.1\ \text{s}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \boxed{2.14\ \text{rev/min}}$$

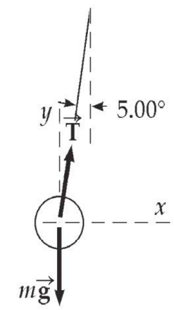
P6.8 ANS. FIG. P6.8 shows the free-body diagram for this problem.

(a) The forces acting on the pendulum in the vertical direction must be in balance since the acceleration of the bob in this direction is zero. From Newton's second law in the *y* direction,

$$\sum F_y = T\cos\theta - mg = 0$$

Solving for the tension T gives

$$T = \frac{mg}{\cos\theta} = \frac{(80.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 5.00°} = 787\text{ N}$$



ANS. FIG. P6.8

In vector form,

$$\vec{\mathbf{T}} = T\sin\theta\hat{\mathbf{i}} + T\cos\theta\hat{\mathbf{j}}$$

$$= \boxed{(68.6\text{ N})\hat{\mathbf{i}} + (784\text{ N})\hat{\mathbf{j}}}$$

(b) From Newton's second law in the x direction,

$$\sum F_x = T\sin\theta = ma_c$$

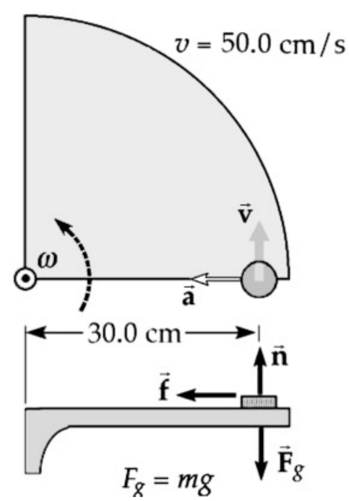
which gives

$$a_c = \frac{T\sin\theta}{m} = \frac{(787\text{ N})\sin 5.00°}{80.0\text{ kg}} = \boxed{0.857\text{ m/s}^2}$$

toward the center of the circle.

The length of the wire is unnecessary information. We could, on the other hand, use it to find the radius of the circle, the speed of the bob, and the period of the motion.

P6.9 ANS. FIG. P6.9 shows the constant maximum speed of the turntable and the centripetal acceleration of the coin.



ANS. FIG. P6.9

(a) The force of $\boxed{\text{static friction}}$ causes the centripetal acceleration.

(b) From ANS. FIG. P6.9,

$$ma\hat{\mathbf{i}} = f\,\hat{\mathbf{i}} + n\hat{\mathbf{j}} + mg(-\hat{\mathbf{j}})$$

$$\sum F_y = 0 = n - mg$$

thus, $n = mg$ and

$$\sum F_r = m\frac{v^2}{r} = f = \mu n = \mu mg$$

Then,

$$\mu = \frac{v^2}{rg} = \frac{(50.0\text{ cm/s})^2}{(30.0\text{ cm})(980\text{ cm/s}^2)} = \boxed{0.085\ 0}$$

P6.10 We solve for the tensions in the two strings:

$$F_g = mg = (4.00\ \text{kg})(9.80\ \text{m/s}^2) = 39.2\ \text{N}$$

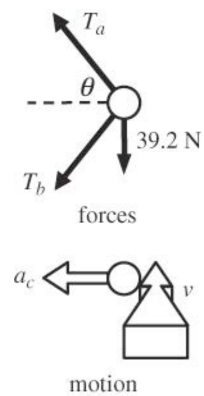
The angle θ is given by

$$\theta = \sin^{-1}\left(\frac{1.50\ \text{m}}{2.00\ \text{m}}\right) = 48.6°$$

The radius of the circle is then

$$r = (2.00\ \text{m})\cos 48.6° = 1.32\ \text{m}$$

Applying Newton's second law,



ANS. FIG. P6.10

$$\sum F_x = ma_x = \frac{mv^2}{r}$$

$$T_a \cos 48.6° + T_b \cos 48.6° = \frac{(4.00\ \text{kg})(3.00\ \text{m/s})^2}{1.32\ \text{m}}$$

$$T_a + T_b = \frac{27.27\ \text{N}}{\cos 48.6°} = 41.2\ \text{N} \qquad \textbf{[1]}$$

$$\sum F_y = ma_y: \quad T_a \sin 48.6° - T_b \sin 48.6° - 39.2\ \text{N} = 0$$

$$T_a - T_b = \frac{39.2\ \text{N}}{\sin 48.6°} = 52.3\ \text{N} \qquad \textbf{[2]}$$

To solve simultaneously, we add the equations in T_a and T_b:

$$(T_a + T_b) + (T_a - T_b) = 41.2\ \text{N} + 52.3\ \text{N}$$

$$T_a = \frac{93.8\ \text{N}}{2} = 46.9\ \text{N}$$

This means that $T_b = 41.2\ \text{N} - T_a = -5.7\ \text{N}$, which we may interpret as meaning the lower string pushes rather than pulls!

The situation is impossible because the speed of the object is too small, requiring that the lower string act like a rod and push rather than like a string and pull.

To answer the **What if?**, we go back to equation [2] above and substitute *mg* for the weight of the object. Then,

$$\sum F_y = ma_y: \quad T_a \sin 48.6° - T_b \sin 48.6° - mg = 0$$

$$T_a - T_b = \frac{(4.00\ \text{kg})g}{\sin 48.6°} = 5.33g$$

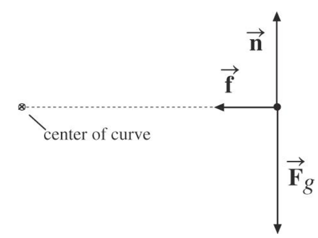
We then add this equation to equation [2] to obtain

$$(T_a + T_b) + (T_a - T_b) = 41.2 \text{ N} + 5.33g$$

or $$T_a = 20.6 \text{ N} + 2.67g \text{ and } T_b = 41.2 \text{ N} - T_a = 41.2 \text{ N} - 2.67g$$

For this situation to be possible, T_b must be > 0, or $g < 7.72 \text{ m/s}^2$. This is certainly the case on the surface of the Moon and on Mars.

P6.11 Call the mass of the egg crate m. The forces on it are its weight $F_g = mg$ vertically down, the normal force n of the truck bed vertically up, and static friction f_s directed to oppose relative sliding motion of the crate on the truck bed. The friction force is directed radially inward. It is the only horizontal force on the crate, so it must provide the centripetal acceleration. When the truck has maximum speed, friction f_s will have its maximum value with $f_s = \mu_s n$.



ANS. FIG. P6.11

Newton's second law in component form becomes

$$\sum F_y = ma_y \quad \text{giving} \quad n - mg = 0 \quad \text{or} \quad n = mg$$

$$\sum F_x = ma_x \quad \text{giving} \quad f_s = ma_r$$

From these three equations,

$$\mu_s n \le \frac{mv^2}{r} \quad \text{and} \quad \mu_s mg \le \frac{mv^2}{r}$$

The mass divides out. The maximum speed is then

$$v \le \sqrt{\mu_s rg} = \sqrt{0.600(35.0 \text{ m})(9.80 \text{ m/s}^2)} \quad \rightarrow \quad v \le \boxed{14.3 \text{ m/s}}$$

Section 6.2 Nonuniform Circular Motion

P6.12 (a) The external forces acting on the water are

$\boxed{\text{the gravitational force}}$

and $\boxed{\text{the contact force exerted on the water by the pail}}$.

(b) The $\boxed{\text{contact force exerted by the pail}}$ is the most important in causing the water to move in a circle. If the gravitational force acted alone, the water would follow the parabolic path of a projectile.

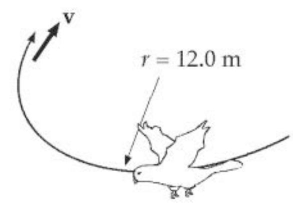
(c) When the pail is inverted at the top of the circular path, it cannot hold the water up to prevent it from falling out. If the water is not to spill, the pail must be moving fast enough that the required centripetal force is at least as large as the gravitational force. That is, we must have

$$m\frac{v^2}{r} \geq mg \quad \text{or} \quad v \geq \sqrt{rg} = \sqrt{(1.00\,\text{m})(9.80\,\text{m/s}^2)} = \boxed{3.13\ \text{m/s}}$$

(d) If the pail were to suddenly disappear when it is at the top of the circle and moving at 3.13 m/s, $\boxed{\text{the water would follow the parabolic path of a projectile}}$ launched with initial velocity components of $v_{xi} = 3.13$ m/s, $v_{yi} = 0$.

P6.13 (a) The hawk's centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{(4.00\ \text{m/s})^2}{12.0\ \text{m}} = \boxed{1.33\ \text{m/s}^2}$$



ANS. FIG. P6.13

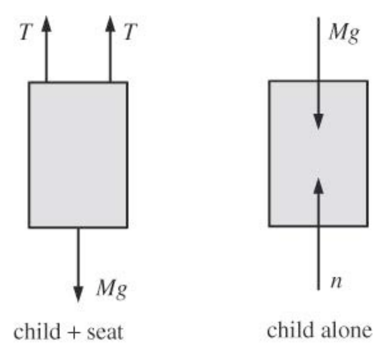
(b) The magnitude of the acceleration vector is

$$a = \sqrt{a_c^2 + a_t^2}$$
$$= \sqrt{(1.33\ \text{m/s}^2)^2 + (1.20\ \text{m/s}^2)^2} = \boxed{1.79\ \text{m/s}^2}$$

at an angle

$$\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right) = \tan^{-1}\left(\frac{1.33\ \text{m/s}^2}{1.20\ \text{m/s}^2}\right) = \boxed{48.0°\ \text{inward}}$$

6.14 We first draw a force diagram that shows the forces acting on the child-seat system and apply Newton's second law to solve the problem. The child's path is an arc of a circle, since the top ends of the chains are fixed. Then at the lowest point the child's motion is changing in direction: He moves with centripetal acceleration even as his speed is not changing and his tangential acceleration is zero.



ANS. FIG. P6.14

(a) ANS. FIG. P6.14 shows that the only forces acting on the system of child + seat are the tensions in the two chains and the weight of the boy:

$$\sum F = F_{net} = 2T - mg = ma = \frac{mv^2}{r}$$

with

$$F_{net} = 2T - mg = 2(350 \text{ N}) - (40.0 \text{ kg})(9.80 \text{ m/s}^2) = 308 \text{ N}$$

solving for v gives

$$v = \sqrt{\frac{F_{net} r}{m}} = \sqrt{\frac{(308 \text{ N})(3.00 \text{ m})}{40.0 \text{ kg}}} = \boxed{4.81 \text{ m/s}}$$

(b) The normal force from the seat on the child accelerates the child in the same way that the total tension in the chain accelerates the child-seat system. Therefore, $n = 2T = \boxed{700 \text{ N}}$.

P6.15 See the forces acting on seat (child) in ANS. FIG. P6.14.

(a) $\sum F = 2T - Mg = \dfrac{Mv^2}{R}$

$$v^2 = (2T - Mg)\left(\frac{R}{M}\right)$$

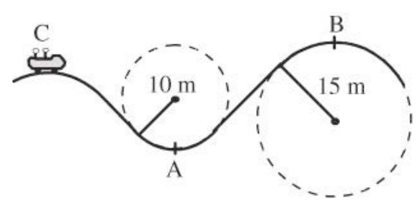
$$\boxed{v = \sqrt{(2T - Mg)\left(\frac{R}{M}\right)}}$$

(b) $n - Mg = F = \dfrac{Mv^2}{R}$

$$\boxed{n = Mg + \frac{Mv^2}{R}}$$

P6.16 (a) We apply Newton's second law at point A, with $v = 20.0$ m/s, n = force of track on roller coaster, and $R = 10.0$ m:

$$\sum F = \frac{Mv^2}{R} = n - Mg$$



ANS. FIG. P6.16

From this we find

$$n = Mg + \frac{Mv^2}{R} = (500 \text{ kg})(9.80 \text{ m/s}^2) + \frac{(500 \text{ kg})(20.0 \text{ m/s}^2)}{10.0 \text{ m}}$$

$$n = 4\,900 \text{ N} + 20\,000 \text{ N} = \boxed{2.49 \times 10^4 \text{ N}}$$

(b) At point B, the centripetal acceleration is now downward, and Newton's second law now gives

$$\sum F = n - Mg = -\frac{Mv^2}{R}$$

The maximum speed at B corresponds to the case where the rollercoaster begins to fly off the track, or when $n = 0$. Then,

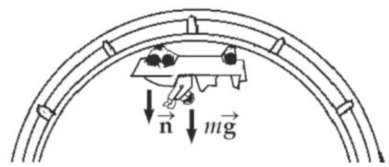
$$-Mg = -\frac{Mv_{max}^2}{R}$$

which gives

$$v_{max} = \sqrt{Rg} = \sqrt{(15.0 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{12.1 \text{ m/s}}$$

P6.17 (a) $a_c = \dfrac{v^2}{r}$

$$r = \frac{v^2}{a_c} = \frac{(13.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{8.62 \text{ m}}$$



ANS. FIG. P6.17

(b) Let n be the force exerted by the rail.

Newton's second law gives

$$Mg + n = \frac{Mv^2}{r}$$

$$n = M\left(\frac{v^2}{r} - g\right) = M(2g - g) = \boxed{Mg, \text{ downward}}$$

(c) $a_c = \dfrac{v^2}{r}$, or $a_c = \dfrac{(13.0 \text{ m/s})^2}{20.0 \text{ m}} = \boxed{8.45 \text{ m/s}^2}$

(d) If the force exerted by the rail is n_1,

then $n_1 + Mg = \dfrac{Mv^2}{r} = Ma_c$

$$n_1 = M(a_c - g) \text{ which is } < 0 \text{ since } a_c = 8.45 \text{ m/s}^2$$

Thus, the normal force would have to point away from the center of the curve. Unless they have belts, the riders will fall from the cars.

In a teardrop-shaped loop, the radius of curvature r decreases, causing the centripetal acceleration to increase. The speed would decrease as the car rises (because of gravity), but the overall effect is that the required centripetal force increases, meaning the normal force increases--there is less danger if not wearing a seatbelt.

P6.18 (a) Consider radial forces on the object, taking inward as positive.

$$\sum F_r = ma_r: \quad T - mg\cos\theta = \frac{mv^2}{r}$$

Solving for the tension gives

$$\begin{aligned} T &= mg\cos\theta + \frac{mv^2}{r} \\ &= (0.500 \text{ kg})(9.80 \text{ m/s}^2)\cos 20.0° \\ &\qquad + (0.500 \text{ kg})(8.00 \text{ m/s})^2/2.00 \text{ m} \\ &= 4.60 \text{ N} + 16.0 \text{ N} = \boxed{20.6 \text{ N}} \end{aligned}$$

(b) We already found the radial component of acceleration,

$$a_r = \frac{v^2}{r} = \frac{(8.00 \text{ m/s})^2}{2.00 \text{ m}} = \boxed{32.0 \text{ m/s}^2 \text{ inward}}$$

Consider the tangential forces on the object:

$$\sum F_t = ma_t: \quad mg\sin\theta = ma_t$$

Solving for the tangential component of acceleration gives

$$\begin{aligned} a_t &= g\sin\theta = (9.80 \text{ m/s}^2)\sin 20.0° \\ &= \boxed{3.35 \text{ m/s}^2 \text{ downward tangent to the circle}} \end{aligned}$$

(c) The magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(32.0 \text{ m/s}^2)^2 + (3.35 \text{ m/s}^2)^2} = 32.2 \text{ m/s}^2$$

at an angle of

$$\tan^{-1}\left(\frac{3.35 \text{ m/s}^2}{32.0 \text{ m/s}^2}\right) = 5.98°$$

Thus, the acceleration is

$$\boxed{32.2 \text{ m/s}^2 \text{ inward and below the cord at } 5.98°}$$

(d) $\boxed{\text{No change.}}$

(e) If the object is swinging down it is gaining speed, and if the object is swinging up it is losing speed, but the forces are the same; therefore, its acceleration is regardless of the direction of swing.

P6.19 Let the tension at the lowest point be T. From Newton's second law, $\sum F = ma$ and

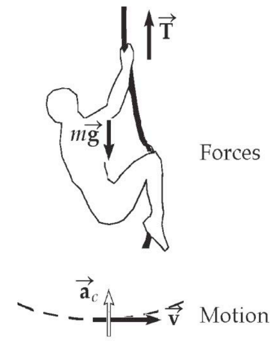
$$T - mg = ma_c = \frac{mv^2}{r}$$

$$T = m\left(g + \frac{v^2}{r}\right)$$

$$T = (85.0\text{ kg})\left[9.80\text{ m/s}^2 + \frac{(8.00\text{ m/s})^2}{10.0\text{ m}}\right]$$

$$= 1.38\text{ kN} > 1\,000\text{ N}$$

He doesn't make it across the river because the vine breaks.



ANS. FIG. P6.19

Section 6.3 Motion in Accelerated Frames

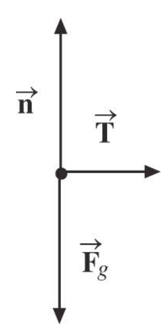
P6.20 (a) From $\sum F_x = Ma$, we obtain

$$a = \frac{T}{M} = \frac{18.0\text{ N}}{5.00\text{ kg}} = \boxed{3.60\text{ m/s}^2} \text{ to the right}$$

(b) If v = const, $a = 0$, so $\boxed{T = 0}$. (This is also an equilibrium situation.)

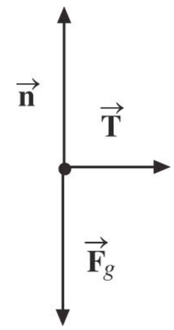
(c) Someone in the car (noninertial observer) claims that the forces on the mass along x are T and a fictitious force ($-Ma$).

(d) Someone at rest outside the car (inertial observer) claims that T is the only force on M in the x direction.



ANS. FIG. P6.20

P6.21 The only forces acting on the suspended object are the force of gravity $m\vec{\mathbf{g}}$ and the force of tension T forward and upward at angle θ with the vertical, as shown in the free-body diagram in ANS. FIG. P6.21. Applying Newton's second law in the x and y directions,



ANS. FIG. P6.21

$$\sum F_x = T\sin\theta = ma \qquad \textbf{[1]}$$

$$\sum F_y = T\cos\theta - mg = 0$$

or $$T\cos\theta = mg \qquad \textbf{[2]}$$

(a) Dividing equation [1] by [2] gives

$$\tan\theta = \frac{a}{g} = \frac{3.00\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 0.306$$

Solving for θ, $\theta = \boxed{17.0°}$

(b) From equation [1],

$$T = \frac{ma}{\sin\theta} = \frac{(0.500\ \text{kg})(3.00\ \text{m/s}^2)}{\sin(17.0°)} = \boxed{5.12\ \text{N}}$$

P6.22 In an inertial reference frame, the girl is accelerating horizontally inward at

$$\frac{v^2}{r} = \frac{(5.70\ \text{m/s})^2}{2.40\ \text{m}} = 13.5\ \text{m/s}^2$$

In her own noninertial frame, her head feels a horizontally outward fictitious force equal to its mass times this acceleration. Together this force and the weight of her head add to have a magnitude equal to the mass of her head times an acceleration of

$$\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2} = \sqrt{(9.80\ \text{m/s}^2)^2 + (13.5\ \text{m/s}^2)^2} = 16.7\ \text{m/s}^2$$

This is larger than g by a factor of $\dfrac{16.7\ \text{m/s}}{9.80\ \text{m/s}} = 1.71$.

Thus, the force required to lift her head is larger by this factor, or the required force is

$$F = 1.71(55.0\ \text{N}) = \boxed{93.8\ \text{N}}$$

P6.23 The scale reads the upward normal force exerted by the floor on the passenger. The maximum force occurs during upward acceleration (when starting an upward trip or ending a downward trip). The minimum normal force occurs with downward acceleration. For each respective situation,

$$\sum F_y = ma_y \quad \text{becomes for starting} \quad +591\ \text{N} - mg = +ma$$

$$\text{and for stopping} \quad +391\ \text{N} - mg = -ma$$

where a represents the magnitude of the acceleration.

(a) These two simultaneous equations can be added to eliminate a and solve for mg:

$$+\ 591\ \text{N} - mg + 391\ \text{N} - mg = 0$$

or $\quad 982\ \text{N} - 2mg = 0$

$$F_g = mg = \frac{982\ \text{N}}{2} = \boxed{491\ \text{N}}$$

(b) From the definition of weight, $m = \dfrac{F_g}{g} = \dfrac{491\ \text{N}}{9.80\ \text{m/s}^2} = \boxed{50.1\ \text{kg}}$

(c) Substituting back gives $\quad +591\ \text{N} - 491\ \text{N} = (50.1\ \text{kg})a$, or

$$a = \frac{100\ \text{N}}{50.1\ \text{kg}} = \boxed{2.00\ \text{m/s}^2}$$

P6.24 Consider forces on the backpack as it slides in the Earth frame of reference.

$$\sum F_y = ma_y: \quad +n - mg = ma,\ n = m(g+a),\ f_k = \mu_k m(g+a)$$

$$\sum F_x = ma_x: \quad -\mu_k m(g+a) = ma_x$$

The motion across the floor is described by

$$L = vt + \frac{1}{2}a_x t^2 = vt - \frac{1}{2}\mu_k(g+a)t^2$$

We solve for μ_k:

$$vt - L = \frac{1}{2}\mu_k(g+a)t^2$$

$$\boxed{\mu_k = \frac{2(vt-L)}{(g+a)t^2}}$$

P6.25 The water moves at speed

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.120\ \text{m})}{7.25\ \text{s}} = 0.104\ \text{m/s}$$

The top layer of water feels a downward force of gravity *mg* and an outward fictitious force in the turntable frame of reference,

$$\frac{mv^2}{r} = \frac{m\left(0.104\ \text{m/s}\right)^2}{0.12\ \text{m}} = m\,9.01\times 10^{-2}\ \text{m/s}^2$$

It behaves as if it were stationary in a gravity field pointing downward and outward at

$$\tan^{-1}\left(\frac{0.090\ 1\ \text{m/s}^2}{9.8\ \text{m/s}^2}\right) = \boxed{0.527°}$$

Its surface slopes upward toward the outside, making this angle with the horizontal.

Section 6.4 Motion in the Presence of Velocity-Dependent Resistive Forces

P6.26 (a) $\rho = \dfrac{m}{V}$, $A = 0.020\ 1\ \text{m}^2$, $R = \dfrac{1}{2}\rho_{\text{air}} A D v_T^2 = mg$

$$m = \rho_{\text{bead}} V = 0.830\ \text{g/cm}^3\left[\frac{4}{3}\pi\left(8.00\ \text{cm}\right)^3\right] = 1.78\ \text{kg}$$

Assuming a drag coefficient of $D = 0.500$ for this spherical object, and taking the density of air at 20°C from the endpapers, we have

$$v_T = \sqrt{\frac{2\left(1.78\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{0.500\left(1.20\ \text{kg/m}^3\right)\left(0.020\ 1\ \text{m}^2\right)}} = \boxed{53.8\ \text{m/s}}$$

(b) From $v_f^2 = v_i^2 + 2gh = 0 + 2gh$, we solve for *h*:

$$h = \frac{v_f^2}{2g} = \frac{\left(53.8\ \text{m/s}\right)^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{148\ \text{m}}$$

P6.27 With 100 km/h = 27.8 m/s, the resistive force is

$$R = \frac{1}{2}D\rho A v^2 = \frac{1}{2}\left(0.250\right)\left(1.20\ \text{kg/m}^3\right)\left(2.20\ \text{m}^2\right)\left(27.8\ \text{m/s}\right)^2$$
$$= 255\ \text{N}$$

$$a = -\frac{R}{m} = -\frac{255\ \text{N}}{1\ 200\ \text{kg}} = \boxed{-0.212\ \text{m/s}^2}$$

P6.28 Given $m = 80.0$ kg, $v_T = 50.0$ m/s, we write

$$mg = \frac{D\rho A v_T^2}{2}$$

which gives

$$\frac{D\rho A}{2} = \frac{mg}{v_T^2} = 0.314 \text{ kg/m}$$

(a) At $v = 30.0$ m/s,

$$a = g - \frac{D\rho A v^2/2}{m} = 9.80 \text{ m/s}^2 - \frac{(0.314 \text{ kg/m})(30.0 \text{ m/s})^2}{80.0 \text{ kg}}$$

$$= \boxed{6.27 \text{ m/s}^2 \text{ downward}}$$

(b) At $v = 50.0$ m/s, terminal velocity has been reached.

$$\sum F_y = 0 = mg - R$$

$$\Rightarrow R = mg = (80.0 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{784 \text{ N directed up}}$$

(c) At $v = 30.0$ m/s,

$$\frac{D\rho A v^2}{2} = (0.314 \text{ kg/m})(30.0 \text{ m/s})^2 = \boxed{283 \text{ N upward}}$$

P6.29 Since the upward velocity is constant, the resultant force on the ball is zero. Thus, the upward applied force equals the sum of the gravitational and drag forces (both downward):

$$F = mg + bv$$

The mass of the copper ball is

$$m = \frac{4\pi\rho r^3}{3} = \left(\frac{4}{3}\right)\pi\left(8.92\times10^3 \text{ kg/m}^3\right)\left(2.00\times10^{-2} \text{ m}\right)^3$$

$$= 0.299 \text{ kg}$$

The applied force is then

$$F = mg + bv = (0.299 \text{ kg})(9.80 \text{ m/s}^2) + (0.950 \text{ kg/s})\left(9.00\times10^{-2} \text{ m/s}\right)$$

$$= \boxed{3.01 \text{ N}}$$

P6.30 (a) The acceleration of the Styrofoam is given by

$$a = g - Bv$$

When $v = v_T$, $a = 0$ and $g = Bv_T \rightarrow B = \dfrac{g}{v_T}$

The Styrofoam falls 1.50 m at constant speed v_T in 5.00 s.

Thus,

$$v_T = \frac{h}{\Delta t} = \frac{1.50 \text{ m}}{5.00 \text{ s}} = 0.300 \text{ m/s}$$

Then

$$\boxed{B = \frac{g}{v_T} = \frac{9.80 \text{ m/s}^2}{0.300 \text{ m/s}} = 32.7 \text{ s}^{-1}}$$

(b) At $t = 0$, $v = 0$, and $a = g = \boxed{9.80 \text{ m/s}^2 \text{ down}}$

(c) When $v = 0.150$ m/s,

$$\begin{aligned} a &= g - Bv \\ &= 9.80 \text{ m/s}^2 - \left(32.7 \text{ s}^{-1}\right)\left(0.150 \text{ m/s}\right) \\ &= \boxed{4.90 \text{ m/s}^2 \text{ down}} \end{aligned}$$

P6.31 We have a particle under a net force in the special case of a resistive force proportional to speed, and also under the influence of the gravitational force.

(a) The speed v varies with time according to Equation 6.6,

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = v_T\left(1 - e^{-t/\tau}\right)$$

where $v_T = mg/b$ is the terminal speed. Hence,

$$b = \frac{mg}{v_T} = \frac{\left(3.00 \times 10^{-3} \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)}{2.00 \times 10^{-2} \text{ m/s}} = \boxed{1.47 \text{ N} \cdot \text{s/m}}$$

(b) To find the time interval for v to reach $0.632v_T$, we substitute $v = 0.632v_T$ into Equation 6.6, giving

$$0.632v_T = v_T\left(1 - e^{-bt/m}\right) \qquad \text{or} \qquad 0.368 = e^{-(1.47t/0.003\,00)}$$

Solve for t by taking the natural logarithm of each side of the equation:

$$\ln(0.368) = -\frac{1.47\text{ t}}{3.00 \times 10^{-3}} \qquad \text{or} \qquad -1 = -\frac{1.47\text{ t}}{3.00 \times 10^{-3}}$$

or $$t = -\left(\frac{m}{b}\right)\ln(0.368) = \boxed{2.04\times10^{-3}\ \text{s}}$$

(c) At terminal speed, $R = v_T b = mg$. Therefore,

$$R = v_T b = mg = \left(3.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = \boxed{2.94\times10^{-2}\ \text{N}}$$

P6.32 We write

$$-kmv^2 = -\frac{1}{2}D\rho Av^2$$

so

$$k = \frac{D\rho A}{2m} = \frac{0.305\left(1.20\ \text{kg/m}^3\right)\left(4.2\times10^{-3}\ \text{m}^2\right)}{2(0.145\ \text{kg})} = 5.3\times10^{-3}/\text{m}$$

solving for the velocity as the ball crosses home plate gives

$$v = v_i e^{-kx} = (40.2\ \text{m/s})e^{-\left(5.3\times10^{-3}/\text{m}\right)(18.3\ \text{m})} = \boxed{36.5\ \text{m/s}}$$

P6.33 We start with Newton's second law,

$$\sum F = ma$$

substituting,

$$-kmv^2 = m\frac{dv}{dt}$$

$$-kdt = \frac{dv}{v^2}$$

$$-k\int_0^t dt = \int_{v_i}^{v} v^{-2}\,dv$$

integrating both sides gives

$$-k(t-0) = \left.\frac{v^{-1}}{-1}\right|_{v_i}^{v} = -\frac{1}{v} + \frac{1}{v_i}$$

$$\frac{1}{v} = \frac{1}{v_i} + kt = \frac{1+v_ikt}{v_i}$$

$$\boxed{v = \frac{v_i}{1+v_ikt}}$$

P6.34 (a) Since the window is vertical, the normal force is horizontal and is given by $n = 4.00$ N. To find the vertical component of the force, we note that the force of kinetic friction is given by

$$f_k = \mu_k n = 0.900(4.00 \text{ N}) = 3.60 \text{ N upward}$$

to oppose downward motion. Newton's second law then becomes

$$\sum F_y = ma_y: \quad +3.6 \text{ N} - (0.16 \text{ kg})(9.8 \text{ m/s}^2) + P_y = 0$$

$$P_y = -2.03 \text{ N} = \boxed{2.03 \text{ N down}}$$

(b) Now, with the increased downward force, Newton's second law gives

$$\sum F_y = ma_y:$$

$$+3.60 \text{ N} - (0.160 \text{ kg})(9.80 \text{ m/s}^2) - 1.25(2.03 \text{ N}) = 0.160 \text{ kg } a_y$$

then

$$a_y = -0.508 \text{ N}/0.16 \text{ kg} = -3.18 \text{ m/s}^2 = \boxed{3.18 \text{ m/s}^2 \text{ down}}$$

(c) At terminal velocity,

$$\sum F_y = ma_y: \quad +(20.0 \text{ N}\cdot\text{s/m})v_T - (0.160 \text{ kg})(9.80 \text{ m/s}^2) - 1.25(2.03 \text{ N}) = 0$$

Solving for the terminal velocity gives

$$v_T = 4.11 \text{ N}/(20 \text{ N}\cdot\text{s/m}) = \boxed{0.205 \text{ m/s down}}$$

P6.35 (a) We must fit the equation $v = v_i e^{-ct}$ to the two data points:

At $t = 0$, $v = 10.0$ m/s, so $v = v_i e^{-ct}$ becomes

$$10.0 \text{ m/s} = v_i e^0 = (v_i)(1)$$

which gives $v_i = 10.0$ m/s

At $t = 20.0$ s, $v = 5.00$ m/s so the equation becomes

$$5.00 \text{ m/s} = (10.0 \text{ m/s})e^{-c(20.0 \text{ s})}$$

giving $\quad 0.500 = e^{-c(20.0 \text{ s})}$

or $\quad -20.0c = \ln\left(\frac{1}{2}\right) \rightarrow c = -\frac{\ln\left(\frac{1}{2}\right)}{20.0} = \boxed{3.47 \times 10^{-2} \text{ s}^{-1}}$

(b) At $t = 40.0$ s

$$v = (10.0 \text{ m/s})e^{-40.0c} = (10.0 \text{ m/s})(0.250) = \boxed{2.50 \text{ m/s}}$$

(c) The acceleration is the rate of change of the velocity:

$$a = \frac{dv}{dt} = \frac{d}{dt}v_i e^{-ct} = v_i\left(e^{-ct}\right)(-c) = -c\left(v_i e^{-ct}\right)$$
$$= \boxed{-cv}$$

Thus, the acceleration is a negative constant times the speed.

P6.36 In $R = \frac{1}{2}D\rho Av^2$, we estimate that the coefficient of drag for an open palm is $D = 1.00$, the density of air is $\rho = 1.20\ \text{kg/m}^3$, the area of an open palm is $A = (0.100\ \text{m})(0.160\ \text{m}) = 1.60 \times 10^{-2}\ \text{m}^2$, and $v = 29.0$ m/s (65 miles per hour). The resistance force is then

$$R = \frac{1}{2}(1.00)(1.20\ \text{kg/m}^3)(1.60 \times 10^{-2}\ \text{m}^2)(29.0\ \text{m/s})^2 = 8.07\ \text{N}$$

or $R \sim \boxed{10^1\ \text{N}}$

Additional Problems

P6.37 Because the car travels at a constant speed, it has no tangential acceleration, but it does have centripetal acceleration because it travels along a circular arc. The direction of the centripetal acceleration is toward the center of curvature, and the direction of velocity is tangent to the curve.

<u>Point A</u>

direction of velocity: $\boxed{\text{East}}$

direction of the centripetal acceleration: $\boxed{\text{South}}$

<u>Point B</u>

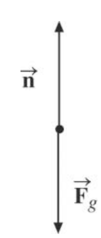
direction of velocity: $\boxed{\text{South}}$

direction of the centripetal acceleration: $\boxed{\text{West}}$

P6.38 The free-body diagram of the passenger is shown in ANS. FIG. P6.38. From Newton's second law,

$$\sum F_y = ma_y$$

$$n - mg = \frac{mv^2}{r}$$



ANS. FIG. P6.38

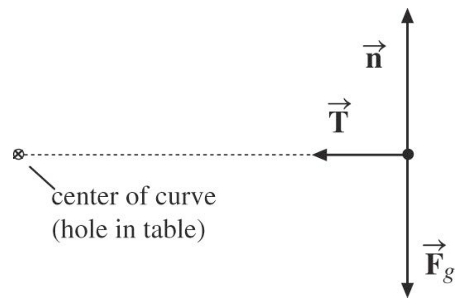
which gives

$$n = mg + \frac{mv^2}{r}$$

$$= (50\ \text{kg})(9.80\ \text{m/s}^2) + \frac{(50.0\ \text{kg})(19\ \text{m/s})^2}{25\ \text{m}}$$

$$= \boxed{1.2 \times 10^3\ \text{N}}$$

P6.39 The free-body diagram of the rock is shown in ANS. FIG. P6.39. Take the *x* direction inward toward the center of the circle. The mass of the rock does not change. We know when $r_1 = 2.50$ m, $v_1 = 20.4$ m/s, and $T_1 = 50.0$ N. To find T_2 when $r_2 = 1.00$ m, and $v_2 = 51.0$ m/s, we use Newton's second law in the horizontal direction:



ANS. FIG. P6.39

$$\sum F_x = ma_x$$

In both cases,

$$T_1 = \frac{mv_1^2}{r_1} \quad \text{and} \quad T_2 = \frac{mv_2^2}{r_2}$$

Taking the ratio of the two tensions gives

$$\frac{T_2}{T_1} = \frac{v_2^2}{v_1^2}\frac{r_1}{r_2} = \left(\frac{51.0\ \text{m/s}}{20.4\ \text{m/s}}\right)^2 \left(\frac{2.50\ \text{m}}{1.00\ \text{m}}\right) = 15.6$$

then

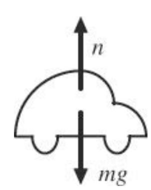
$$T_2 = 15.6T_1 = 15.6(50.0\ \text{N}) = \boxed{781\ \text{N}}$$

We assume the tension in the string is not altered by friction from the hole in the table.

P6.40 (a) We first convert the speed of the car to SI units:

$$v = (30\ \text{km/h})\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\,000\ \text{m}}{1\ \text{km}}\right)$$

$$= 8.33\ \text{m/s}$$



ANS. FIG. P6.40

Newton's second law in the vertical direction then gives

$$\sum F_y = ma_y: \qquad +n - mg = -\frac{mv^2}{r}$$

Solving for the normal force,

$$n = m\left(g - \frac{v^2}{r}\right)$$
$$= (1\,800 \text{ kg})\left[9.80 \text{ m/s}^2 - \frac{(8.33 \text{ m/s})^2}{20.4 \text{ m}}\right]$$
$$= \boxed{1.15 \times 10^4 \text{ N up}}$$

(b) At the maximum speed, the weight of the car is just enough to provide the centripetal force, so $n = 0$. Then $mg = \dfrac{mv^2}{r}$ and

$$v = \sqrt{gr} = \sqrt{(9.80 \text{ m/s}^2)(20.4 \text{ m})} = \boxed{14.1 \text{ m/s}} = 50.9 \text{ km/h}$$

P6.41 (a) The free-body diagram in ANS. FIG. P6.40 shows the forces on the car in the vertical direction. Newton's second law then gives

$$\sum F_y = ma_y = \frac{mv^2}{R}$$

$$mg - n = \frac{mv^2}{R} \qquad n = \boxed{mg - \frac{mv^2}{R}}$$

(b) When $n = 0$, $\qquad mg = \dfrac{mv^2}{R}$

Then, $v = \boxed{\sqrt{gR}}$

A more gently curved bump, with larger radius, allows the car to have a higher speed without leaving the road. This speed is proportional to the square root of the radius.

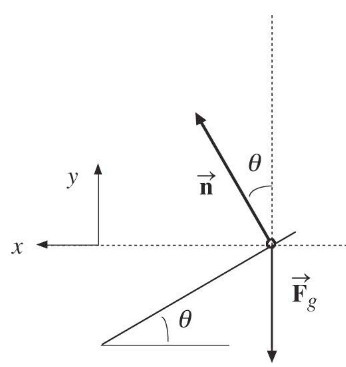
P6.42 The free-body diagram for the object is shown in ANS. FIG. P6.42. The object travels in a circle of radius $r = L\cos\theta$ about the vertical rod.

Taking inward toward the center of the circle as the positive x direction, we have

$$\sum F_x = ma_x: \quad n\sin\theta = \frac{mv^2}{r}$$

$$\sum F_y = ma_y:$$

$$n\cos\theta - mg = 0 \rightarrow n\cos\theta = mg$$



ANS. FIG. P6.42

Dividing, we find

$$\frac{n\sin\theta}{n\cos\theta}=\frac{mv^2/r}{gr} \quad \to \quad \tan\theta=\frac{v^2}{gr}$$

Solving for v gives

$$v^2 = gr\tan\theta$$

$$v^2 = g(L\cos\theta)\tan\theta$$

$$\boxed{v=(gL\sin\theta)^{1/2}}$$

P6.43 Let v_i represent the speed of the object at time 0. We have

$$\int_{v_i}^{v}\frac{dv}{v}=-\frac{b}{m}\int_i^t dt \qquad \ln v\Big|_{v_i}^{v}=-\frac{b}{m}t\Big|_i^t$$

$$\ln v-\ln v_i=-\frac{b}{m}(t-0) \qquad \ln(v/v_i)=-\frac{bt}{m}$$

$$v/v_i=e^{-bt/m} \qquad \boxed{v=v_ie^{-bt/m}}$$

From its original value, the speed decreases rapidly at first and then more and more slowly, asymptotically approaching zero.

$\boxed{\text{In this model the object keeps losing speed forever. It travels a finite distance in stopping.}}$

The distance it travels is given by

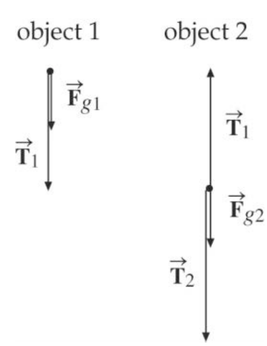
$$\int_0^r dr = v_i\int_0^t e^{-bt/m}dt$$

$$r=-\frac{m}{b}v_i\int_0^t e^{-bt/m}\left(-\frac{b}{m}dt\right)=-\frac{m}{b}v_i\,e^{-bt/m}\Big|_0^t$$

$$=-\frac{m}{b}v_i\left(e^{-bt/m}-1\right)=\frac{mv_i}{b}\left(1-e^{-bt/m}\right)$$

As t goes to infinity, the distance approaches $\frac{mv_i}{b}(1-0)=mv_i/b$.

P6.44 The radius of the path of object 1 is twice that of object 2. Because the strings are always "collinear," both objects take the same time interval to travel around their respective circles; therefore, the speed of object 1 is twice that of object 2.



ANS. FIG. P6.44

The free-body diagrams are shown in ANS. FIG. P6.44. We are given $m_1 = 4.00$ kg, $m_2 = 3.00$ kg, $v = 4.00$ m/s, and $\ell = 0.500$ m.

Taking down as the positive direction, we have

Object 1: $T_1 + m_1 g = \dfrac{m_1 v_1^2}{r_1}$, where $v_1 = 2v$, $r_1 = 2\ell$.

Object 2: $T_2 - T_1 + m_2 g = \dfrac{m_2 v_2^2}{r_2}$, where $v_2 = v$, $r_2 = 2\ell$.

(a) From above:

$$T_1 = \frac{m_1 v_1^2}{r_1} - m_1 g = m_1\left(\frac{v_1{}^2}{r_1} - g\right)$$

$$T_1 = (4.00\text{ kg})\left[\frac{\left[2(4.00\text{ m/s})\right]^2}{2(0.500\text{ m})} - 9.80\text{ m/s}^2\right]$$

$$T_1 = 216.8\text{ N} = \boxed{217\text{ N}}$$

(b) From above:

$$T_2 = T_1 + \frac{m_2 v_2^2}{r_2} - m_2 g$$

$$T_2 = T_1 + m_2\left(\frac{v_2{}^2}{r_2} - g\right)$$

$$T_2 = T_1 + (3.00\text{ kg})\left[\frac{(4.00\text{ m/s})^2}{0.500\text{ m}} - 9.80\text{ m/s}^2\right]$$

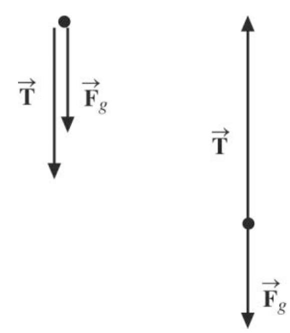
$$T_2 = 216.8\text{ N} + 66.6\text{ N} = 283.4\text{ N} = \boxed{283\text{ N}}$$

(c) $\boxed{\text{From above, } T_2 > T_1 \text{ always, so string 2 will break first.}}$

P6.45 (a) At each point on the vertical circular path, two forces are acting on the ball (see ANS. FIG. P6.45):

(1) $\boxed{\text{The downward gravitational force}}$ with constant magnitude $F_g = mg$

(2) $\boxed{\text{The tension force in the string, always directed toward the center of the path}}$



ANS. FIG. P6.45

(b) ANS. FIG. P6.45 shows the forces acting on the ball when it is at the highest point on the path (left-hand diagram) and when it is at the bottom of the circular path (right-hand diagram). Note that the gravitational force has the same magnitude and direction at each point on the circular path. The tension force varies in magnitude at different points and is always directed toward the center of the path.

(c) At the top of the circle, $F_c = mv^2/r = T + F_g$, or

$$T = \frac{mv^2}{r} - F_g = \frac{mv^2}{r} - mg = m\left(\frac{v^2}{r} - g\right)$$

$$= (0.275\text{ kg})\left[\frac{(5.20\text{ m/s})^2}{0.850\text{ m}} - 9.80\text{ m/s}^2\right] = \boxed{6.05\text{ N}}$$

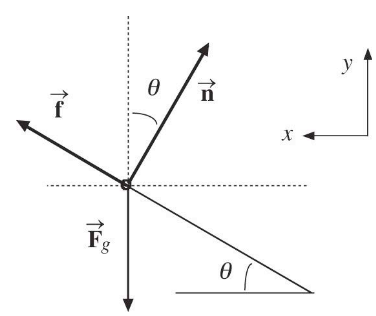
(d) At the bottom of the circle, $F_c = mv^2/r = T - F_g = T - mg$, and solving for the speed gives

$$v^2 = \frac{r}{m}(T - mg) = r\left(\frac{T}{m} - g\right) \quad \text{and} \quad v = \sqrt{r\left(\frac{T}{m} - g\right)}$$

If the string is at the breaking point at the bottom of the circle, then $T = 22.5$ N, and the speed of the object at this point must be

$$v = \sqrt{(0.850\text{ m})\left(\frac{22.5\text{ N}}{0.275\text{ kg}} - 9.80\text{ m/s}^2\right)} = \boxed{7.82\text{ m/s}}$$

P6.46 The free-body diagram is shown on the right, where it is assumed that friction points up the incline, otherwise, the child would slide down the incline. The net force is directed left toward the center of the circular path in which the child travels. The radius of this path is $R = d\cos\theta$.



ANS. FIG. P6.46

Three forces act on the child, a normal force, static friction, and gravity. The relations of their force components are:

$$\sum F_x: \quad f_s\cos\theta - n\sin\theta = mv^2/R \qquad \textbf{[1]}$$

$$\sum F_y: \quad f_s\sin\theta + n\cos\theta - mg = 0 \rightarrow$$

$$f_s\sin\theta + n\cos\theta = mg \qquad \textbf{[2]}$$

Solve for the static friction and normal force.

To solve for static friction, multiply equation [1] by cos θ and equation [2] by sin θ and add:

$$\cos\theta\left[f_s\cos\theta - n\sin\theta\right] + \sin\theta\left[f_s\sin\theta - n\cos\theta\right]$$
$$= \cos\theta\left(\frac{mv^2}{R}\right) + \sin\theta\left(mg\right)$$

$$f_s = mg\sin\theta + \left(\frac{mv^2}{R}\right)\cos\theta$$

To solve for the normal force, multiply equation [1] by –sin θ and equation [2] by cos θ and add:

$$-\sin\theta\left[f_s\cos\theta - n\sin\theta\right] + \cos\theta\left[f_s\sin\theta - n\cos\theta\right]$$
$$= -\sin\theta\left(\frac{mv^2}{R}\right) + \cos\theta\left(mg\right)$$

$$n = mg\cos\theta - \left(\frac{mv^2}{R}\right)\sin\theta$$

In the above, we have used $\sin^2\theta + \cos^2\theta = 1$.

If the above equations are to be consistent, static friction and the normal force must satisfy the condition $f_s \le \mu_s n$; this means

$$(mg)\sin\theta + (mv^2/R)\cos\theta \le \mu_s[(mg)\cos\theta - (mv^2/R)\sin\theta] \rightarrow$$
$$v^2(\cos\theta + \mu_s\sin\theta) \le gR(\mu_s\cos\theta - \sin\theta)$$

Using this result, and that $R = d\cos\theta$, we have the requirement that

$$v \le \sqrt{\frac{gd\cos\theta(\mu_s\cos\theta - \sin\theta)}{(\cos\theta + \mu_s\sin\theta)}}$$

If this condition cannot be met, if v is too large, the physical situation cannot exist.

The values given in the problem are $d = 5.32$ m, $\mu_s = 0.700$, $\theta = 20.0°$, and $v = 3.75$ m/s. Check whether the given value of v satisfies the above condition:

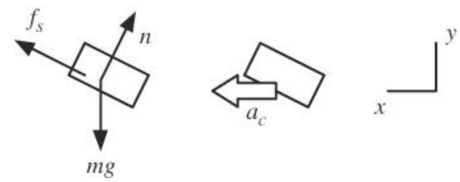
$$\sqrt{\frac{(9.80\text{ m/s}^2)(5.32\text{ m})\cos 20.0°\left[(0.700)\cos 20.0° - \sin 20.0°\right]}{(\cos 20.0° + 0.700\sin 20.0°)}}$$
$$= 3.62\text{ m/s}$$

The situation is impossible because the speed of the child given in the problem is too large: static friction could not keep the child in place on the incline.

P6.47 (a) The speed of the bag is

$$\frac{2\pi(7.46\text{ m})}{38\text{ s}} = 1.23\text{ m/s}$$

The total force on it must add to



ANS. FIG. P6.47

$$\begin{aligned} ma_c &= \frac{mv^2}{r} \\ &= \frac{(30\text{ kg})(1.23\text{ m/s})^2}{7.46\text{ m}} = 6.12\text{ N} \end{aligned}$$

Newton's second law gives

$$\sum F_x = ma_x: \quad f_s\cos 20.0° - n\sin 20.0° = 6.12\text{ N}$$
$$\sum F_y = ma_y: \quad f_s\sin 20.0° + n\cos 20.0° - (30.0\text{ kg})(9.80\text{ m/s}^2) = 0$$

Solving for the normal force gives

$$n = \frac{f_s\cos 20.0° - 6.12\text{ N}}{\sin 20.0°}$$

Substituting,

$$f_s\sin 20.0° + f_s\frac{\cos^2 20.0°}{\sin 20.0°} - (6.12\text{ N})\frac{\cos 20.0°}{\sin 20.0°} = 294\text{ N}$$
$$f_s(2.92) = 294\text{ N} + 16.8\text{ N}$$
$$f_s = \boxed{106\text{ N}}$$

(b) The speed of the bag is now

$$v = \frac{2\pi(7.94\text{ m})}{34\text{ s}} = 1.47\text{ m/s}$$

which corresponds to a total force of

$$\begin{aligned} ma_c &= \frac{mv^2}{r} \\ &= \frac{(30\text{ kg})(1.47\text{ m/s})^2}{7.94\text{ m}} = 8.13\text{ N} \end{aligned}$$

Newton's second law then gives

$$f_s\cos 20 - n\sin 20 = 8.13\text{ N}$$
$$f_s\sin 20 + n\cos 20 = 294\text{ N}$$

Solving for n,

$$n = \frac{f_s\cos 20.0° - 8.13\text{ N}}{\sin 20.0°}$$

Substituting,

$$f_s \sin 20.0° + f_s \frac{\cos^2 20.0°}{\sin 20.0°} - (8.13\ \text{N})\frac{\cos 20.0°}{\sin 20.0°} = 294\ \text{N}$$

$$f_s(2.92) = 294\ \text{N} + 22.4\ \text{N}$$

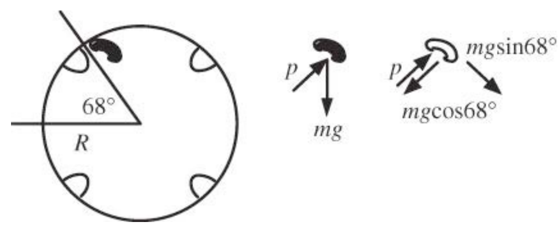
$$f_s = 108\ \text{N}$$

$$n = \frac{(108\ \text{N})\cos 20.0° - 8.13\ \text{N}}{\sin 20.0°} = 273\ \text{N}$$

$$\mu_s = \frac{f_s}{n} = \frac{108\ \text{N}}{273\ \text{N}} = \boxed{0.396}$$

P6.48 When the cloth is at a lower angle θ, the radial component of $\sum F = ma$ reads

$$n + mg\sin\theta = \frac{mv^2}{r}$$



ANS. FIG. P6.48

At $\theta = 68.0°$, the normal force drops to zero and $g\sin 68° = \frac{v^2}{r}$:

$$v = \sqrt{rg\sin 68°} = \sqrt{(0.33\ \text{m})(9.8\ \text{m/s}^2)\sin 68°} = 1.73\ \text{m/s}$$

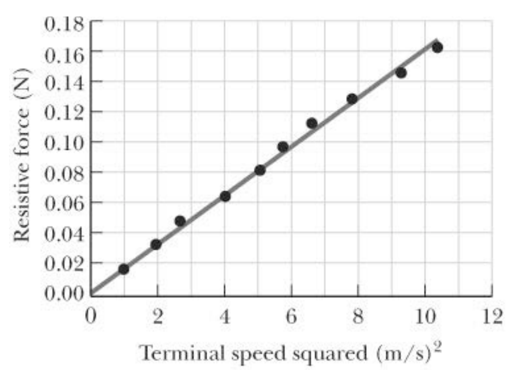
The rate of revolution is

$$\text{angular speed} = (1.73\ \text{m/s})\left(\frac{1\ \text{rev}}{2\pi r}\right)\left(\frac{2\pi r}{2\pi(0.33\ \text{m})}\right)$$

$$= \boxed{0.835\ \text{rev/s}} = 50.1\ \text{rev/min}$$

P6.49 The graph in Figure 6.16b is shown in ANS. FIG. P6.49.

(a) The graph line is straight, so we may use any two points on it to find the slope. It is convenient to take the origin as one point, and we read ($9.9\ \text{m}^2/\text{s}^2$, 0.16 N) as the coordinates of another point. Then the slope is



ANS. FIG. P6.49

$$\text{slope} = \frac{0.160\ \text{N} - 0}{9.9\ \text{m}^2/\text{s}^2} = \boxed{0.016\,2\ \text{kg/m}}$$

(b) In $R = \frac{1}{2}D\rho Av^2$, we identify the vertical-axis variable as *R* and the horizontal-axis variable as v^2. Then the slope is

$$\text{slope} = \frac{R}{v^2} = \frac{\frac{1}{2}D\rho Av^2}{v^2} = \boxed{\frac{1}{2}D\rho A}$$

(c) We follow the directions in the problem statement:

$$\frac{1}{2}D\rho A = 0.0162\ \text{kg/m}$$

$$D = \frac{2(0.0162\ \text{kg/m})}{(1.20\ \text{kg/m}^3)\pi(0.105\ \text{m})^2} = \boxed{0.778}$$

(d) From the table, the eighth point is at force

$$mg = 8(1.64\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2) = 0.129\ \text{N}$$

and horizontal coordinate $(2.80\ \text{m/s})^2$. The vertical coordinate of the line is here

$$(0.016\,2\ \text{kg/m})(2.80\ \text{m/s})^2 = 0.127\ \text{N}$$

The scatter percentage is

$$\frac{0.129\ \text{N} - 0.127\ \text{N}}{0.127\ \text{N}} = \boxed{1.5\%}$$

(e) The interpretation of the graph can be stated thus:

For stacked coffee filters falling at terminal speed, a graph of air resistance force as a function of squared speed demonstrates that the force is proportional to the speed squared within the experimental uncertainty estimated as 2%. This proportionality agrees with that described by the theoretical equation $R = \frac{1}{2}D\rho Av^2$. The value of the constant slope of the graph implies that the drag coefficient for coffee filters is $D = 0.78 \pm 2\%$.

P6.50 (a) The forces acting on the ice cube are the Earth's gravitational force, straight down, and the basin's normal force, upward and inward at 35.0° with the vertical. We choose the x and y axes to be horizontal and vertical, so that the acceleration is purely in the x direction. Then

$$\sum F_x = ma_x: \quad n\sin 35° = mv^2/R$$

$$\sum F_y = ma_y: \quad n\cos 35° - mg = 0$$

Dividing eliminates the normal force:

$$n\sin 35.0°/n\cos 35.0° = mv^2/Rmg$$

$$\tan 35.0° = v^2/Rg$$

$$\boxed{v = \sqrt{Rg\tan 35.0°} = \sqrt{\left(6.86\text{ m/s}^2\right)R}}$$

(b) $\boxed{\text{The mass is unnecessary.}}$

(c) The answer to (a) indicates that the speed is proportional to the square root of the radius, so $\boxed{\text{increasing the radius will make the required speed increase.}}$

(d) The period of revolution is given by

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{Rg\tan 35.0°}} = \left(2.40\text{ s}/\sqrt{\text{m}}\right)\sqrt{R}$$

$\boxed{\text{When the radius increases, the period increases.}}$

(e) $\boxed{\text{On a larger circle, the ice cube's speed is proportional to } \sqrt{R} \text{ but the distance it travels is proportional to } R\text{, so the time interval required is proportional to } R/\sqrt{R} = \sqrt{R}.}$

P6.51 Take the positive x axis up the hill. Newton's second law in the x direction then gives

$$\sum F_x = ma_x: \quad +T\sin\theta - mg\sin\phi = ma$$

from which we obtain

$$a = \frac{T}{m}\sin\theta - g\sin\phi \qquad \textbf{[1]}$$

In the y direction,

$$\sum F_y = ma_y: \quad +T\cos\theta - mg\cos\phi = 0$$

Solving for the tension gives

$$T = \frac{mg\cos\phi}{\cos\theta} \qquad \textbf{[2]}$$

Substituting for *T* from [2] into [1] gives

$$a = \frac{g\cos\phi\sin\theta}{\cos\theta} - g\sin\phi$$

$$a = \boxed{g(\cos\phi\tan\theta - \sin\phi)}$$

P6.52 (a) We first convert miles per hour to feet per second:

$$v = (300 \text{ mi/h})\left(\frac{88.0 \text{ ft/s}}{60.0 \text{ mi/h}}\right) = 440 \text{ ft/s} \text{ at the top of the loop}$$

and $v = 450$ mi/h $= 660$ ft/s at the bottom of the loop.

At the lowest point, his seat exerts an upward force; therefore, his weight seems to increase. His apparent weight is

$$F_g' = mg + m\frac{v^2}{r} = 160 \text{ lb} + \left(\frac{160 \text{ lb}}{32.0 \text{ ft/s}^2}\right)\frac{(660 \text{ ft/s})^2}{1\,200 \text{ ft}} = \boxed{1\,975 \text{ lb}}$$

(b) At the highest point, the force of the seat on the pilot is directed down and

$$F_g' = mg - m\frac{v^2}{r} = 160 \text{ lb} - \left(\frac{160 \text{ lb}}{32.0 \text{ ft/s}^2}\right)\frac{(440 \text{ ft/s})^2}{1\,200 \text{ ft}} = \boxed{-647 \text{ lb}}$$

Since the plane is upside down, the seat exerts this downward force as a normal force.

(c) When $F_g' = 0$, then $mg = \frac{mv^2}{R}$. If we vary the aircraft's R and v such that this equation is satisfied, then the pilot feels weightless.

P6.53 (a) The only horizontal force on the car is the force of friction, with a maximum value determined by the surface roughness (described by the coefficient of static friction) and the normal force (here equal to the gravitational force on the car).

(b) From Newton's second law in one dimension,

$$\sum F_x = ma_x: \quad -f = ma \quad \rightarrow \quad a = -\frac{f}{m} = (v^2 - v_0^2)/2(x - x_0)$$

solving for the stopping distance gives

$$x - x_0 = \frac{m\left(v^2 - v_0^2\right)}{2f} = \frac{(1\ 200\text{ kg})\left[0^2 - (20.0\text{ m/s})^2\right]}{2(-7\ 000\text{ N})} = \boxed{34.3\text{ m}}$$

(c) Newton's second law now gives

$$f = \frac{mv^2}{r}$$

or $$r = \frac{mv^2}{f} = \frac{(1\ 200\text{ kg})(20.0\text{ m/s})^2}{7\ 000\text{ N}} = \boxed{68.6\text{ m}}$$

A top view shows that you can avoid running into the wall by turning through a quarter-circle, if you start at least this far away from the wall.

(d) Braking is better. You should not turn the wheel. If you used any of the available friction force to change the direction of the car, it would be unavailable to slow the car, and the stopping distance would be longer.

(e) The conclusion is true in general. The radius of the curve you can barely make is twice your minimum stopping distance.

P6.54 (a) Since the object of mass m_2 is in equilibrium, $\sum F_y = T - m_2 g = 0$

or $T = \boxed{m_2 g}$.

(b) The tension in the string provides the required centripetal acceleration of the puck.

Thus, $F_c = T = \boxed{m_2 g}$.

(c) From $F_c = \dfrac{m_1 v^2}{R}$,

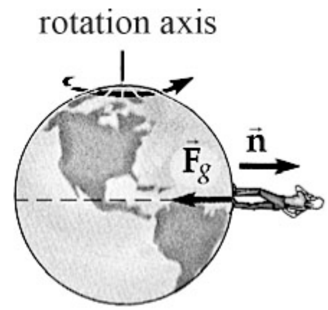
we have $$v = \sqrt{\frac{RF_c}{m_1}} = \boxed{\sqrt{\left(\frac{m_2}{m_1}\right) gR}}$$

(d) $\boxed{\text{The puck will spiral inward, gaining speed as it does so.}}$ It gains speed because the extra-large string tension pulls at an angle of less than 90° to the direction of the inward-spiraling velocity, producing forward tangential acceleration as well as inward radial acceleration of the puck.

(e) $\boxed{\text{The puck will spiral outward, slowing down as it does so.}}$

P6.55 (a) The gravitational force exerted by the planet on the person is

$$mg = (75.0\text{ kg})(9.80\text{ m/s}^2)$$
$$= \boxed{735\text{ N}}\text{ down}$$



ANS. FIG. P6.55

Let n represent the force exerted on the person by a scale, which is an upward force whose size is her "apparent weight." The true weight is mg down. For the person at the equator, summing up forces on the object in the direction towards the Earth's center gives $\sum F = ma$:

$$mg - n = ma_c$$

where $a_c = v^2/R_E = 0.033\,7\text{ m/s}^2$

is the centripetal acceleration directed toward the center of the Earth.

Thus, we can solve part (c) before part (b) by noting that

$$n = m(g - a_c) < mg$$

(c) or $mg = n + ma_c > n$.

$\boxed{\begin{array}{l}\text{The gravitational force is greater. The normal force is smaller,}\\ \text{just as one experiences at the top of a moving ferris wheel.}\end{array}}$

(b) If $m = 75.0$ kg and $g = 9.80\text{ m/s}^2$, at the equator we have

$$n = m(g - a_c) = (75.0\text{ kg})(9.800\text{ m/s}^2 - 0.033\,7\text{ m/s}^2) = \boxed{732\text{ N}}$$

P6.56 (a) $v = v_i + kx$ implies the acceleration is

$$\boxed{a = \frac{dv}{dt} = 0 + k\frac{dx}{dt} = +kv}$$

(b) The total force is

$$\sum F = ma = m(+kv)$$

As a vector, the force is parallel or antiparallel to the velocity:

$$\boxed{\sum \vec{\mathbf{F}} = km\vec{\mathbf{v}}}$$

(c) For k positive, some feedback mechanism could be used to impose such a force on an object for a while. The object's speed rises exponentially.

(d) For k negative, think of a duck landing on a lake, where the water exerts a resistive force on the duck proportional to its speed.

P6.57 (a) As shown in the free-body diagram on the right, the mass at the end of the chain is in vertical equilibrium. Thus,

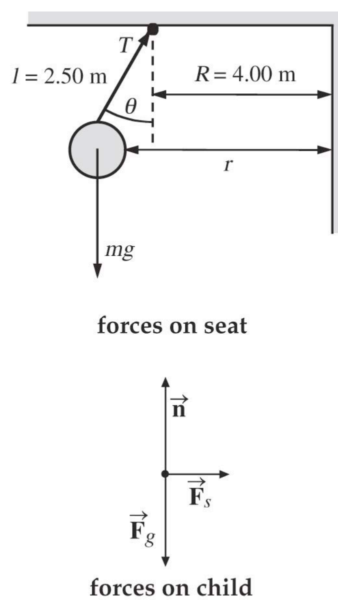
$$T\cos\theta = mg \qquad \textbf{[1]}$$

Horizontally, the mass is accelerating toward the center of a circle of radius *r*:

$$T\sin\theta = ma_r = \frac{mv^2}{r} \qquad \textbf{[2]}$$

Here, *r* is the sum of the radius of the circular platform $R = D/2 = 4.00$ m and $2.50 \sin\theta$:

$$r = (2.50\sin\theta + 4.00)\ \text{m}$$
$$r = (2.50\sin 28.0° + 4.00)\ \text{m}$$
$$= 5.17\ \text{m}$$



ANS. FIG. P6.57

We solve for the tension *T* from [1]:

$$T\cos\theta = mg \rightarrow T = \frac{mg}{\cos\theta}$$

and substitute into [2] to obtain

$$\tan\theta = \frac{a_r}{g} = \frac{v^2}{gr}$$

$$v^2 = gr\tan\theta = (9.80\ \text{m/s}^2)(5.17\ \text{m})(\tan 28.0°)$$

$$v = \boxed{5.19\ \text{m/s}}$$

(b) The free-body diagram for the child is shown in ANS. FIG. P6.57.

(c) $T = \frac{mg}{\cos\theta} = \frac{(40.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 28.0°} = \boxed{444\text{ N}}$

P6.58 (a) The putty, when dislodged, rises and returns to the original level in time t. To find t, we use $v_f = v_i + at$: i.e., $-v = +v - gt$ or $t = \frac{2v}{g}$, where v is the speed of a point on the rim of the wheel.

If R is the radius of the wheel, $v = \frac{2\pi R}{t}$, so $t = \frac{2v}{g} = \frac{2\pi R}{v}$.

Thus, $v^2 = \pi R g$ and $v = \boxed{\sqrt{\pi R g}}$.

(b) The putty is dislodged when F, the force holding it to the wheel, is

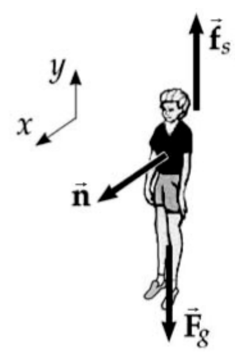
$$F = \frac{mv^2}{R} = \boxed{m\pi g}$$

P6.59 (a) The wall's normal force pushes inward:

$$\sum F_{\text{inward}} = ma_{\text{inward}}$$

becomes

$$n = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2 Rm}{T^2}$$



ANS. FIG. P6.59

The friction and weight balance:

$$\sum F_{\text{upward}} = ma_{\text{upward}}$$

becomes

$$+f - mg = 0$$

so with the person just ready to start sliding down,

$$f_s = \mu_s n = mg$$

Substituting,

$$\mu_s n = \mu_s \frac{4\pi^2 Rm}{T^2} = mg$$

Solving,

$$T^2 = \frac{4\pi^2 R\mu_s}{g}$$

gives

$$\boxed{T = \sqrt{\frac{4\pi^2 R\mu_s}{g}}}$$

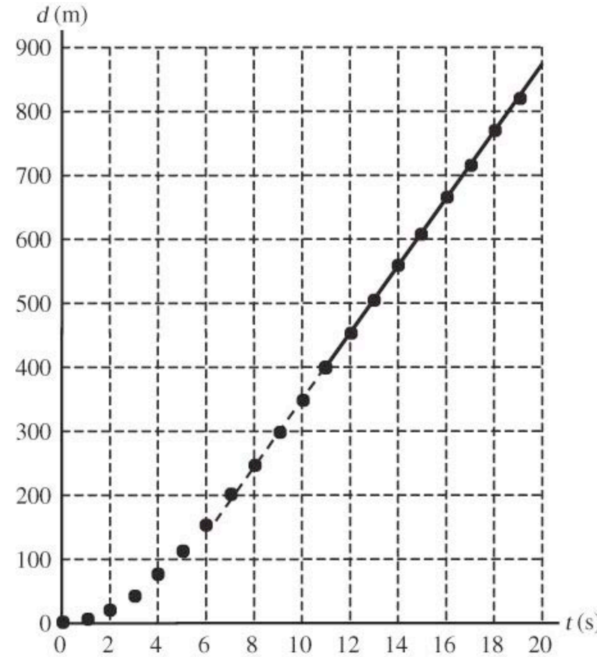
(b) The gravitational and friction forces remain constant. (Static friction adjusts to support the weight.) The normal force increases. The person remains in motion with the wall.

(c) The gravitational force remains constant. The normal and friction forces decrease. The person slides relative to the wall and downward into the pit.

P6.60 (a)

t(s)	d (m)	t(s)	d (m)
1.00	4.88	11.0	399
2.00	18.9	12.0	452
3.00	42.1	13.0	505
4.00	43.8	14.0	558
5.00	112	15.0	611
6.00	154	16.0	664
7.00	199	17.0	717
8.00	246	18.0	770
9.00	296	19.0	823
10.0	347	20.0	876

(b)



(c) A straight line fits the points from t = 11.0 s to 20.0 s quite precisely. Its slope is the terminal speed.

$$v_T = \text{slope} = \frac{876 \text{ m} - 399 \text{ m}}{20.0 \text{ s} - 11.0 \text{ s}} = \boxed{53.0 \text{ m/s}}$$

P6.61 (a) If the car is about to slip *down* the incline, f is directed up the incline.

$$\sum F_y = n\cos\theta + f\sin\theta - mg = 0$$

where $f = \mu_s n$. Substituting,

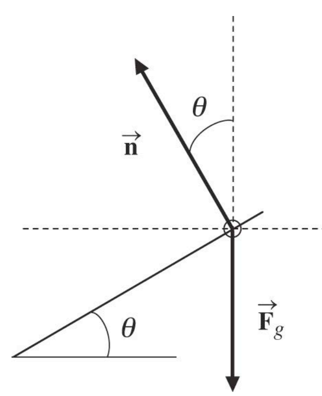
$$n = \frac{mg}{\cos\theta\left(1 + \mu_s \tan\theta\right)}$$

and $$f = \frac{\mu_s mg}{\cos\theta\left(1 + \mu_s \tan\theta\right)}$$

Then, $\sum F_x = n\sin\theta - f\cos\theta = m\dfrac{v_{min}^2}{R}$

yields

$$\boxed{v_{min} = \sqrt{\frac{Rg\left(\tan\theta - \mu_s\right)}{1 + \mu_s \tan\theta}}}$$



ANS. FIG. P6.61

When the car is about to slip *up* the incline, f is directed down the incline.

Then,

$$\sum F_y = n\cos\theta - f\sin\theta - mg = 0, \quad \text{with } f = \mu_s n$$

This yields

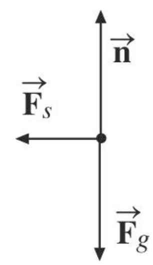
$$n = \frac{mg}{\cos\theta\left(1-\mu_s\tan\theta\right)} \quad \text{and} \quad f = \frac{\mu_s mg}{\cos\theta\left(1-\mu_s\tan\theta\right)}$$

In this case, $\sum F_x = n\sin\theta + f\cos\theta = m\frac{v_{max}^2}{R}$, which gives

$$\boxed{v_{max} = \sqrt{\frac{Rg\left(\tan\theta+\mu_s\right)}{1-\mu_s\tan\theta}}}$$

(b) If $v_{min} = \sqrt{\frac{Rg\left(\tan\theta-\mu_s\right)}{1+\mu_s\tan\theta}} = 0$, then $\boxed{\mu_s = \tan\theta}$.

P6.62 There are three forces on the child, a vertical normal force, a horizontal force (combination of friction and a horizontal force from a seat belt), and gravity.



ANS. FIG. P6.62

$$\sum F_x: \quad F_s = mv^2/R$$

$$\sum F_y: \quad n - mg = 0 \rightarrow n = mg$$

The magnitude of the net force is

$$F_{net} = \sqrt{\left(mv^2/R\right)^2 + \left(mg\right)^2}$$

with a direction of

$$\theta = \tan^{-1}\left[\frac{mg}{mv^2/R}\right] = \tan^{-1}\left[\frac{gR}{v^2}\right] \text{ above the horizontal}$$

For m = 40.0 kg and R = 10.0 m:

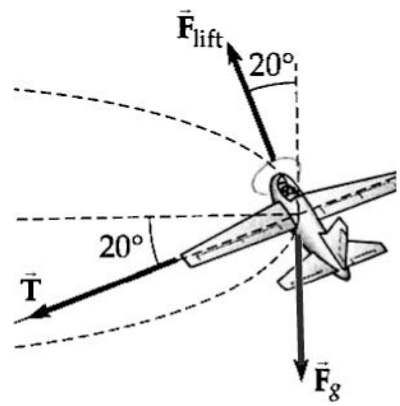
$$F_{net} = \sqrt{\left[\frac{(40.0\text{ kg})(3.00\text{ m/s})^2}{10.0\text{ m}}\right]^2 + \left[(40.0\text{ kg})(9.80\text{ m/s}^2)\right]^2}$$

$$\boxed{F_{net} = 394\text{ N}}$$

direction: $\theta = \tan^{-1}\left[\frac{(9.80\text{ m/s}^2)(10.0\text{ m})}{(3.00\text{ m/s})^2}\right] \rightarrow \boxed{\theta = 84.7°}$

P6.63 The plane's acceleration is toward the center of the circle of motion, so it is horizontal. The radius of the circle of motion is (60.0 m) cos 20.0° = 56.4 m and the acceleration is

$$a_c = \frac{v^2}{r} = \frac{(35\ \text{m/s})^2}{56.4\ \text{m}} = 21.7\ \text{m/s}^2$$



ANS. FIG. P6.63

We can also calculate the weight of the airplane:

$$F_g = mg = (0.750\ \text{kg})(9.80\ \text{m/s}^2) = 7.35\ \text{N}$$

We define our axes for convenience. In this case, two of the forces—one of them our force of interest—are directed along the 20.0° line. We define the x axis to be directed in the $+\vec{T}$ direction, and the y axis to be directed in the direction of lift. With these definitions, the x component of the centripetal acceleration is

$$a_{cx} = a_c \cos 20.0°$$

and $\sum F_x = ma_x$ yields $T + F_g \sin 20.0° = ma_{cx}$

Solving for T,

$$T = ma_{cx} - F_g \sin 20.0°$$

Substituting,

$$T = (0.750\ \text{kg})(21.7\ \text{m/s}^2)\cos 20.0° - (7.35\ \text{N})\sin 20.0°$$

Computing,

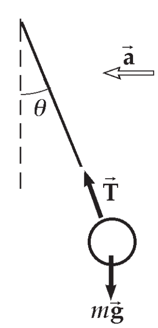
$$T = 15.3\ \text{N} - 2.51\ \text{N} = \boxed{12.8\ \text{N}}$$

***P6.64** (a) While the car negotiates the curve, the accelerometer is at the angle θ.

Horizontally: $T\sin\theta = \dfrac{mv^2}{r}$

Vertically: $T\cos\theta = mg$

where r is the radius of the curve, and v is the speed of the car.



ANS. FIG. P6.64

By division, $\tan\theta = \dfrac{v^2}{rg}$

Then

$$a_c = \frac{v^2}{r} = g\tan\theta:$$

$$a_c = \left(9.80\ \text{m/s}^2\right)\tan 15.0°$$

$$a_c = \boxed{2.63\ \text{m/s}^2}$$

(b) $r = \dfrac{v^2}{a_c}$ gives $r = \dfrac{(23.0\ \text{m/s})^2}{2.63\ \text{m/s}^2} = \boxed{201\ \text{m}}$

(c) $v^2 = rg\tan\theta = (201\ \text{m})\left(9.80\ \text{m/s}^2\right)\tan 9.00°$

$$v = \boxed{17.7\ \text{m/s}}$$

Challenge Problems

P6.65 We find the terminal speed from

$$v = \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right] \qquad \textbf{[1]}$$

where $\exp(x) = e^x$ is the exponential function.

At $t \to \infty$: $v \to v_T = \dfrac{mg}{b}$

At $t = 5.54$ s: $0.500v_T = v_T\left[1 - \exp\left(\dfrac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right)\right]$

Solving,

$$\exp\left(\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right) = 0.500$$

$$\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}} = \ln 0.500 = -0.693$$

$$b = \frac{(9.00\ \text{kg})(0.693)}{5.54\ \text{s}} = 1.13\ \text{kg/s}$$

(a) From $v_T = \dfrac{mg}{b}$, we have

$$v_T = \frac{(9.00\text{ kg})(9.80\text{ m/s}^2)}{1.13\text{ kg/s}} = \boxed{78.3\text{ m/s}}$$

(b) We substitute $0.750v_T$ on the left-hand side of equation [1]:

$$0.750v_T = v_T\left[1-\exp\left(\frac{-1.13t}{9.00\text{ s}}\right)\right]$$

and solve for t:

$$\exp\left(\frac{-1.13t}{9.00\text{ s}}\right) = 0.250$$

$$t = \frac{9.00(\ln 0.250)}{-1.13}\text{ s} = \boxed{11.1\text{ s}}$$

(c) We differentiate equation [1] with respect to time,

$$\frac{dx}{dt} = \left(\frac{mg}{b}\right)\left[1-\exp\left(-\frac{bt}{m}\right)\right]$$

then, integrate both sides

$$\int_{x_0}^{x} dx = \int_0^t \left(\frac{mg}{b}\right)\left[1-\exp\left(\frac{-bt}{m}\right)\right]dt$$

$$x - x_0 = \frac{mgt}{b} + \left(\frac{m^2g}{b^2}\right)\exp\left(\frac{-bt}{m}\right)\Big|_0^t$$

$$= \frac{mgt}{b} + \left(\frac{m^2g}{b^2}\right)\left[\exp\left(\frac{-bt}{m}\right)-1\right]$$

At $t = 5.54$ s,

$$x = (9.00\text{ kg})(9.80\text{ m/s}^2)\left(\frac{5.54\text{ s}}{1.13\text{ kg/s}}\right) + \left(\frac{(9.00\text{ kg})^2(9.80\text{ m/s}^2)}{(1.13\text{ kg/s})^2}\right)\left[\exp(-0.693)-1\right]$$

$$x = 434\text{ m} + 626\text{ m}(-0.500) = \boxed{121\text{ m}}$$

P6.66 (a) From Problem 6.33,

$$v = \frac{dx}{dt} = \frac{v_i}{1 + v_i kt}$$

$$\int_0^x dx = \int_0^t v_i \frac{dt}{1 + v_i kt} = \frac{1}{k}\int_0^t \frac{v_i k dt}{1 + v_i kt}$$

$$x\Big|_0^x = \frac{1}{k}\ln\left(1 + v_i kt\right)\Big|_0^t$$

$$x - 0 = \frac{1}{k}\left[\ln\left(1 + v_i kt\right) - \ln 1\right]$$

$$\boxed{x = \frac{1}{k}\ln\left(1 + v_i kt\right)}$$

(b) We have $\ln\left(1 + v_i kt\right) = kx$

$$1 + v_i kt = e^{kx} \quad \text{so} \quad v = \frac{v_i}{1 + v_i kt} = \frac{v_i}{e^{kx}} = \boxed{v_i e^{-kx} = v}$$

P6.67 Let the *x* axis point eastward, the *y* axis upward, and the *z* axis point southward.

(a) The range is $Z = \dfrac{v_i^2 \sin 2\theta_i}{g}$

The initial speed of the ball is therefore

$$v_i = \sqrt{\frac{gZ}{\sin 2\theta_i}} = \sqrt{\frac{\left(9.80 \text{ m/s}^2\right)\left(285 \text{ m}\right)}{\sin 96.0°}} = 53.0 \text{ m/s}$$

The time the ball is in the air is found from $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ as

$$0 = \left(53.0 \text{ m/s}\right)\left(\sin 48.0°\right)t - \left(4.90 \text{ m/s}^2\right)t^2$$

giving $t = \boxed{8.04 \text{ s}}$.

(b) $$v_{xi} = \frac{2\pi R_e \cos\phi_i}{86\,400 \text{ s}} = \frac{2\pi\left(6.37 \times 10^6 \text{ m}\right)\cos 35.0°}{86\,400 \text{ s}} = \boxed{379 \text{ m/s}}$$

(c) 360° of latitude corresponds to a distance of $2\pi R_e$, so 285 m is a change in latitude of

$$\Delta\phi = \left(\frac{S}{2\pi R_e}\right)\left(360°\right) = \left(\frac{285 \text{ m}}{2\pi\left(6.37 \times 10^6 \text{ m}\right)}\right)\left(360°\right)$$

$$= 2.56 \times 10^{-3} \text{ degrees}$$

The final latitude is then

$$\phi_f = \phi_i - \Delta\phi = 35.0° - 0.002\,56° = 34.997\,4°$$

The cup is moving eastward at a speed

$$v_{xf} = \frac{2\pi R_e \cos\phi_f}{86\,400\text{ s}}$$

which is larger than the eastward velocity of the tee by

$$\Delta v_x = v_{xf} - v_{xi} = \left(\frac{2\pi R_e}{86\,400\text{ s}}\right)\left[\cos\phi_f - \cos\phi_i\right]$$

$$= \left(\frac{2\pi R_e}{86\,400\text{ s}}\right)\left[\cos(\phi - \Delta\phi) - \cos\phi_i\right]$$

$$= \left(\frac{2\pi R_e}{86\,400\text{ s}}\right)\left[\cos\phi_i \cos\Delta\phi + \sin\phi_i \sin\Delta\phi - \cos\phi_i\right]$$

Since $\Delta\phi$ is such a small angle, $\cos\Delta\phi \approx 1$ and

$$\Delta v_x \approx \left(\frac{2\pi R_e}{86\,400\text{ s}}\right)\sin\phi_i \sin\Delta\phi$$

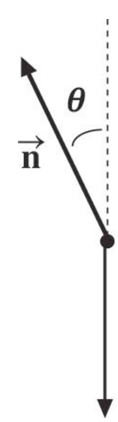
$$\Delta v_x \approx \left[\frac{2\pi(6.37\times 10^6\text{ m})}{86\,400\text{ s}}\right]\sin 35.0° \sin 0.002\,56°$$

$$= \boxed{1.19\times 10^{-2}\text{ m/s}}$$

(d) $\Delta x = (\Delta v_x)t = (1.19\times 10^{-2}\text{ m/s})(8.04\text{ s}) = 0.095\,5\text{ m} = \boxed{9.55\text{ cm}}$

P6.68 (a) We let *R* represent the radius of the hoop and *T* represent the period of its rotation. The bead moves in a circle with radius $r = R\sin\theta$ at a speed of

$$v = \frac{2\pi r}{T} = \frac{2\pi R\sin\theta}{T}$$



ANS. FIG. P6.68

The normal force has an inward radial component of $n\sin\theta$ and an upward component of $n\cos\theta$.

$$\sum F_y = ma_y: \quad n\cos\theta - mg = 0$$

or

$$n = \frac{mg}{\cos\theta}$$

Then $\sum F_x = n\sin\theta = m\dfrac{v^2}{r}$ becomes

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{m}{R\sin\theta}\left(\frac{2\pi R\sin\theta}{T}\right)^2$$

which reduces to $\dfrac{g\sin\theta}{\cos\theta} = \dfrac{4\pi^2 R\sin\theta}{T^2}$

This has two solutions: $\sin\theta = 0 \quad\Rightarrow\quad \theta = 0°$ **[1]**

and $\cos\theta = \dfrac{gT^2}{4\pi^2 R}$ **[2]**

If R = 15.0 cm and T = 0.450 s, the second solution yields

$$\cos\theta = \frac{(9.80\ \text{m/s}^2)(0.450\ \text{s})^2}{4\pi^2(0.150\ \text{m})} = 0.335 \quad \text{or} \quad \theta = 70.4°$$

Thus, in this case, the bead can ride at two positions: $\boxed{\theta = 70.4°}$ and $\boxed{\theta = 0°}$.

(b) At this slower rotation, solution [2] above becomes

$$\cos\theta = \frac{(9.80\ \text{m/s}^2)(0.850\ \text{s})^2}{4\pi^2(0.150\ \text{m})} = 1.20\text{, which is impossible.}$$

In this case, the bead can ride only at the bottom of the loop, $\boxed{\theta = 0°}$.

(c) $\boxed{\text{There is only one solution for (b) because the period is too large.}}$

(d) The equation that the angle must satisfy has two solutions whenever $4\pi^2 R > gT^2$ but only the solution 0° otherwise. The loop's rotation must be faster than a certain threshold value in order for the bead to move away from the lowest position. Zero is always a solution for the angle.

(e) From the derivation of the solution in (a), there are never more than two solutions.

P6.69 At terminal velocity, the accelerating force of gravity is balanced by friction drag:

$$mg = arv + br^2v^2$$

(a) With $r = 10.0\ \mu\text{m}$, $mg = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$

For water, $m = \rho V = 1\,000\ \text{kg/m}^3\left[\frac{4}{3}\pi\left(10^{-5}\ \text{m}\right)^3\right]$

$$mg = 4.11\times10^{-11} = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$$

Assuming v is small, ignore the second term on the right hand side: $\boxed{v = 0.013\ 2\ \text{m/s}}$

(b) With $r = 100\ \mu\text{m}$, $mg = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$

Here we cannot ignore the second term because the coefficients are of nearly equal magnitude.

$$mg = 4.11\times10^{-8} = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$$

Taking the positive root,

$$v = \frac{-3.10 + \sqrt{(3.10)^2 + 4(0.870)(4.11)}}{2(0.870)} = \boxed{1.03\ \text{m/s}}$$

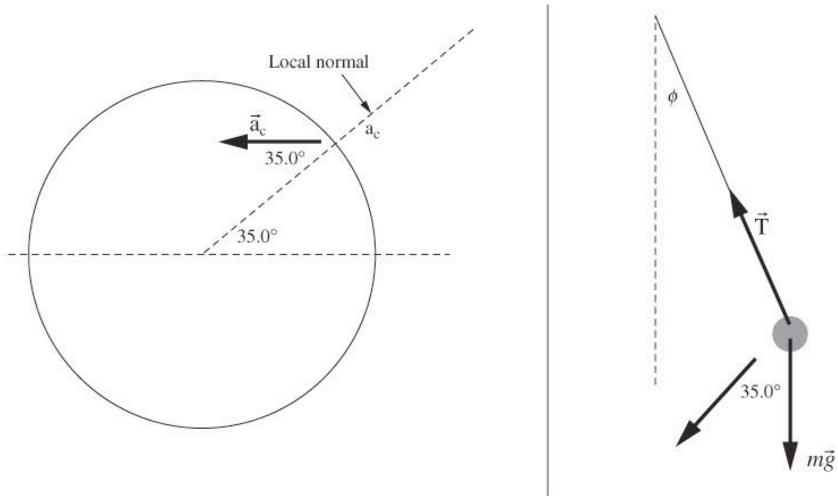
(c) With $r = 1.00\ \text{mm}$, $mg = \left(3.10\times10^{-7}\right)v + \left(0.870\times10^{-6}\right)v^2$

Assuming $v > 1$ m/s, and ignoring the first term:

$$4.11\times10^{-5} = \left(0.870\times10^{-6}\right)v^2 \qquad v = \boxed{6.87\ \text{m/s}}$$

P6.70 At a latitude of 35°, the centripetal acceleration of a plumb bob is directed at 35° to the local normal, as can be seen from the following diagram below at left.

Therefore, if we look at a diagram of the forces on the plumb bob and its acceleration with the local normal in a vertical orientation, we see the second diagram in ANS. FIG. P6.70:



ANS. FIG. P6.70

We first find the centripetal acceleration of the plumb bob. The first figure shows that the radius of the circular path of the plumb bob is *R* cos 35.0°, where *R* is the radius of the Earth. The acceleration is

$$a_c = \frac{v^2}{r} = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 R\cos 35.0°}{T^2}$$
$$= \frac{4\pi^2\left(6.37\times 10^6\ \text{m}\right)\cos 35.0°}{\left(86\ 400\ \text{s}\right)^2} = 0.027\ 6\ \text{m/s}^2$$

Apply the particle under a net force model to the plumb bob in both *x* and *y* directions in the second diagram:

$$x:\ T\sin\phi = ma_c\sin 35.0°$$
$$y:\ mg - T\cos\phi = ma_c\cos 35.0°$$

Divide the equations:

$$\tan\phi = \frac{a_c\sin 35.0°}{g - a_c\cos 35.0°}$$

$$\tan\phi = \frac{\left(0.027\ 6\ \text{m/s}^2\right)\sin 35.0°}{9.80\ \text{m/s}^2 - \left(0.027\ 6\ \text{m/s}^2\right)\cos 35.0°} = 1.62\times 10^{-3}$$

$$\phi = \tan^{-1}\left(1.62\times 10^{-3}\right) = \boxed{0.092\ 8°}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P6.2 (a) 1.65×10^3 m/s; (b) 6.84×10^3 s

P6.4 215 N, horizontally inward

P6.6 (a) $\left(-0.233\hat{\mathbf{i}} + 0.163\hat{\mathbf{j}}\right)$ m/s^2; (b) 6.53 m/s, $\left(-0.181\hat{\mathbf{i}} + 0.181\hat{\mathbf{j}}\right)$ m/s^2

P6.8 (a) $(68.6\text{ N})\hat{\mathbf{i}} + (784\text{ N})\hat{\mathbf{j}}$; (b) $a = 0.857$ m/s^2

P6.10 The situation is impossible because the speed of the object is too small, requiring that the lower string act like a rod and push rather than like a string and pull.

P6.12 (a) the gravitational force and the contact force exerted on the water by the pail; (b) contact force exerted by the pail; (c) 3.13 m/s; (d) the water would follow the parabolic path of a projectile

P6.14 (a) 4.81 m/s; (b) 700 N

P6.16 (a) 2.49×10^4 N; (b) 12.1 m/s

P6.18 (a) 20.6 N; (b) 32.0 m/s^2 inward, 3.35 m/s^2 downward tangent to the circle; (c) 32.2 m/s^2 inward and below the cord at 5.98°; (d) no change; (e) acceleration is regardless of the direction of swing

P6.20 (a) 3.60 m/s^2; (b) $T = 0$; (c) noninertial observer in the car claims that the forces on the mass along x are T and a fictitious force $(-Ma)$; (d) inertial observer outside the car claims that T is the only force on M in the x direction

P6.22 93.8 N

P6.24 $\dfrac{2(vt - L)}{(g + a)t^2}$

P6.26 (a) 53.8 m/s; (b) 148 m

P6.28 (a) 6.27 m/s^2 downward; (b) 784 N directed up; (c) 283 N upward

P6.30 (a) 32.7 s^{-1}; (b) 9.80 m/s^2 down; (c) 4.90 m/s^2 down

P6.32 36.5 m/s

P6.34 (a) 2.03 N down; (b) 3.18 m/s^2 down; (c) 0.205 m/s down

P6.36 10^1 N

P6.38 1.2×10^3 N

P6.40 (a) 1.15×10^4 N up; (b) 14.1 m/s

P6.42 See Problem 6.42 for full derivation.

P6.44 (a) 217 N; (b) 283 N; (c) $T_2 > T_1$ always, so string 2 will break first

P6.46 The situation is impossible because the speed of the child given in the problem is too large: static friction could not keep the child in place on the incline

P6.48 0.835 rev/s

P6.50 (a) $v = \sqrt{Rg\tan 35.0°} = \sqrt{(6.86\text{ m/s}^2)R}$; (b) the mass is unnecessary; (c) increasing the radius will make the required speed increase; (d) when the radius increases, the period increases; (e) the time interval required is proportional to $R/\sqrt{R} = \sqrt{R}$

P6.52 (a) 1 975 lb; (b) −647 lb; (c) When $F_g' = 0$, then $mg = \dfrac{mv^2}{R}$.

P6.54 (a) m_2g; (b) m_2g; (c) $\sqrt{\left(\dfrac{m_2}{m_1}\right)gR}$; (d) The puck will spiral inward, gaining speed as it does so; (e) The puck will spiral outward, slowing down as it does so

P6.56 (a) $a = +kv$; (b) $\sum \vec{\mathbf{F}} = km\vec{\mathbf{v}}$; (c) some feedback mechanism could be used to impose such a force on an object; (d) think of a duck landing on a lake, where the water exerts a resistive force on the duck proportional to its speed

P6.58 (a) $\sqrt{\pi Rg}$; (b) $m\pi g$

P6.60 (a) See table in P6.60 (a); (b) See graph in P6.60 (b); (c) 53.0 m/s

P6.62 84.7°

P6.64 (a) 2.63 m/s^2; (b) 201 m; (c) 17.7 m/s

P6.66 (a) $x = \dfrac{1}{k}\ln(1 + v_ikt)$; (b) $v = v_ie^{-kx}$

P6.68 (a) $\theta = 70.4°$ and $\theta = 0°$; (b) $\theta = 0°$; (c) the period is too large; (d) Zero is always a solution for the angle; (e) there are never more than two solutions

P6.70 0.092 8°

7

Energy of a System

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ7.1 Answer (c). Assuming that the cabinet has negligible speed during the operation, all of the work Alex does is used in increasing the gravitational potential energy of the cabinet-Earth system. However, in addition to increasing the gravitational potential energy of the cabinet-Earth system by the same amount as Alex did, John must do work overcoming the friction between the cabinet and ramp. This means that the total work done by John is greater than that done by Alex.

OQ7.2 Answer (d). The work–energy theorem states that $W_{net} = \Delta K = K_f - K_i$. Thus, if $W_{net} = 0$, then $K_f - K_i$ or $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, which leads to the conclusion that the speed is unchanged ($v_f = v_i$). The velocity of the particle involves both magnitude (speed) and direction. The work–energy theorem shows that the magnitude or speed is unchanged

when $W_{net} = 0$, but makes no statement about the direction of the velocity.

OQ7.3 Answer (a). The work done on the wheelbarrow by the worker is

$$W = (F\cos\theta)\Delta x = (50\text{ N})(5.0\text{ m}) = +250\text{ J}$$

OQ7.4 Answer (c). The system consisting of the cart's fixed, initial kinetic energy is the mechanical energy that can be transformed due to friction from the surface. Therefore, the loss of mechanical energy is $\Delta E_{mech} = -f_k d = -(6\text{ N})(0.06\text{ m}) = 0.36\text{ J}.$ This product must remain the same in all cases. For the cart rolling through gravel, $-(9\text{ N})(d) = 0.36\text{ J}$ tells us $d = 4$ cm.

OQ7.5 The answer is a > b = e > d > c. Each dot product has magnitude $(1)\cdot(1)\cdot\cos\theta$, where θ is the angle between the two factors. Thus for (a) we have cos 0 = 1. For (b) and (e), cos 45° = 0.707. For (c), cos 180° = –1. For (d), cos 90° = 0.

OQ7.6 Answer (c). The net work needed to accelerate the object from $v = 0$ to v is

$$W_1 = KE_{1f} - KE_{1i} = \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 = \frac{1}{2}mv^2$$

The work required to accelerate the object from speed v to speed $2v$ is

$$W_2 = KE_{2f} - KE_{2i} = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2}m\left(4v^2 - v^2\right) = 3\left(\frac{1}{2}mv^2\right) = 3W_1$$

OQ7.7 Answer (e). As the block falls freely, only the conservative gravitational force acts on it. Therefore, mechanical energy is conserved, or $KE_f + PE_f = KE_i + PE_i$. Assuming that the block is released from rest ($KE_i = 0$), and taking $y = 0$ at ground level ($PE_f = 0$), we have that

$$KE_f = PE_i \quad \text{or} \quad \frac{1}{2}mv_f^2 = mgy \quad \text{and} \quad y_i = \frac{v_f^2}{2g}$$

Thus, to double the final speed, it is necessary to increase the initial height by a factor of four.

OQ7.8 (i) Answer (b). Tension is perpendicular to the motion. (ii) Answer (c). Air resistance is opposite to the motion.

OQ7.9 Answer (e). Kinetic energy is proportional to mass.

OQ7.10 (i) Answers (c) and (e). The force of block on spring is equal in magnitude and opposite to the force of spring on block.

(ii) Answers (c) and (e). The spring tension exerts equal-magnitude forces toward the center of the spring on objects at both ends.

OQ7.11 Answer (a). Kinetic energy is proportional to squared speed. Doubling the speed makes an object's kinetic energy four times larger.

OQ7.12 Answer (b). Since the rollers on the ramp used by David were frictionless, he did not do any work overcoming nonconservative forces as he slid the block up the ramp. Neglecting any change in kinetic energy of the block (either because the speed was constant or was essentially zero during the lifting process), the work done by either Mark or David equals the increase in the gravitational potential energy of the block-Earth system as the block is lifted from the ground to the truck bed. Because they lift identical blocks through the same vertical distance, they do equal amounts of work.

OQ7.13 (i) Answer: a = b = c = d. The gravitational acceleration is quite precisely constant at locations separated by much less than the radius of the planet.

(ii) Answer: c = d > a = b. The mass but not the elevation affects the gravitational force.

(iii) Answer: c > b = d > a. Gravitational potential energy of the object-Earth system is proportional to mass times height.

OQ7.14 Answer (d). $4.00\text{ J} = \frac{1}{2}k(0.100\text{ m})^2$. Therefore, $k = 800$ N/m and to stretch the spring to 0.200 m requires extra work

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\text{ J} = 12.0\text{ J}$$

OQ7.15 Answer (a). The system consisting of the cart's fixed, initial kinetic energy is the mechanical energy that can be transformed due to friction from the surface. Therefore, the loss of mechanical energy is $\Delta E_{mech} = -f_k d = -(6\text{ N})(0.06\text{ m}) = 0.36\text{ J}$. This product must remain the same in all cases. For the cart rolling through gravel, $-(f_k)(0.18\text{ m}) = 0.36\text{ J}$ tells us $f_k = 2$ N.

OQ7.16 Answer (c). The ice cube is in neutral equilibrium. Its zero acceleration is evidence for equilibrium.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ7.1 Yes. The floor of a rising elevator does work on a passenger. A normal force exerted by a stationary solid surface does no work.

CQ7.2 Yes. Object 1 exerts some forward force on object 2 as they move through the same displacement. By Newton's third law, object 2 exerts an equal-size force in the opposite direction on object 1. In $W = F\Delta r\cos\theta$, the factors F and Δr are the same, and θ differs by 180°, so object 2 does −15.0 J of work on object 1. The energy transfer is 15 J from object 1 to object 2, which can be counted as a change in energy of −15 J for object 1 and a change in energy of +15 J for object 2.

CQ7.3 It is sometimes true. If the object is a particle initially at rest, the net work done on the object is equal to its final kinetic energy. If the object is not a particle, the work could go into (or come out of) some other form of energy. If the object is initially moving, its initial kinetic energy must be added to the total work to find the final kinetic energy.

CQ7.4 The scalar product of two vectors is positive if the angle between them is between 0° and 90°, including 0°. The scalar product is negative when $90° < \theta \le 180°$.

CQ7.5 No. Kinetic energy is always positive. Mass and squared speed are both positive.

CQ7.6 Work is only done in accelerating the ball from rest. The work is done over the effective length of the pitcher's arm—the distance his hand moves through windup and until release. He extends this distance by taking a step forward.

CQ7.7 (a) Positive work is done by the chicken on the dirt.

(b) The person does no work on anything in the environment. Perhaps some extra chemical energy goes through being energy transmitted electrically and is converted into internal energy in his brain; but it would be very hard to quantify "extra."

(c) Positive work is done on the bucket.

(d) Negative work is done on the bucket.

(e) Negative work is done on the person's torso.

CQ7.8 (a) Not necessarily. It does if it makes the object's speed change, but not if it only makes the direction of the velocity change.

(b) Yes, according to Newton's second law.

CQ7.9 The gravitational energy of the key-Earth system is lowest when the key is on the floor letter-side-down. The average height of particles in

the key is lowest in that configuration. As described by $F = -dU/dx$, a force pushes the key downhill in potential energy toward the bottom of a graph of potential energy versus orientation angle. Friction removes mechanical energy from the key-Earth system, tending to leave the key in its minimum-potential energy configuration.

CQ7.10 There is no violation. Choose the book as the system. You did positive work (average force and displacement are in same direction) and the Earth did negative work (average force and displacement are in opposite directions) on the book. The average force you exerted just counterbalanced the weight of the book. The total work on the book is zero, and is equal to its overall change in kinetic energy.

CQ7.11 $k' = 2k$. Think of the original spring as being composed of two half-springs. The same force F that stretches the whole spring by x stretches each of the half-springs by $x/2$; therefore, the spring constant for each of the half-springs is $k' = [F/(x/2)] = 2(F/x) = 2k$.

CQ7.12 A graph of potential energy versus position is a straight horizontal line for a particle in neutral equilibrium. The graph represents a constant function.

CQ7.13 Yes. As you ride an express subway train, a backpack at your feet has no kinetic energy as measured by you since, according to you, the backpack is not moving. In the frame of reference of someone on the side of the tracks as the train rolls by, the backpack is moving and has mass, and thus has kinetic energy.

CQ7.14 Force of tension on a ball moving in a circle on the end of a string. Normal force and gravitational force on an object at rest or moving across a level floor.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 7.2 Work Done by a Constant Force

P7.1 (a) The 35-N force applied by the shopper makes a 25° angle with the displacement of the cart (horizontal). The work done on the cart by the shopper is then

$$W_{\text{shopper}} = (F\cos\theta)\Delta x = (35.0\text{ N})(50.0\text{ m})\cos 25.0°$$
$$= \boxed{1.59 \times 10^3\text{ J}}$$

(b) The force exerted by the shopper is now completely horizontal and will be equal to the friction force, since the cart stays at a constant velocity. In part (a), the shopper's force had a downward

vertical component, increasing the normal force on the cart, and thereby the friction force. Because there is no vertical component here, the friction force will be less, and the $\boxed{\text{the force is smaller}}$ than before.

(c) Since the horizontal component of the force is less in part (b), the work performed by the shopper on the cart over the same 50.0-m distance is $\boxed{\text{the same}}$ as in part (b).

P7.2 (a) The work done on the raindrop by the gravitational force is given by

$$W = mgh = \left(3.35\times10^{-5}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(100\text{ m}\right) = \boxed{3.28\times10^{-2}\text{ J}}$$

(b) Since the raindrop is falling at constant velocity, all forces acting on the drop must be in balance, and $R = mg$, so

$$W_{\text{air resistance}} = \boxed{-3.28\times10^{-2}\text{ J}}$$

P7.3 (a) The work done by a constant force is given by

$$W = Fd\cos\theta$$

where θ is the angle between the force and the displacement of the object. In this case, $F = -mg$ and $\theta = 180°$, giving

$$W = (281.5\text{ kg})(9.80\text{ m/s}^2)[(17.1\text{ cm})(1\text{ m}/10^2\text{ cm})] = \boxed{472\text{ J}}$$

(b) If the object moved upward at constant speed, the net force acting on it was zero. Therefore, the magnitude of the upward force applied by the lifter must have been equal to the weight of the object:

$$F = mg = (281.5\text{ kg})(9.80\text{ m/s}^2) = 2.76\times10^3\text{ N} = \boxed{2.76\text{ kN}}$$

P7.4 Assuming the mass is lifted at constant velocity, the total upward force exerted by the two men equals the weight of the mass: $F_{\text{total}} = mg = (653.2\text{ kg})(9.80\text{ m/s}^2) = 6.40\times10^3\text{ N}$. They exert this upward force through a total upward displacement of 96 inches (4 inches per lift for each of 24 lifts). The total work would then be

$$W_{\text{total}} = (6.40\times10^3\text{ N})[(96\text{ in})(0.025\,4\text{ m}/1\text{ in})] = \boxed{1.56\times10^4\text{ J}}$$

P7.5 We apply the definition of work by a constant force in the first three parts, but then in the fourth part we add up the answers. The total (net) work is the sum of the amounts of work done by the individual forces, and is the work done by the total (net) force. This identification is not represented by an equation in the chapter text, but is something

you know by thinking about it, without relying on an equation in a list.

The definition of work by a constant force is $W = F\Delta r\cos\theta$.

(a) The applied force does work given by

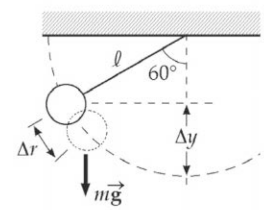
$$W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$$

(b), (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

P7.6 **METHOD ONE**

Let ϕ represent the instantaneous angle the rope makes with the vertical as it is swinging up from $\phi_i = 0$ to $\phi_f = 60°$. In an incremental bit of motion from angle ϕ to $\phi + d\phi$, the definition of radian measure implies that $\Delta r = (12.0\text{ m})d\phi$. The angle θ between the incremental displacement and the force of gravity is $\theta = 90° + \phi$. Then



ANS. FIG. P7.6

$$\cos\theta = \cos(90° + \phi) = -\sin\phi$$

The work done by the gravitational force on Spiderman is

$$W = \int_i^f F\cos\theta\, dr = \int_{\phi=0}^{\phi=60°} mg(-\sin\phi)(12.0\text{ m})d\phi$$

$$= -mg(12.0\text{ m})\int_0^{60°} \sin\phi\, d\phi$$

$$= (-80.0\text{ kg})(9.80\text{ m/s}^2)(12\text{ m})(-\cos\phi)\Big|_0^{60°}$$

$$= (-784\text{ N})(12.0\text{ m})(-\cos 60° + 1) = \boxed{-4.70\times 10^3\text{ J}}$$

METHOD TWO

The force of gravity on Spiderman is $mg = (80\text{ kg})(9.8\text{ m/s}^2) = 784\text{ N}$ down. Only his vertical displacement contributes to the work gravity does. His original y coordinate below the tree limb is –12 m. His final y coordinate is $(-12.0\text{ m})\cos 60.0° = -6.00\text{ m}$. His change in elevation is $-6.00\text{ m} - (-12.0\text{ m})$. The work done by gravity is

$$W = F\Delta r\cos\theta = (784\text{ N})(6.00\text{ m})\cos 180° = \boxed{-4.70\text{ kJ}}$$

Section 7.3 The Scalar Product of Two Vectors

P7.7

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right)\cdot\left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

$$\begin{aligned}\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} &= A_xB_x\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}\right) + A_xB_y\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}\right) + A_xB_z\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{k}}\right)\\ &\quad + A_yB_x\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}\right) + A_yB_y\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}\right) + A_yB_z\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{k}}\right)\\ &\quad + A_zB_x\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{i}}\right) + A_zB_y\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{j}}\right) + A_zB_z\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}\right)\end{aligned}$$

And since $\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$ and $\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 0$,

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \boxed{A_xB_x + A_yB_y + A_zB_z}$$

P7.8 $A = 5.00$; $B = 9.00$; $\theta = 50.0°$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = AB\cos\theta = (5.00)(9.00)\cos 50.0° = \boxed{28.9}$$

P7.9

$$\vec{\mathbf{A}}-\vec{\mathbf{B}} = \left(3.00\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}\right) - \left(-\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} + 5.00\hat{\mathbf{k}}\right) = 4.00\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6.00\hat{\mathbf{k}}$$

$$\begin{aligned}\vec{\mathbf{C}}\cdot\left(\vec{\mathbf{A}}-\vec{\mathbf{B}}\right) &= \left(2.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right)\cdot\left(4.00\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6.00\hat{\mathbf{k}}\right) = 0 + (-2.00) + (+18.0)\\ &= \boxed{16.0}\end{aligned}$$

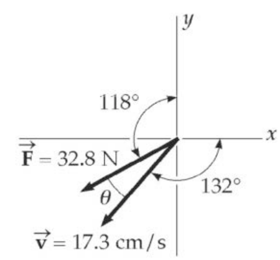
P7.10 We must first find the angle between the two vectors. It is

$$\begin{aligned}\theta &= (360° - 132°) - (118° + 90.0°)\\ &= 20.0°\end{aligned}$$

Then

$$\begin{aligned}\vec{\mathbf{F}}\cdot\vec{\mathbf{r}} &= Fr\cos\theta\\ &= (32.8\ \text{N})(0.173\ \text{m})\cos 20.0°\end{aligned}$$

or $\vec{\mathbf{F}}\cdot\vec{\mathbf{r}} = 5.33\ \text{N}\cdot\text{m} = \boxed{5.33\ \text{J}}$



ANS. FIG. P7.10

P7.11 (a) We use the mathematical representation of the definition of work.

$$\begin{aligned}W = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}} &= F_x x + F_y y = (6.00)(3.00)\ \text{N}\cdot\text{m} + (-2.00)(1.00)\ \text{N}\cdot\text{m}\\ &= \boxed{16.0\ \text{J}}\end{aligned}$$

(b) $\theta = \cos^{-1}\left(\dfrac{\vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}}{F\Delta r}\right)$

$$\begin{aligned}&= \cos^{-1}\frac{16\ \text{N}\cdot\text{m}}{\sqrt{(6.00\ \text{N})^2 + (-2.00\ \text{N})^2}\cdot\sqrt{(3.00\ \text{m})^2 + (1.00\ \text{m})^2}}\\ &= \boxed{36.9°}\end{aligned}$$

P7.12 (a) $\vec{\mathbf{A}} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$

$\vec{\mathbf{B}} = 4.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}$

$$\theta = \cos^{-1}\left(\frac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \cos^{-1}\left(\frac{12.0 + 8.00}{\sqrt{13.0}\cdot\sqrt{32.0}}\right) = \boxed{11.3°}$$

(b) $\vec{\mathbf{A}} = -2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$

$\vec{\mathbf{B}} = 3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$$\cos\theta = \left(\frac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \frac{-6.00 - 16.0}{\sqrt{20.0}\cdot\sqrt{29.0}} \quad\rightarrow\quad \theta = \boxed{156°}$$

(c) $\vec{\mathbf{A}} = \hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\vec{\mathbf{B}} = 3.00\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}}$

$$\theta = \cos^{-1}\left(\frac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \cos^{-1}\left(\frac{-6.00 + 8.00}{\sqrt{9.00}\cdot\sqrt{25.0}}\right) = \boxed{82.3°}$$

P7.13 Let θ represent the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Turning by 25.0° makes the dot product larger, so the angle between $\vec{\mathbf{C}}$ and $\vec{\mathbf{B}}$ must be smaller. We call it $\theta - 25.0°$. Then we have

$$5A\cos\theta = 30 \quad \text{and} \quad 5A\cos(\theta - 25.0°) = 35$$

Then

$$A\cos\theta = 6 \quad \text{and} \quad A(\cos\theta\cos 25.0° + \sin\theta\sin 25.0°) = 7$$

Dividing,

$$\cos 25.0° + \tan\theta\sin 25.0° = 7/6$$

or $\tan\theta = (7/6 - \cos 25.0°)/\sin 25.0° = 0.616$

Which gives $\theta = 31.6°$. Then the direction angle of A is

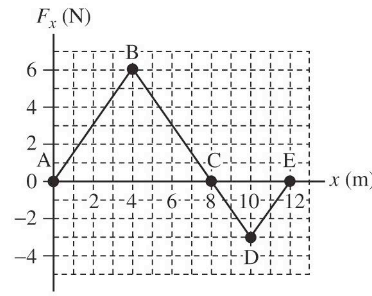
$$60.0° - 31.6° = 28.4°$$

Substituting back,

$$A\cos 31.6° = 6 \quad \text{so} \quad \vec{\mathbf{A}} = \boxed{7.05\text{ m at }28.4°}$$

Section 7.4 Work Done by a Varying Force

P7.14 $W = \int_i^f F dx =$ area under curve from x_i to x_f



ANS. FIG. P7.14

(a) $x_i = 0$ and $x_f = 800$ m

$$W_{0\to 8} = \text{area of triangle ABC}$$

$$= \left(\frac{1}{2}\right) \text{AC} \times \text{height}$$

$$W_{0\to 8} = \left(\frac{1}{2}\right) \times 8.00 \text{ m} \times 6.00 \text{ N}$$

$$= \boxed{24.0 \text{ J}}$$

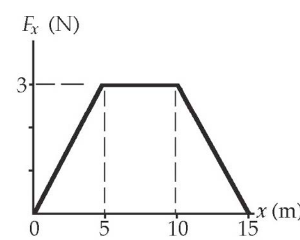
(b) $x_i = 8.00$ m and $x_f = 10.0$ m

$$W_{8\to 10} = \text{area of } \Delta\text{CDE} = \left(\frac{1}{2}\right) \text{CE} \times \text{height},$$

$$W_{8\to 10} = \left(\frac{1}{2}\right) \times (2.00 \text{ m}) \times (-3.00 \text{ N}) = \boxed{-3.00 \text{ J}}$$

(c) $W_{0\to 10} = W_{0\to 8} + W_{0\to 10} = 24.0 + (-3.00) = \boxed{21.0 \text{ J}}$

P7.15 We use the graphical representation of the definition of work. *W* equals the area under the force-displacement curve. This definition is still written $W = \int F_x dx$ but it is computed geometrically by identifying triangles and rectangles on the graph.



ANS. FIG. P7.15

(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00 \text{ N})(5.00 \text{ m})}{2} = \boxed{7.50 \text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$, $W = (3.00 \text{ N})(5.00 \text{ m}) = \boxed{15.0 \text{ J}}$

(c) For the region $10.00 \le x \le 15.0$, $W = \frac{(3.00 \text{ N})(5.00 \text{ m})}{2} = \boxed{7.50 \text{ J}}$

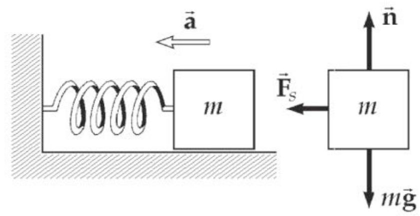
(d) For the region $0 \le x \le 15.0$, $W = (7.50 + 7.50 + 15.0) \text{ J} = \boxed{30.0 \text{ J}}$

P7.16 $\sum F_x = ma_x$: $kx = ma$

$$k = \frac{ma}{x}$$

$$= \frac{(4.70 \times 10^{-3}\ \text{kg})\,(0.800)\,(9.80\ \text{m/s}^2)}{0.500 \times 10^{-2}\text{m}}$$

$$= \boxed{7.37\ \text{N/m}}$$



ANS. FIG. P7.16

P7.17 When the load of mass $M = 4.00$ kg is hanging on the spring in equilibrium, the upward force exerted by the spring on the load is equal in magnitude to the downward force that the Earth exerts on the load, given by $w = Mg$. Then we can write Hooke's law as $Mg = +kx$. The spring constant, force constant, stiffness constant, or Hooke's-law constant of the spring is given by

$$k = \frac{F}{y} = \frac{Mg}{y} = \frac{(4.00\ \text{kg})(9.80\ \text{m/s}^2)}{2.50 \times 10^{-2}\ \text{m}} = 1.57 \times 10^3\ \text{N/m}$$

(a) For the 1.50-kg mass,

$$y = \frac{mg}{k} = \frac{(1.50\ \text{kg})(9.80\ \text{m/s}^2)}{1.57 \times 10^3\ \text{N/m}} = 0.009\ 38\ \text{m} = \boxed{0.938\ \text{cm}}$$

(b) Work $= \frac{1}{2}ky^2 = \frac{1}{2}\left(1.57 \times 10^3\ \text{N/m}\right)\left(4.00 \times 10^{-2}\ \text{m}\right)^2 = \boxed{1.25\ \text{J}}$

P7.18 In $F = -kx$, F refers to the size of the force that the spring exerts on each end. It pulls down on the doorframe in part (a) in just as real a sense as it pulls on the second person in part (b).

(a) Consider the upward force exerted by the bottom end of the spring, which undergoes a downward displacement that we count as negative:

$$k = -F/x = -(7.50\ \text{kg})(9.80\ \text{m/s}^2)/(-0.415\ \text{m} + 0.350\ \text{m})$$

$$= -73.5\ \text{N}/(-0.065\ \text{m}) = \boxed{1.13\ \text{kN/m}}$$

(b) Consider the end of the spring on the right, which exerts a force to the left:

$$x = -F/k = -(-190\ \text{N})/(1130\ \text{N/m}) = 0.168\ \text{m}$$

The length of the spring is then

$$0.350\ \text{m} + 0.168\ \text{m} = \boxed{0.518\ \text{m} = 51.8\ \text{cm}}$$

P7.19 (a) Spring constant is given by $F = kx$:

$$k = \frac{F}{x} = \frac{230 \text{ N}}{0.400 \text{ m}} = \boxed{575 \text{ N/m}}$$

(b) Work $= F_{avg}x = \left(\frac{230 \text{ N} - 0}{2}\right)(0.400 \text{ m}) = \boxed{46.0 \text{ J}}$

P7.20 The same force makes both light springs stretch.

(a) The hanging mass moves down by

$$x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= (1.5 \text{ kg})(9.8 \text{ m/s}^2)\left(\frac{1}{1\,200 \text{ N/m}} + \frac{1}{1\,800 \text{ N/m}}\right)$$

$$= \boxed{2.04 \times 10^{-2} \text{ m}}$$

(b) We define the effective spring constant as

$$k = \frac{F}{x} = \frac{mg}{mg(1/k_1 + 1/k_2)} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$

$$= \left(\frac{1}{1\,200 \text{ N/m}} + \frac{1}{1\,800 \text{ N/m}}\right)^{-1} = \boxed{720 \text{ N/m}}$$

P7.21 (a) The force mg is the tension in each of the springs. The bottom of the upper (first) spring moves down by distance $x_1 = |F|/k_1 = mg/k_1$. The top of the second spring moves down by this distance, and the second spring also stretches by $x_2 = mg/k_2$. The bottom of the lower spring then moves down by distance

$$x_{total} = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = \boxed{mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$$

(b) From the last equation we have

$$mg = \frac{x_1 + x_2}{\frac{1}{k_1} + \frac{1}{k_2}}$$

This is of the form

$$|F| = \left(\frac{1}{1/k_1 + 1/k_2}\right)(x_1 + x_2)$$

The downward displacement is opposite in direction to the upward force the springs exert on the load, so we may write $F = -k_{eff}\, x_{\text{total}}$, with the effective spring constant for the pair of springs given by

$$k_{eff} = \boxed{\frac{1}{1/k_1 + 1/k_2}}$$

P7.22 $$[k] = \left[\frac{F}{x}\right] = \frac{\text{N}}{\text{m}} = \frac{\text{kg}\cdot\text{m/s}^2}{\text{m}} = \boxed{\frac{\text{kg}}{\text{s}^2}}$$

P7.23 (a) $\boxed{\text{If the weight of the first tray stretches all four springs by a distance equal to the thickness of the tray, then the proportionality expressed by Hooke's law guarantees that each additional tray will have the same effect, so that the top surface of the top tray can always have the same elevation above the floor if springs with the right spring constant are used.}}$

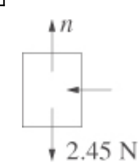
(b) The weight of a tray is $(0.580\text{ kg})(9.8\text{ m/s}^2) = 5.68\text{ N}$. The force $\frac{1}{4}(5.68\text{ N}) = 1.42\text{ N}$ should stretch one spring by 0.450 cm, so its spring constant is

$$k = \frac{|F_s|}{x} = \frac{1.42\text{ N}}{0.004\,5\text{ m}} = \boxed{316\text{ N/m}}$$

(c) $\boxed{\text{We did not need to know the length or width of the tray.}}$

P7.24 The spring exerts on each block an outward force of magnitude

$$|F_s| = kx = (3.85\text{ N/m})(0.08\text{ m}) = 0.308\text{ N}$$



ANS. FIG. P7.24

Take the $+x$ direction to the right. For the light block on the left, the vertical forces are given by

$$F_g = mg = (0.250\text{ kg})(9.80\text{ m/s}^2) = 2.45\text{ N}$$

and $\sum F_y = 0$

so $n - 2.45\text{ N} = 0 \quad \rightarrow \quad n = 2.45\text{ N}$

Similarly, for the heavier block,

$$n = F_g = (0.500\text{ kg})(9.80\text{ m/s}^2) = 4.90\text{ N}$$

(a) For the block on the left,

$$\sum F_x = ma_x: \quad -0.308 \text{ N} = (0.250 \text{ kg})a$$

$$a = \boxed{-1.23 \text{ m/s}^2}$$

For the heavier block,

$$+0.308 \text{ N} = (0.500 \text{ kg})a$$

$$a = \boxed{0.616 \text{ m/s}^2}$$

(b) For the block on the left, $f_k = \mu_k n = 0.100(2.45 \text{ N}) = 0.245 \text{ N}$.

$$\sum F_x = ma_x$$

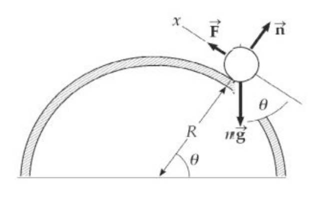
$$-0.308 \text{ N} + 0.245 \text{ N} = (0.250 \text{ kg})a$$

$$a = \boxed{-0.252 \text{ m/s}^2 \text{ if the force of static friction is not too large}}.$$

For the block on the right, $f_k = \mu_k n = 0.490$ N. The maximum force of static friction would be larger, so no motion would begin and the acceleration is $\boxed{\text{zero}}$.

(c) Left block: $f_k = 0.462(2.45 \text{ N}) = 1.13$ N. The maximum static friction force would be larger, so the spring force would produce no motion of this block or of the right-hand block, which could feel even more friction force. For both, $a = \boxed{0}$.

P7.25 (a) The radius to the object makes angle θ with the horizontal. Taking the x axis in the direction of motion tangent to the cylinder, the object's weight makes an angle θ with the $-x$ axis. Then,



ANS. FIG. P7.25

$$\sum F_x = ma_x$$

$$F - mg\cos\theta = 0$$

$$F = \boxed{mg\cos\theta}$$

(b) $W = \int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$

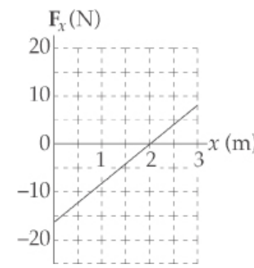
We use radian measure to express the next bit of displacement as $dr = Rd\theta$ in terms of the next bit of angle moved through:

$$W = \int_0^{\pi/2} mg\cos\theta R\, d\theta = mgR\sin\theta\Big|_0^{\pi/2} = mgR(1-0) = \boxed{mgR}$$

P7.26 The force is given by $F_x = (8x - 16)$ N.

(a) See ANS. FIG. P7.26 to the right.

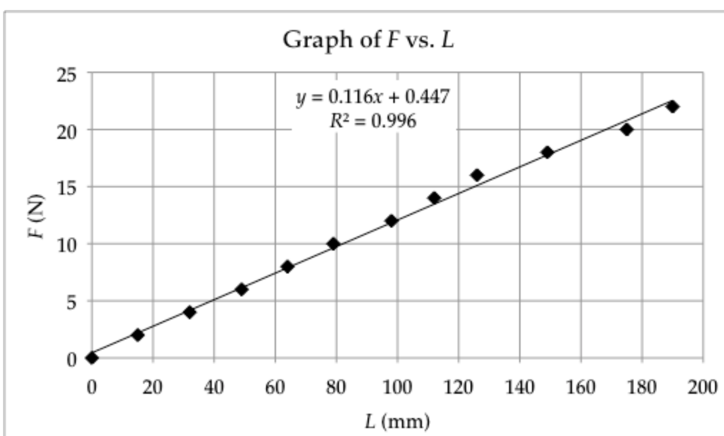
(b) $$W_{\text{net}} = \frac{-(2.00\text{ m})(16.0\text{ N})}{2} + \frac{(1.00\text{ m})(8.00\text{ N})}{2} = \boxed{-12.0\text{ J}}$$



ANS. FIG. P7.26

P7.27 (a)

F (N)	L (mm)	F (N)	L (mm)
0.00	0.00	12.0	98.0
2.00	15.0	14.0	112
4.00	32.0	16.0	126
6.00	49.0	18.0	149
8.00	64.0	20.0	175
10.0	79.0	22.0	190



ANS FIG. P7.27(a)

(b) By least-squares fitting, its slope is 0.116 N/mm = $\boxed{116\text{ N/m}}$.

(c) To draw the straight line we use all the points listed and also the origin. If the coils of the spring touched each other, a bend or nonlinearity could show up at the bottom end of the graph. If the spring were stretched "too far," a nonlinearity could show up at the top end. But there is no visible evidence for a bend in the graph near either end.

(d) In the equation $F = kx$, the spring constant k is the slope of the F-versus-x graph.

$$\boxed{k = 116 \text{ N/m}}$$

(e) $F = kx = (116 \text{ N/m})(0.105 \text{ m}) = \boxed{12.2 \text{ N}}$

P7.28 (a) We find the work done by the gas on the bullet by integrating the function given:

$$W = \int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

$$W = \int_0^{0.600 \text{ m}} \left(15\,000 \text{ N} + 10\,000x \text{ N/m} - 25\,000x^2 \text{ N/m}^2\right) dx \cos 0°$$

$$W = 15\,000x + \frac{10\,000x^2}{2} - \frac{25\,000x^3}{3}\Bigg|_0^{0.600 \text{ m}}$$

$$W = 9.00 \text{ kJ} + 1.80 \text{ kJ} - 1.80 \text{ kJ} = \boxed{9.00 \text{ kJ}}$$

(b) Similarly,

$$W = (15.0 \text{ kN})(1.00 \text{ m}) + \frac{(10.0 \text{ kN/m})(1.00 \text{ m})^2}{2} - \frac{\left(25.0 \text{ kN/m}^2\right)(1.00 \text{ m})^3}{3}$$

$$W = 11.67 \text{ kJ} = \boxed{11.7 \text{ kJ}}$$

(c) $\dfrac{11.7 \text{ kJ} - 9.00 \text{ kJ}}{9.00 \text{ kJ}} \times 100\% = 29.6\%$

$\boxed{\text{The work is greater by 29.6\%.}}$

P7.29 $W = \int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_0^{5 \text{ m}} \left(4x\hat{\mathbf{i}} + 3y\hat{\mathbf{j}}\right) \text{N} \cdot dx\hat{\mathbf{i}}$

$$\int_0^{5 \text{ m}} (4 \text{ N/m})x\,dx + 0 = (4 \text{ N/m})\frac{x^2}{2}\Bigg|_0^{5 \text{ m}} = \boxed{50.0 \text{ J}}$$

P7.30 We read the coordinates of the two specified points from the graph as

$$a = (5\text{ cm}, -2\text{ N}) \text{ and } b = (25\text{ cm}, 8\text{ N})$$

We can then write u as a function of v by first finding the slope of the curve:

$$\text{slope} = \frac{u_b - u_a}{v_b - v_a} = \frac{8\text{ N} - (-2\text{ N})}{25\text{ cm} - 5\text{ cm}} = 0.5\text{ N/cm}$$

The y intercept of the curve can be found from $u = mv + b$, where $m = 0.5$ N/cm is the slope of the curve, and b is the y intercept. Plugging in point a, we obtain

$$u = mv + b$$
$$-2\text{ N} = (0.5\text{ N/cm})(5\text{ cm}) + b$$
$$b = -4.5\text{ N}$$

Then,

$$u = mv + b = (0.5\text{ N/cm})v - 4.5\text{ N}$$

(a) Integrating the function above, suppressing units, gives

$$\int_a^b u\,dv = \int_5^{25}(0.5v - 4.5)dv = \left[0.5v^2/2 - 4.5v\right]_5^{25}$$
$$= 0.25(625 - 25) - 4.5(25 - 5)$$
$$= 150 - 90 = 60\text{ N}\cdot\text{cm} = \boxed{0.600\text{ J}}$$

(b) Reversing the limits of integration just gives us the negative of the quantity:

$$\int_b^a u\,dv = \boxed{-0.600\text{ J}}$$

(c) This is an entirely different integral. It is larger because all of the area to be counted up is positive (to the right of $v = 0$) instead of partly negative (below $u = 0$).

$$\int_a^b v\,du = \int_{-2}^{8}(2u + 9)du = \left[2u^2/2 + 9u\right]_{-2}^{8}$$
$$= 64 - (-2)^2 + 9(8 + 2)$$
$$= 60 + 90 = 150\text{ N}\cdot\text{cm} = \boxed{1.50\text{ J}}$$

Section 7.5 Kinetic Energy and the Work-Kinetic Energy Theorem

P7.31 $\vec{\mathbf{v}}_i = (6.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}})$ m/s^2

(a) $v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{37.0}$ m/s

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3.00 \text{ kg})(37.0 \text{ m}^2/\text{s}^2) = \boxed{55.5 \text{ J}}$$

(b) $\vec{\mathbf{v}}_f = 8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$

$$v_f^2 = \vec{\mathbf{v}}_f \cdot \vec{\mathbf{v}}_f = 64.0 + 16.0 = 80.0 \text{ m}^2/\text{s}^2$$

$$\Delta K = K_f - K_i = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{3.00}{2}(80.0) - 55.5 = \boxed{64.5 \text{ J}}$$

P7.32 (a) Since the applied force is horizontal, it is in the direction of the displacement, giving $\theta = 0°$. The work done by this force is then

$$W_{F_0} = (F_0 \cos\theta)\Delta x = F_0(\cos 0)\Delta x = F_0 \Delta x$$

and

$$F_0 = \frac{W_{F_0}}{\Delta x} = \frac{350 \text{ J}}{12.0 \text{ m}} = \boxed{29.2 \text{ N}}$$

(b) If the applied force is greater than 29.2 N, the crate would accelerate in the direction of the force, so its $\boxed{\text{speed would increase}}$ with time.

(c) If the applied force is less than 29.2 N, the $\boxed{\text{crate would slow down and come to rest.}}$

P7.33 (a) $K_A = \frac{1}{2}(0.600 \text{ kg})(2.00 \text{ m/s})^2 = \boxed{1.20 \text{ J}}$

(b) $\frac{1}{2}mv_B^2 = K_B$: $v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{(2)(7.50 \text{ J})}{0.600 \text{ kg}}} = \boxed{5.00 \text{ m/s}}$

(c) $\sum W = \Delta K = K_B - K_A = \frac{1}{2}m(v_B^2 - v_A^2) = 7.50 \text{ J} - 1.20 \text{ J} = \boxed{6.30 \text{ J}}$

P7.34 (a) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 5.00$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.50\text{ J})}{4.00\text{ kg}}} = \boxed{1.94\text{ m/s}}$$

(b) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 10.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(22.5\text{ J})}{4.00\text{ kg}}} = \boxed{3.35\text{ m/s}}$$

(c) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 15.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(30.0\text{ J})}{4.00\text{ kg}}} = \boxed{3.87\text{ m/s}}$$

P7.35 Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall. Let $d = 5.00$ m represent the distance over which the driver falls freely, and $h = 0.12$ the distance it moves the piling.

$$\sum W = \Delta K: \quad W_{\text{gravity}} + W_{\text{beam}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

so $$(mg)(h+d)\cos 0° + (\bar{F})(d)\cos 180° = 0 - 0$$

Thus,

$$\bar{F} = \frac{(mg)(h+d)}{d} = \frac{(2\,100\text{ kg})(9.80\text{ m/s}^2)(5.12\text{ m})}{0.120\text{ m}}$$

$$= \boxed{8.78\times 10^5\text{ N}}$$

The force on the pile driver is $\boxed{\text{upward.}}$

P7.36 (a) $v_f = 0.096(3.00\times 10^8\text{ m/s}) = 2.88\times 10^7\text{ m/s}$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(9.11\times 10^{-31}\text{ kg})(2.88\times 10^7\text{ m/s})^2 = \boxed{3.78\times 10^{-16}\text{ J}}$$

(b) $K_i + W = K_f : 0 + F\Delta r\cos\theta = K_f$

$$F(0.028\text{ m})\cos 0° = 3.78\times 10^{-16}\text{ J}$$

$$F = \boxed{1.35\times 10^{-14}\text{ N}}$$

(c) $\sum F = ma$: $\quad a = \dfrac{\sum F}{m} = \dfrac{1.35\times10^{-14}\text{ N}}{9.11\times10^{-31}\text{ kg}} = \boxed{1.48\times10^{+16}\text{ m/s}^2}$

(d) $v_{xf} = v_{xi} + a_x t$: $\quad 2.88\times10^{7}\text{ m/s} = 0 + \left(1.48\times10^{16}\text{ m/s}^2\right)t$

$t = \boxed{1.94\times10^{-9}\text{ s}}$

P7.37 (a) $K_i + \sum W = K_f = \dfrac{1}{2}mv_f^2$

$$0 + \sum W = \frac{1}{2}(15.0\times10^{-3}\text{ kg})(780\text{ m/s})^2 = \boxed{4.56\text{ kJ}}$$

(b) As shown in part (a), the net work performed on the bullet is $\boxed{4.56\text{ kJ}.}$

(c) $F = \dfrac{W}{\Delta r\cos\theta} = \dfrac{4.56\times10^{3}\text{ J}}{(0.720\text{ m})\cos0°} = \boxed{6.34\text{ kN}}$

(d) $a = \dfrac{v_f^2 - v_i^2}{2x_f} = \dfrac{(780\text{ m/s})^2 - 0}{2(0.720\text{ m})} = \boxed{422\text{ km/s}^2}$

(e) $\sum F = ma = (15\times10^{-3}\text{ kg})(422\times10^{3}\text{ m/s}^2) = \boxed{6.34\text{ kN}}$

(f) $\boxed{\text{The forces are the same. The two theories agree.}}$

P7.38 (a) As the bullet moves the hero's hand, work is done on the bullet to decrease its kinetic energy. The average force is opposite to the displacement of the bullet:

$$W_{net} = F_{avg}\Delta x\cos\theta = -F_{avg}\Delta x = \Delta K$$

$$F_{avg} = \frac{\Delta K}{-\Delta x} = \frac{0 - \frac{1}{2}\left(7.80\times10^{-3}\text{ kg}\right)\left(575\text{ m/s}\right)^2}{-0.055\ 0\text{ m}}$$

$$\boxed{F_{avg} = 2.34\times10^{4}\text{ N, opposite to the direction of motion}}$$

(b) If the average force is constant, the bullet will have a constant acceleration and its average velocity while stopping is $\bar{v} = (v_f + v_i)/2$. The time required to stop is then

$$\Delta t = \frac{\Delta x}{\bar{v}} = \frac{2(\Delta x)}{v_f + v_i} = \frac{2(5.50\times10^{-2}\text{ m})}{0 + 575\text{ m/s}} = \boxed{1.91\times10^{-4}\text{ s}}$$

P7.39 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}m(v_x{}^2 + v_y{}^2)$

$$= \frac{1}{2}(5.75\text{ kg})[(5.00\text{ m/s})^2 + (-3.00\text{ m/s})^2] = \boxed{97.8\text{ J}}$$

(b) We know $F_x = ma_x$ and $F_y = ma_y$. At $t = 0$, $x_i = y_i = 0$, and $v_{xi} = 5.00$ m/s, $v_{yi} = -3.00$ m/s; at $t = 2.00$ s, $x_f = 8.50$ m, $y_f = 5.00$ m.

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$a_x = \frac{2(x_f - x_i - v_{xi}t)}{t^2} = \frac{2[8.50\text{ m} - 0 - (5.00\text{ m/s})(2.00\text{ s})]}{(2.00\text{ s})^2}$$

$$= -0.75\text{ m/s}^2$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$a_y = \frac{2(y_f - y_i - v_{yi}t)}{t^2} = \frac{2[5.00\text{ m} - 0 - (-3.00\text{ m/s})(2.00\text{ s})]}{(2.00\text{ s})^2}$$

$$= 5.50\text{ m/s}^2$$

$$F_x = ma_x = (5.75\text{ kg})(-0.75\text{ m/s}^2) = -4.31\text{ N}$$

$$F_y = ma_y = (5.75\text{ kg})(5.50\text{ m/s}^2) = 31.6\text{ N}$$

$$\boxed{\vec{\mathbf{F}} = \left(-4.31\hat{\mathbf{i}} + 31.6\hat{\mathbf{j}}\right)\text{ N}}$$

(c) We can obtain the particle's speed at $t = 2.00$ s from the particle under constant acceleration model, or from the nonisolated system model. From the former,

$$v_{xf} = v_{xi} + a_x t = (5.00\text{ m/s}) + \left(-0.75\text{ m/s}^2\right)(2.00\text{ s}) = 3.50\text{ m/s}$$

$$v_{xf} = v_{yi} + a_y t = (-3.00\text{ m/s}) + \left(5.50\text{ m/s}^2\right)(2.00\text{ s}) = 8.00\text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(3.50\text{ m/s})^2 + (8.00\text{ m/s})^2} = \boxed{8.73\text{ m/s}}$$

From the nonisolated system model,

$$\sum W = \Delta K: \quad W_{ext} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The work done by the force is given by

$$W_{ext} = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = F_x\Delta r_x + F_y\Delta r_y$$

$$= (-4.31\text{ N})(8.50\text{ m}) + (31.6\text{ N})(5.00\text{ m}) = 121\text{ J}$$

then,

$$\frac{1}{2}mv_f^2 = W_{ext} + \frac{1}{2}mv_i^2 = 121\text{ J} + 97.8\text{ J} = 219\text{ J}$$

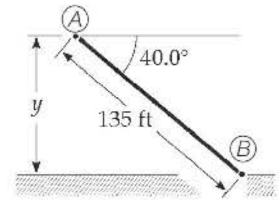
which gives

$$v_f = \sqrt{\frac{2(219\text{ J})}{5.75\text{ kg}}} = \boxed{8.73\text{ m/s}}$$

Section 7.6 Potential Energy of a System

P7.40 (a) With our choice for the zero level for potential energy of the car-Earth system when the car is at point Ⓑ,

$$\boxed{U_B = 0}$$



ANS. FIG. P7.40

When the car is at point Ⓐ, the potential energy of the car-Earth system is given by

$$U_A = mgy$$

where y is the vertical height above zero level. With 135 ft = 41.1 m, this height is found as:

$$y = (41.1\text{ m})\sin 40.0° = 26.4\text{ m}$$

Thus,

$$U_A = (1\,000\text{ kg})(9.80\text{ m/s}^2)(26.4\text{ m}) = \boxed{2.59 \times 10^5\text{ J}}$$

The change in potential energy of the car-Earth system as the car moves from Ⓐ to Ⓑ is

$$U_B - U_A = 0 - 2.59 \times 10^5\text{ J} = \boxed{-2.59 \times 10^5\text{ J}}$$

(b) With our choice of the zero configuration for the potential energy of the car-Earth system when the car is at point Ⓐ, we have $\boxed{U_A = 0}$. The potential energy of the system when the car is at point Ⓑ is given by $U_B = mgy$, where y is the vertical distance of point Ⓑ below point Ⓐ. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number.

Thus,

$$U_B = (1\,000 \text{ kg})\left(9.80 \text{ m/s}^2\right)(-26.5 \text{ m}) = \boxed{-2.59\times10^5 \text{ J}}$$

The change in potential energy when the car moves from Ⓐ to Ⓑ is

$$U_B - U_A = -2.59\times10^5 \text{ J} - 0 = \boxed{-2.59\times10^5 \text{ J}}$$

P7.41 Use $U = mgy$, where y is measured relative to a reference level. Here, we measure y to be relative to the top edge of the well, where we take $y = 0$.

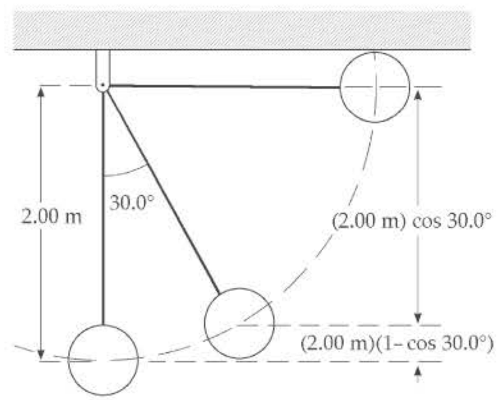
(a) $y = 1.3$ m: $U = mgy = (0.20 \text{ kg})(9.80 \text{ m/s}^2)(1.3 \text{ m}) = \boxed{+2.5 \text{ J}}$

(b) $y = -5.0$ m: $U = mgy = (0.20 \text{ kg})(9.80 \text{ m/s}^2)(-5.0 \text{ m}) = \boxed{-9.8 \text{ J}}$

(c) $\Delta U = U_f - U_i = (-9.8 \text{ J}) - (2.5 \text{ J}) = -12.3 = \boxed{-12 \text{ J}}$

P7.42 (a) We take the zero configuration of system potential energy with the child at the lowest point of the arc. When the swing is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,

$$\begin{aligned} U_g &= mgy \\ &= (400 \text{ N})(2.00 \text{ m}) \\ &= \boxed{800 \text{ J}} \end{aligned}$$



ANS. FIG. P7.42

(b) From the sketch, we see that at an angle of 30.0° the child is at a vertical height of (2.00 m) (1 – cos 30.0°) above the lowest point of the arc. Thus,

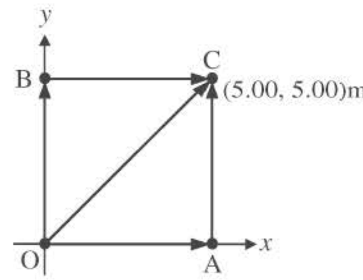
$$U_g = mgy = (400 \text{ N})(2.00 \text{ m})(1 - \cos 30.0°) = \boxed{107 \text{ J}}$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.

Section 7.7 Conservative and Nonconservative Forces

P7.43 The gravitational force is downward:

$$F_g = mg = (4.00\text{ kg})(9.80\text{ m/s}^2) = 39.2\text{ N}$$



ANS. FIG. P7.43

(a) Work along OAC = work along OA + work along AC

$$= F_g(\text{OA})\cos 90.0° + F_g(\text{AC})\cos 180°$$
$$= (39.2\text{ N})(5.00\text{ m})(0) + (39.2\text{ N})(5.00\text{ m})(-1)$$
$$= \boxed{-196\text{ J}}$$

(b) W along OBC = W along OB + W along BC

$$= (39.2\text{ N})(5.00\text{ m})\cos 180° + (39.2\text{ N})(5.00\text{ m})\cos 90.0°$$
$$= \boxed{-196\text{ J}}$$

(c) Work along OC = F_g(OC) cos 135°

$$= (39.2\text{ N})\left(5.00\times\sqrt{2}\text{ m}\right)\left(-\frac{1}{\sqrt{2}}\right) = \boxed{-196\text{ J}}$$

(d) $\boxed{\text{The results should all be the same, since the gravitational force is conservative.}}$

P7.44 (a) $W = \int \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$, and if the force is constant, this can be written as

$$W = \vec{\mathbf{F}}\cdot\int d\vec{\mathbf{r}} = \boxed{\vec{\mathbf{F}}\cdot\left(\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i\right)\text{, which depends only on the end points,}}$$

$\boxed{\text{and not on the path}}$.

(b) $W = \int \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = \int\left(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\right)\cdot\left(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}\right)$

$$= (3.00\text{ N})\int_0^{5.00\text{ m}} dx + (4.00\text{ N})\int_0^{5.00\text{ m}} dy$$

$$W = (3.00\text{ N})x\Big|_0^{5.00\text{ m}} + (4.00\text{ N})y\Big|_0^{5.00\text{ m}} = 15.0\text{ J} + 20.0\text{ J} = \boxed{35.0\text{ J}}$$

The same calculation applies for all paths.

P7.45 In the following integrals, remember that

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}=\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}=1 \quad \text{and} \quad \hat{\mathbf{i}}\cdot\hat{\mathbf{j}}=0$$

(a) The work done on the particle in its first section of motion is

$$W_{OA}=\int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} 2y\,dx$$

and since along this path, $y=0$, that means $W_{OA}=0$.

In the next part of its path,

$$W_{AC}=\int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} x^2\,dy$$

For $x=5.00$ m, $W_{AC}=125$ J

and $W_{OAC}=0+125=\boxed{125\text{ J}}$.

(b) Following the same steps,

$$W_{OB}=\int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} x^2\,dy$$

Since along this path, $x=0$, that means $W_{OB}=0$.

$$W_{BC}=\int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} 2y\,dx$$

Since $y=5.00$ m, $W_{BC}=50.0$ J.

$$W_{OAC}=0+125=\boxed{125\text{ J}}$$

(c) $W_{OC}=\int\left(dx\hat{\mathbf{i}}+dy\hat{\mathbf{j}}\right)\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int\left(2y\,dx+x^2dy\right)$

Since $x=y$ along OC, $W_{OC}=\int_0^{5.00\text{ m}}\left(2x+x^2\right)dx=\boxed{66.7\text{ J}}$

(d) $\boxed{\text{F is nonconservative.}}$

(e) $\boxed{\text{The work done on the particle depends on the path followed by the particle.}}$

P7.46 Along each step of motion, to overcome friction you must push with a force of 3.00 N in the direction of travel along the path, so in the expression for work, $\cos\theta=\cos 0°=1$.

(a) $W=(3.00\text{ N})(5.00\text{ m})(1)+(3.00\text{ N})(5.00\text{ m})(1)=\boxed{30.0\text{ J}}$

(b) The distance CO is $(5.00^2 + 5.00^2)^{1/2}$ m = 7.07 m

$$W = (3.00\text{ N})(5.00\text{ m})(1) + (3.00\text{ N})(5.00\text{ m})(1) + (3.00\text{ N})(7.07\text{ m})(1) = \boxed{51.2\text{ J}}$$

(c) $W = (3.00\text{ N})(7.07\text{ m})(1) + (3.00\text{ N})(7.07\text{ m})(1) = \boxed{42.4\text{ J}}$

(d) Friction is a nonconservative force.

Section 7.8 Relationship Between Conservative Forces and Potential Energy

P7.47 We use the relation of force to potential energy as the force is the negative derivative of the potential energy with respect to distance:

$$U(r) = \frac{A}{r}$$

$$F_r = -\frac{\partial U}{\partial r} = -\frac{d}{dr}\left(\frac{A}{r}\right) = \boxed{\frac{A}{r^2}}$$

If *A* is positive, the positive value of radial force indicates a force of repulsion.

P7.48 We need to be very careful in identifying internal and external work on the book-Earth system. The first 20.0 J, done by the librarian on the system, is external work, so the system now contains an additional 20.0 J compared to the initial configuration. When the book falls and the system returns to the initial configuration, the 20.0 J of work done by the gravitational force from the Earth is *internal* work. This work only transforms the gravitational potential energy of the system to kinetic energy. It does *not* add more energy to the system. Therefore, the book hits the ground with 20.0 J of kinetic energy. The book-Earth system now has zero gravitational potential energy, for a total energy of 20.0 J, which is the energy put into the system by the librarian.

P7.49
$$F_x = -\frac{\partial U}{\partial x} = -\frac{\partial(3x^3y - 7x)}{\partial x} = -(9x^2y - 7) = 7 - 9x^2y$$

$$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial(3x^3y - 7x)}{\partial y} = -(3x^3 - 0) = -3x^3$$

Thus, the force acting at the point (x, y) is

$$\vec{F} = F_x\hat{i} + F_y\hat{j} = \boxed{(7 - 9x^2y)\hat{i} - 3x^3\hat{j}}$$

P7.50 (a) We use Equation 7.27 relating the potential energy of the system to the conservative force acting on the particle, with $U_i = 0$:

$$U = U_f - U_i = U_f - 0$$

$$= -\int_0^x \left(-Ax + Bx^2\right)dx = A\frac{x^2}{2} - B\frac{x^3}{3}\bigg|_0^x = \boxed{\frac{Ax^2}{2} - \frac{Bx^3}{3}}$$

(b) From (a), $U(2.00\text{ m}) = 2A - 2.67B$, and $U(3.00\text{ m}) = 4.5A - 9B$.

$$\boxed{\Delta U = (4.5A - 9B) - (2A - 2.67B) = 2.5A - 6.33B}$$

(c) If we consider the particle alone as a system, the change in its kinetic energy is the work done by the force on the particle: $W = \Delta K$. For the entire system of which this particle is a member, this work is internal work and equal to the negative of the change in potential energy of the system:

$$\boxed{\Delta K = -\Delta U = -2.5A + 6.33B}$$

P7.51 (a) For a particle moving along the x axis, the definition of work by a variable force is

$$W_F = \int_{x_i}^{x_f} F_x dx$$

Here $F_x = (2x + 4)$ N, $x_i = 1.00$ m, and $x_f = 5.00$ m.

So

$$W_F = \int_{1.00\text{ m}}^{5.00\text{ m}} (2x + 4)dx\ \text{N}\cdot\text{m} = x^2 + 4x\Big]_{1.00\text{ m}}^{5.00\text{ m}}\ \text{N}\cdot\text{m}$$

$$= \left(5^2 + 20 - 1 - 4\right)\text{ J} = \boxed{40.0\text{ J}}$$

(b) The change in potential energy of the system is the negative of the internal work done by the conservative force on the particle: $\Delta U = -W_{int} = \boxed{-40.0\text{ J}}$

(c) From $\Delta K = K_f - \dfrac{mv_1^2}{2}$, we obtain

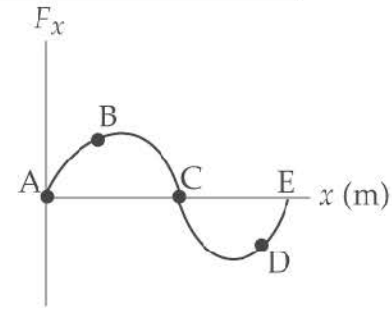
$$K_f = \Delta K + \frac{mv_1^2}{2} = 40.0\text{ J} + \frac{(5.00\text{ kg})(3.00\text{ m/s})^2}{2} = \boxed{62.5\text{ J}}$$

Section 7.9 Energy Diagrams and Equilibrium of a System

P7.52 (a) $\boxed{F_x \text{ is zero at points A, C, and E; } F_x \text{ is positive at point B and negative at point D.}}$

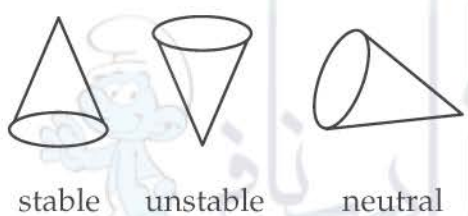
(b) $\boxed{\text{A and E are unstable, and C is stable.}}$

(c) ANS. FIG. P7.52 shows the curve for F_x vs. x for this problem.



ANS. FIG. P7.52

P7.53 The figure below shows the three equilibrium configurations for a right circular cone.



ANS. FIG. P7.53

Additional Problems

P7.54 (a) $$\vec{\mathbf{F}} = -\frac{d}{dx}\left(-x^3 + 2x^2 + 3x\right)\hat{\mathbf{i}}$$

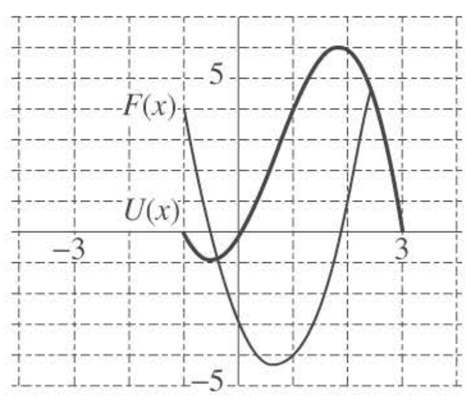
$$= \boxed{\left(3x^2 - 4x - 3\right)\hat{\mathbf{i}}}$$

(b) $F = 0$ when

$x = \boxed{1.87 \text{ and } -0.535}$.

(c) The stable point is at $x = -0.535$, point of minimum $U(x)$.

The unstable point is at $x = 1.87$, maximum in $U(x)$.



ANS. FIG. P7.54

P7.55 Initially, the ball's velocity is

$$\vec{\mathbf{v}} = (40.0 \text{ m/s}) \cos 30.0°\hat{\mathbf{i}} + (40.0 \text{ m/s}) \sin 30.0°\hat{\mathbf{j}}$$

At its apex, the ball's velocity is

$$\vec{\mathbf{v}} = (40.0 \text{ m/s}) \cos 30.0°\hat{\mathbf{i}} + 0\hat{\mathbf{j}} = (34.6 \text{ m/s})\hat{\mathbf{i}}$$

The ball's kinetic energy of the ball at this point is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.150 \text{ kg})(34.6 \text{ m/s})^2 = \boxed{90.0 \text{ J}}$$

P7.56 We evaluate $\int_{12.8}^{23.7} \frac{375dx}{x^3 + 3.75x}$ by calculating

$$\frac{375(0.100)}{(12.8)^3 + 3.75(12.8)} + \frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \cdots \frac{375(0.100)}{(23.6)^3 + 3.75(23.6)} = 0.806$$

and

$$\frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \frac{375(0.100)}{(13.0)^3 + 3.75(13.0)} + \cdots \frac{375(0.100)}{(23.7)^3 + 3.75(23.7)} = 0.791$$

The answer must be between these two values. We may find it more precisely by using a value for Δx smaller than 0.100. Thus, we find the integral to be $\boxed{0.799 \text{ N} \cdot \text{m}}$.

P7.57 (a) The equivalent spring constant for the stel balls is

$$k = \frac{|F|}{|x|} = \frac{16\,000 \text{ N}}{0.000\,2 \text{ m}} = \boxed{8 \times 10^7 \text{ N/m}}$$

(b) $\boxed{\text{A time interval}}$. If the interaction occupied no time, the force exerted by each ball on the other could be infinite, and that cannot happen.

(c) We assume that steel has the density of its main constituent, iron, shown in Table 14.1. Then its mass is

$$\rho V = \rho\left(\frac{4}{3}\right)\pi r^3 = \left(\frac{4\pi}{3}\right)(7\ 860 \text{ kg/m}^3)(0.025\ 4 \text{ m/2})^3$$
$$= 0.067\ 4 \text{ kg}$$

its kinetic energy is then

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.067\ 4\ \text{kg})(5\ \text{m/s})^2 = \boxed{0.8\ \text{J}}$$

(d) Imagine one ball running into an infinitely hard wall and bouncing off elastically. The original kinetic energy becomes elastic potential energy

$$0.843\ \text{J} = (1/2)(8\times 10^7\ \text{N/m})\,x^2 \qquad x = 0.145\ \text{mm} \approx \boxed{0.15\,\text{mm}}$$

(e) The ball does not really stop with constant acceleration, but imagine it moving 0.145 mm forward with average speed $(5\ \text{m/s} + 0)/2 = 2.5\ \text{m/s}$. The time interval over which it stops is then

$$0.145\ \text{mm}/(2.5\ \text{m/s}) = 6\times 10^{-5}\ \text{s} \approx \boxed{10^{-4}\ \text{s}}$$

P7.58 The work done by the applied force is

$$W = \int_i^f F_{\text{applied}}\,dx = \int_0^{x_{\max}} -\left[-\left(k_1x + k_2x^2\right)\right]dx$$

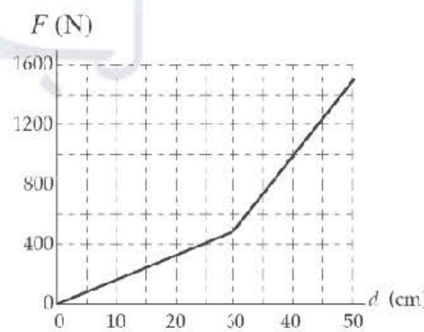
$$= \int_0^{x_{\max}} k_1x\,dx + \int_0^{x_{\max}} k_2x^2\,dx = k_1\left.\frac{x^2}{2}\right|_0^{x_{\max}} + k_2\left.\frac{x^3}{3}\right|_0^{x_{\max}}$$

$$= \boxed{k_1\frac{x_{\max}^2}{2} + k_2\frac{x_{\max}^3}{3}}$$

P7.59 Compare an initial picture of the rolling car with a final picture with both springs compressed. From conservation of energy, we have

$$K_i + \sum W = K_f$$

Work by both springs changes the car's kinetic energy.



ANS. FIG. P7.59

$$K_i + \frac{1}{2}k_1\left(x_{1i}^2 - x_{1f}^2\right) + \frac{1}{2}k_2\left(x_{2i}^2 - x_{2f}^2\right) = K_f$$

Substituting,

$$\frac{1}{2}mv_i^2 + 0 - \frac{1}{2}(1\ 600\ \text{N/m})(0.500\ \text{m})^2 + 0 - \frac{1}{2}(3\ 400\ \text{N/m})(0.200\ \text{m})^2 = 0$$

Which gives

$$\frac{1}{2}(6\,000\text{ kg})v_i^2 - 200\text{ J} - 68.0\text{ J} = 0$$

Solving for v_i,

$$v_i = \sqrt{\frac{2(268\text{ J})}{6\,000\text{ kg}}} = \boxed{0.299\text{ m/s}}$$

P7.60 Apply the work-energy theorem to the ball. The spring is initially compressed by $x_{sp,i} = d = 5.00$ cm. After the ball is released from rest, the spring pushes the ball up the incline the distance d, doing positive work on the ball, and gravity does negative work on the ball as it travels up the incline a distance Δx from its starting point. Solve for Δx.

$$K_i + W_s + W_g = K_f$$

$$\frac{1}{2}mv_i^2 + \left(\frac{1}{2}kx_{sp,i}^2 - \frac{1}{2}kx_{sp,f}^2\right) - mg\Delta x\sin\theta = \frac{1}{2}mv_f^2$$

$$0 + \left(\frac{1}{2}kd^2 - 0\right) - mg\Delta x\sin 10.0° = 0$$

$$\Delta x = \frac{kd^2}{2mg\sin 10.0°} = \frac{(1.20\text{ N/cm})(5.00\text{ cm})(0.0500\text{ m})}{2(0.100\text{ kg})\left(9.80\text{ m/s}^2\right)\sin 10.0°}$$
$$= 0.881\text{ m}$$

Thus, the ball travels up the incline a distance of 0.881 m after it is released.

Applying the work-kinetic energy theorem to the ball, one finds that it momentarily comes to rest at a distance up the incline of only 0.881 m. This distance is much smaller than the height of a professional basketball player, so the ball will not reach the upper end of the incline to be put into play in the machine. The ball will simply stop momentarily and roll back to the spring; not an exciting entertainment for any casino visitor!

P7.61 (a) $\vec{F}_1 = (25.0\text{ N})\left(\cos 35.0°\hat{i} + \sin 35.0°\hat{j}\right) = \boxed{\left(20.5\hat{i} + 14.3\hat{j}\right)\text{ N}}$

$\vec{F}_2 = (42.0\text{ N})\left(\cos 150°\hat{i} + \sin 150°\hat{j}\right) = \boxed{\left(-36.4\hat{i} + 21.0\hat{j}\right)\text{ N}}$

(b) $\sum\vec{F} = \vec{F}_1 + \vec{F}_2 = \boxed{\left(-15.9\hat{i} + 35.3\hat{j}\right)\text{ N}}$

(c) $\vec{a} = \dfrac{\sum \vec{F}}{m} = \boxed{\left(-3.18\hat{i} + 7.07\hat{j}\right) \text{ m/s}^2}$

(d) $\vec{v}_f = \vec{v}_i + \vec{a}t = \left(4.00\hat{i} + 2.50\hat{j}\right) \text{ m/s} + \left(-3.18\hat{i} + 7.07\hat{j}\right)\left(\text{m/s}^2\right)(3.00 \text{ s})$

$$\vec{v}_f = \boxed{\left(-5.54\hat{i} + 23.7\hat{j}\right) \text{ m/s}}$$

(e) $\vec{r}_f = \vec{r}_i + \vec{v}_i t + \dfrac{1}{2}\vec{a}t^2$

$$\vec{r}_f = 0 + \left(4.00\hat{i} + 2.50\hat{j}\right)(\text{m/s})(3.00 \text{ s}) + \frac{1}{2}\left(-3.18\hat{i} + 7.07\hat{j}\right)\left(\text{m/s}^2\right)(3.00 \text{ s})^2$$

$$\Delta\vec{r} = \vec{r}_f = \boxed{\left(-2.30\hat{i} + 39.3\hat{j}\right) \text{ m}}$$

(f) $K_f = \dfrac{1}{2}mv_f^2 = \dfrac{1}{2}(5.00 \text{ kg})\left[(5.54)^2 + (23.7)^2\right]\left(\text{m/s}^2\right) = \boxed{1.48 \text{ kJ}}$

(g) $K_f = \dfrac{1}{2}mv_i^2 + \sum \vec{F} \cdot \Delta\vec{r}$

$$K_f = \frac{1}{2}(5.00 \text{ kg})\left[(4.00)^2 + (2.50)^2\right](\text{m/s})^2 + \left[(-15.9 \text{ N})(-2.30 \text{ m}) + (35.3 \text{ N})(39.3 \text{ m})\right]$$

$$K_f = 55.6 \text{ J} + 1\,426 \text{ J} = \boxed{1.48 \text{ kJ}}$$

(h) The work-kinetic energy theorem is consistent with Newton's second law, used in deriving it.

P7.62 (a) We write

$$F = ax^b$$

$$1\,000 \text{ N} = a(0.129 \text{ m})^b$$

$$5\,000 \text{ N} = a(0.315 \text{ m})^b$$

Dividing the two equations gives

$$5 = \left(\frac{0.315}{0.129}\right)^b = 2.44^b$$

$$\ln 5 = b \ln 2.44$$

$$b = \frac{\ln 5}{\ln 2.44} = \boxed{1.80}$$

$$a = \frac{1\,000\text{ N}}{(0.129\text{ m})^{1.80}} = \boxed{4.01 \times 10^4\text{ N/m}^{1.8}}$$

(b) $$W = \int_i^f F_{\text{applied}}\,dx = \int_0^x ax^b\,dx = \left.\frac{ax^{b+1}}{b+1}\right|_0^x = \frac{ax^{b+1}}{b+1} - 0 = \frac{ax^{b+1}}{b+1}$$

$$W = \frac{(4.01 \times 10^4\text{ N/m}^{1.8})x^{2.8}}{2.80}$$

For $x = 0.250$ m,

$$W = \frac{(4.01 \times 10^4\text{ N/m}^{1.8})(0.250\text{ m})^{2.8}}{2.80}$$

$$= \frac{(4.01 \times 10^4\text{ N/m}^{1.8})(0.250)^{2.8}(\text{m}^{2.8})}{2.80}$$

$$W = \frac{(4.01 \times 10^4\text{ N}\cdot\text{m})(0.250)^{2.8}}{2.80} = \boxed{295\text{ J}}$$

P7.63 The component of the weight force parallel to the incline, $mg\sin\theta$, accelerates the block down the incline through a distance d until it encounters the spring, after which the spring force, pushing up the incline, opposes the weight force and slows the block through a distance x until the block eventually is brought to a momentary stop. The weight force does positive work on the block as it slides down the incline through total distance $(d + x)$, and the spring force does negative work on the block as it slides through distance x. The normal force does no work. Applying the work-energy theorem,

$$K_i + W_g + W_s = K_f$$

$$\frac{1}{2}mv_i^2 + mg\sin\theta(d + x) + \left(\frac{1}{2}kx_{sp,i}^2 - \frac{1}{2}kx_{sp,f}^2\right) = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}mv^2 + mg\sin\theta(d + x) + \left(0 - \frac{1}{2}kx^2\right) = 0$$

Dividing by m, we have

$$\frac{1}{2}v^2 + g\sin\theta(d + x) - \frac{k}{2m}x^2 = 0 \quad\rightarrow$$

$$\frac{k}{2m}x^2 - (g\sin\theta)x - \left[\frac{v^2}{2} + (g\sin\theta)d\right] = 0$$

Solving for x, we have

$$x = \frac{g\sin\theta \pm \sqrt{(g\sin\theta)^2 - 4\left(\frac{k}{2m}\right)\left[-\left(\frac{v^2}{2} + (g\sin\theta)d\right)\right]}}{2\left(\frac{k}{2m}\right)}$$

$$x = \frac{g\sin\theta \pm \sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}$$

Because distance x must be positive,

$$x = \frac{g\sin\theta + \sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}$$

For $v = 0.750$ m/s, $k = 500$ N/m, $m = 2.50$ kg, $\theta = 20.0°$, and $g =$ 9.80 m/s^2, we have $g\sin\theta = (9.80\text{ m/s}^2)\sin 20.0° = 3.35\text{ m/s}^2$ and k/m $= (500\text{ N/m})/(2.50\text{ kg}) = 200\text{ N/m}\cdot\text{kg}$. Suppressing units, we have

$$x = \frac{3.35 + \sqrt{(3.35)^2 + (200)\left[(0.750)^2 + 2(3.35)(0.300)\right]}}{200}$$
$$= \boxed{0.131\text{ m}}$$

P7.64 The component of the weight force parallel to the incline, $mg\sin\theta$, accelerates the block down the incline through a distance d until it encounters the spring, after which the spring force, pushing up the incline, opposes the weight force and slows the block through a distance x until the block eventually is brought to a momentary stop. The weight force does positive work on the block as it slides down the incline through total distance $(d + x)$, and the spring force does negative work on the block as it slides through distance x. The normal force does no work.

Applying the work-energy theorem,

$$K_i + W_g + W_s = K_f$$

$$\frac{1}{2}mv_i^2 + mg\sin\theta(d + x) + \left(\frac{1}{2}kx_{sp,i}^2 - \frac{1}{2}kx_{sp,f}^2\right) = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}mv^2 + mg\sin\theta(d + x) + \left(0 - \frac{1}{2}kx^2\right) = 0$$

Dividing by m, we have

$$\frac{1}{2}v^2 + g\sin\theta(d+x) - \frac{k}{2m}x^2 = 0 \quad \rightarrow$$

$$\frac{k}{2m}x^2 - (g\sin\theta)x - \left[\frac{v^2}{2} + (g\sin\theta)d\right] = 0$$

Solving for x, we have

$$x = \frac{g\sin\theta \pm \sqrt{(g\sin\theta)^2 - 4\left(\frac{k}{2m}\right)\left[-\left(\frac{v^2}{2} + (g\sin\theta)d\right)\right]}}{2\left(\frac{k}{2m}\right)}$$

$$x = \frac{g\sin\theta \pm \sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}$$

Because distance x must be positive,

$$\boxed{x = \frac{g\sin\theta + \sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}}$$

P7.65 (a) The potential energy of the system at point x is given by 5 plus the negative of the work the force does as a particle feeling the force is carried from $x = 0$ to location x.

$$dU = -F dx$$

$$\int_5^U dU = -\int_0^x 8e^{-2x}\,dx$$

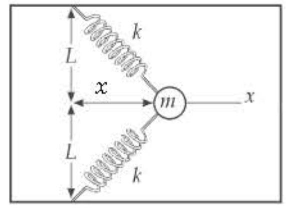
$$U - 5 = -\left(\frac{8}{[-2]}\right)\int_0^x e^{-2x}(-2\,dx)$$

$$U = 5 - \left(\frac{8}{[-2]}\right)e^{-2x}\Big|_0^x = 5 + 4e^{-2x} - 4\cdot 1 = \boxed{1 + 4e^{-2x}}$$

(b) The force must be conservative because the work the force does on the object on which it acts depends only on the original and final positions of the object, not on the path between them. There is a uniquely defined potential energy for the associated force.

Challenge Problems

P7.66 (a) The new length of each spring is $\sqrt{x^2+L^2}$, so its extension is $\sqrt{x^2+L^2}-L$ and the force it exerts is $k\left(\sqrt{x^2+L^2}-L\right)$ toward its fixed end. The y components of the two spring forces add to zero. Their x components (with $\cos\theta=\dfrac{x}{\sqrt{x^2+L^2}}$) add to



ANS. FIG. P7.66

$$\vec{F}=-2k\left(\sqrt{x^2+L^2}-L\right)\frac{x}{\sqrt{x^2+L^2}}\hat{i}$$

$$=\boxed{-2kx\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)\hat{i}}$$

(b) Choose $U=0$ at $x=0$. Then at any point the potential energy of the system is

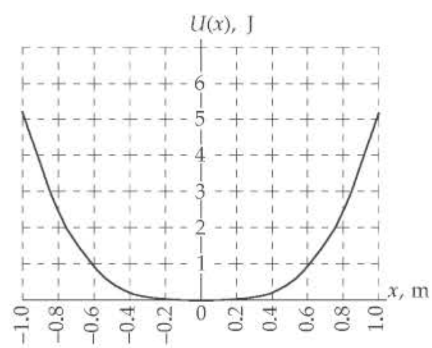
$$U(x)=-\int_0^x F_x\,dx=-\int_0^x\left(-2kx+\frac{2kLx}{\sqrt{x^2+L^2}}\right)dx$$

$$=2k\int_0^x x\,dx-2kL\int_0^x\frac{x}{\sqrt{x^2+L^2}}dx$$

$$U(x)=\boxed{kx^2+2kL\left(L-\sqrt{x^2+L^2}\right)}$$

(c) $U(x)=(40.0\text{ N/m})x^2+(96.0\text{ N})\left(1.20\text{ m}-\sqrt{x^2+1.44\text{ m}^2}\right)$

For negative x, $U(x)$ has the same value as for positive x. The only equilibrium point (i.e., where $F_x=0$) is $\boxed{x=0}$.



ANS FIG. P7.66(c)

(d) If we consider the particle alone as a system, the change in its kinetic energy is the work done by the force of the springs on the particle: $W = \Delta K$. For the entire system of particle and springs, this work is internal work and equal to the negative of the change in potential energy of the system: $\Delta K = -\Delta U$. From part (c), we evaluate U for $x = 0.500$ m:

$$\begin{aligned} U &= (40.0 \text{ N/m})(0.500 \text{ m})^2 \\ &\quad + (96.0 \text{ N})\left(1.20 \text{ m} - \sqrt{(0.500 \text{ m})^2 + 1.44 \text{ m}^2}\right) \\ &= 0.400 \text{ J} \end{aligned}$$

Now find the speed of the particle:

$$\frac{1}{2}mv^2 = -\Delta U$$

$$v = \sqrt{\frac{-2\Delta U}{m}} = \sqrt{\frac{-2}{1.18 \text{ kg}}(0 - 0.400 \text{ J})} = \boxed{0.823 \text{ m/s}}$$

P7.67 (a) We assume the spring lies in the horizontal plane of the motion, then the radius of the puck's motion is $r = L_0 + x$, where $L_0 = 0.155$ m is the unstretched length. The spring force causes the puck's centripetal acceleration:

$$F = mv^2/r \quad \to \quad kx = m(2\pi r/T)^2/r \quad \to \quad kT^2x = 4\pi^2 mr$$

Substituting $r = (L_0 + x)$, we have

$$kT^2x = 4\pi^2 m(L_0 + x)$$

$$kx = \frac{(4\pi^2 mL_0)}{T^2} + \frac{x(4\pi^2 m)}{T^2}$$

$$x\left(k - \frac{4\pi^2 m}{T^2}\right) = \frac{4\pi^2 mL_0}{T^2}$$

$$x = \frac{4\pi^2 mL_0/T^2}{k - 4\pi^2 mL_0/T^2}$$

For $k = 4.30$ N/m, $L_0 = 0.155$ m, and $T = 1.30$ s, we have

$$\begin{aligned} x &= \frac{4\pi^2 m(0.155 \text{ m})/(1.30 \text{ s})^2}{4.30 \text{ N/m} - 4\pi^2 m/(1.30 \text{ s})^2} \\ &= \frac{(3.62 \text{ m/s}^2)m}{4.30 \text{ kg/s}^2 - (23.36/\text{s}^2)m} \\ &= \frac{(3.62 \text{ m})m}{[4.30 \text{ kg} - (23.36)m]}\frac{1/\text{s}^2}{1/\text{s}^2} \end{aligned}$$

$$\boxed{x = \frac{(3.62\text{ m})m}{4.30\text{ kg} - (23.4)m}}$$

(b) For $m = 0.070$ kg,

$$x = \frac{(3.62\text{ m})[0.070\text{ kg}]}{4.30\text{ kg} - 23.36(0.070\text{ kg})} = \boxed{0.095\ 1\text{ m}}$$

(c) We double the puck mass and find

$$x = \frac{(3.6208\text{ m})[0.140\text{ kg}]}{4.30\text{ kg} - 23.360(0.140\text{ kg})} = \boxed{0.492\text{ m}}$$

more than twice as big!

(d) For $m = 0.180$ kg,

$$x = \frac{(3.62\text{ m})[0.180\text{ kg}]}{4.30\text{ kg} - 23.36(0.180\text{ kg})} = \frac{0.652}{0.0952}\text{ m} = \boxed{6.85\text{ m}}$$

We have to get a bigger table!

(e) When the denominator of the fraction goes to zero, the extension becomes infinite. This happens for 4.3 kg – 23.4 $m = 0$; that is for $m = 0.184$ kg. For any larger mass, the spring cannot constrain the motion. The situation is impossible.

(f) The extension is directly proportional to m when m is only a few grams. Then it grows faster and faster, diverging to infinity for $m = 0.184$ kg.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P7.2 (a) 3.28×10^{-2} J; (b) -3.28×10^{-2} J

P7.4 1.56×10^{4} J

P7.6 method one: -4.70×10^{3} J; method two: -4.70 kJ

P7.8 28.9

P7.10 5.33 J

P7.12 (a) 11.3°; (b) 156°; (c) 82.3°

P7.14 (a) 24.0 J; (b) -3.00 J; (c) 21.0 J

P7.16 7.37 N/m

P7.18 (a) 1.13 kN/m; (b) 0.518 m = 51.8 cm

P7.20 (a) 2.04×10^{-2} m; (b) 720 N/m

P7.22 kg/s^2

P7.24 (a) -1.23 m/s^2, 0.616 m/s^2; (b) -0.252 m/s^2 if the force of static friction is not too large, zero; (c) 0

P7.26 (a) See ANS FIG P7.26; (b) -12.0 J

P7.28 (a) 9.00 kJ; (b) 11.7 kJ; (c) The work is greater by 29.6%

P7.30 (a) 0.600 J; (b) -0.600 J; (c) 1.50 J

P7.32 (a) 29.2 N; (b) speed would increase; (c) crate would slow down and come to rest.

P7.34 (a) 1.94 m/s; (b) 3.35 m/s; (c) 3.87 m/s

P7.36 (a) 3.78×10^{-16} J; (b) 1.35×10^{-14} N; (c) $1.48 \times 10^{+16}$ m/s^2; (d) 1.94×10^{-9} s

P7.38 (a) $F_{avg} = 2.34 \times 10^{4}\ \text{N}$, opposite to the direction of motion; (b) 1.91×10^{-4} s

P7.40 (a) $U_B = 0$, 2.59×10^{5} J; (b) $U_A = 0$, -2.59×10^{5} J, -2.59×10^{5} J

P7.42 (a) 800 J; (b) 107 J; (c) $U_g = 0$

P7.44 (a) $\vec{\mathbf{F}} \cdot \left(\vec{\mathbf{r}}_{fi} - \vec{\mathbf{r}}_{i}\right)$, which depends only on end points, and not on the path; (b) 35.0 J

P7.46 (a) 30.0 J; (b) 51.2 J; (c) 42.4 J; (d) Friction is a nonconservative force

P7.48 The book hits the ground with 20.0 J of kinetic energy. The book-Earth now has zero gravitational potential energy, for a total energy of 20.0 J, which is the energy put into the system by the librarian.

P7.50 (a) $\frac{Ax^2}{2}-\frac{Bx^3}{3}$; (b) $\Delta U=(4.5A-9B)-(2A-2.67B)=2.5A-6.33B$;

(c) $\Delta K=-\Delta U=-2.5A+6.33B$

P7.52 (a) F_x is zero at points A, C, and E; F_x is positive at point B and negative at point D; (b) A and E are unstable, and C is stable; (c) See ANS FIG P7.52

P7.54 (a) $(3x^2-4x-3)\hat{\mathbf{i}}$; (b) 1.87 and −0.535; (c) See ANS. FIG. P7.54

P7.56 0.799 N · m

P7.58 $k_1\frac{x_{max}^2}{2}+k_2\frac{x_{max}^3}{3}$

P7.60 The ball will simply stop momentarily and roll back to the spring.

P7.62 (a) $b = 1.80$, $a = 4.01\times10^4\ \text{N/m}^{1.8}$; (b) 295 J

P7.64 $$x=\frac{g\sin\theta\sqrt{(g\sin\theta)^2+\left(\frac{k}{m}\right)\left[v^2+2(g\sin\theta)d\right]}}{k/m}$$

P7.66 (a) $-2kx\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)\hat{\mathbf{i}}$; (b) $kx^2+2kL\left(L-\sqrt{x^2+L^2}\right)$; (c) See ANS. FIG.

P7.66(c), $x = 0$; (d) $v = 0.823$ m/s

8

Conservation of Energy

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ8.1 Answer (a). We assume the light band of the slingshot puts equal amounts of kinetic energy into the missiles. With three times more speed, the bean has nine times more squared speed, so it must have one-ninth the mass.

OQ8.2 (i) Answer (b). Kinetic energy is proportional to mass.

(ii) Answer (c). The slide is frictionless, so $v = (2gh)^{1/2}$ in both cases.

(iii) Answer (a). g for the smaller child and $g\sin\theta$ for the larger.

OQ8.3 Answer (d). The static friction force that each glider exerts on the other acts over no distance relative to the surface of the other glider. The air track isolates the gliders from outside forces doing work. The gliders-Earth system keeps constant mechanical energy.

OQ8.4 Answer (c). Once the athlete leaves the surface of the trampoline, only a conservative force (her weight) acts on her. Therefore, the total mechanical energy of the athlete-Earth system is constant during her flight: $K_f + U_f = K_i + U_i$. Taking the $y = 0$ at the surface of the trampoline, $U_i = mgy_i = 0$. Also, her speed when she reaches maximum

height is zero, or $K_f = 0$. This leaves us with $U_f = K_i$, or $mgy_{max} = \frac{1}{2}mv_i^2$, which gives the maximum height as

$$y_{max} = \frac{v_i^2}{2g} = \frac{(8.5\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 3.7\text{ m}$$

OQ8.5 (a) Yes: a block slides on the floor where we choose $y = 0$.

(b) Yes: a picture on the classroom wall high above the floor.

(c) Yes: an eraser hurtling across the room.

(d) Yes: the block stationary on the floor.

OQ8.6 In order the ranking: c > a = d > b. We have $\frac{1}{2}mv^2 = \mu_k mgd$ so $d = v^2/2\mu_k g$. The quantity v^2/μ_k controls the skidding distance. In the cases quoted respectively, this quantity has the numerical values: (a) 5 (b) 1.25 (c) 20 (d) 5.

OQ8.7 Answer (a). We assume the climber has negligible speed at both the beginning and the end of the climb. Then $K_f = K_i$, and the work done by the muscles is

$$\begin{aligned} W_{nc} &= 0 + (U_f - U_i) = mg(y_f - y_i) \\ &= (70.0\text{ kg})(9.80\text{ m/s}^2)(325\text{ m}) \\ &= 2.23 \times 10^5\text{ J} \end{aligned}$$

The average power delivered is

$$P = \frac{W_{nc}}{\Delta t} = \frac{2.23 \times 10^5\text{ J}}{(95.0\text{ min})(60\text{ s}/1\text{ min})} = 39.1\text{ W}$$

OQ8.8 Answer (d). The energy is internal energy. Energy is never "used up." The ball finally has no elevation and no compression, so the ball-Earth system has no potential energy. There is no stove, so no energy is put in by heat. The amount of energy transferred away by sound is minuscule.

OQ8.9 Answer (c). Gravitational energy is proportional to the mass of the object in the Earth's field.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ8.1 (a) No. They will not agree on the original gravitational energy if they make different $y = 0$ choices. (b) Yes, (c) Yes. They see the same change in elevation and the same speed, so they do agree on the change in gravitational energy and on the kinetic energy.

CQ8.2 The larger engine is unnecessary. Consider a 30-minute commute. If you travel the same speed in each car, it will take the same amount of time, expending the same amount of energy. The extra power available from the larger engine isn't used.

CQ8.3 Unless an object is cooled to absolute zero, then that object will have internal energy, as temperature is a measure of the energy content of matter. Potential energy is not measured for single objects, but for systems. For example, a system comprised of a ball and the Earth will have potential energy, but the ball itself can never be said to have potential energy. An object can have zero kinetic energy, but this measurement is dependent on the reference frame of the observer.

CQ8.4 All the energy is supplied by foodstuffs that gained their energy from the Sun.

CQ8.5 (a) The total energy of the ball-Earth system is conserved. Since the system initially has gravitational energy mgh and no kinetic energy, the ball will again have zero kinetic energy when it returns to its original position. Air resistance will cause the ball to come back to a point slightly below its initial position. (b) If she gives a forward push to the ball from its starting position, the ball will have the same kinetic energy, and therefore the same speed, at its return: the demonstrator will have to duck.

CQ8.6 Yes, if it is exerted by an object that is moving in our frame of reference. The flat bed of a truck exerts a static friction force to start a pumpkin moving forward as it slowly starts up.

CQ8.7
(a) original elastic potential energy into final kinetic energy
(b) original chemical energy into final internal energy
(c) original chemical potential energy in the batteries into final internal energy, plus a tiny bit of outgoing energy transmitted by mechanical waves
(d) original kinetic energy into final internal energy in the brakes
(e) energy input by heat from the lower layers of the Sun, into energy transmitted by electromagnetic radiation
(f) original chemical energy into final gravitational energy

CQ8.8 (a) (i) A campfire converts chemical energy into internal energy, within the system wood-plus-oxygen, and before energy is transferred by heat and electromagnetic radiation into the surroundings. If all the fuel burns, the process can be 100% efficient.

(ii) Chemical-energy-into-internal-energy is also the conversion as iron rusts, and it is the main conversion in mammalian metabolism.

(b) (i) An escalator motor converts electrically transmitted energy into gravitational energy. As the system we may choose motor-plus-escalator-and-riders. The efficiency could be, say 90%, but in many escalators a significant amount of internal energy is generated and leaves the system by heat.

(ii) A natural process, such as atmospheric electric current in a lightning bolt, which raises the temperature of a particular region of air so that the surrounding air buoys it up, could produce the same electricity-to-gravitational energy conversion with low efficiency.

(c) (i) A diver jumps up from a diving board, setting it vibrating temporarily. The material in the board rises in temperature slightly as the visible vibration dies down, and then the board cools off to the constant temperature of the environment. This process for the board-plus-air system can have 100% efficiency in converting the energy of vibration into energy transferred by heat. The energy of vibration is all elastic energy at instants when the board is momentarily at rest at turning points in its motion.

(ii) For a natural process, you could think of the branch of a palm tree vibrating for a while after a coconut falls from it.

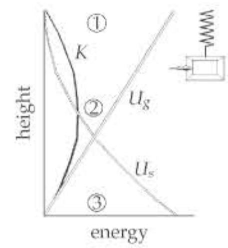
(d) (i) Some of the energy transferred by sound in a shout results in kinetic energy of a listener's eardrum; most of the mechanical-wave energy becomes internal energy as the sound is absorbed by all the surfaces it falls upon.

(ii) We would also assign low efficiency to a train of water waves doing work to shift sand back and forth in a region near a beach.

(e) (i) A demonstration solar car takes in electromagnetic-wave energy in sunlight and turns some fraction of it temporarily into the car's kinetic energy. A much larger fraction becomes internal energy in the solar cells, battery, motor, and air pushed aside.

(ii) Perhaps with somewhat higher net efficiency, the pressure of light from a newborn star pushes away gas and dust in the nebula surrounding it.

CQ8.9 The figure illustrates the relative amounts of the forms of energy in the cycle of the block, where the vertical axis shows position (height) and the horizontal axis shows energy. Let the gravitational energy (U_g) be zero for the configuration of the system when the block is at the lowest point in the motion, point (3). After the block moves downward through position (2), where its kinetic energy (K) is a maximum, its kinetic energy converts into extra elastic potential energy in the spring (U_s). After the block starts moving up at its lower turning point (3), this energy becomes both kinetic energy and gravitational potential energy, and then just gravitational energy when the block is at its greatest height (1) where its elastic potential energy is the least. The energy then turns back into kinetic and elastic potential energy as the block descends, and the cycle repeats.



ANS. FIG. CQ8.9

CQ8.10 Lift a book from a low shelf to place it on a high shelf. The net change in its kinetic energy is zero, but the book-Earth system increases in gravitational potential energy. Stretch a rubber band to encompass the ends of a ruler. It increases in elastic energy. Rub your hands together or let a pearl drift down at constant speed in a bottle of shampoo. Each system (two hands; pearl and shampoo) increases in internal energy.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 8.1 Analysis Model: Nonisolated system (Energy)

P8.1 (a) The toaster coils take in energy by electrical transmission. They increase in internal energy and put out energy by heat into the air and energy by electromagnetic radiation as they start to glow.

$$\boxed{\Delta E_{int} = Q + T_{ET} + T_{ER}}$$

(b) The car takes in energy by matter transfer. Its fund of chemical potential energy increases. As it moves, its kinetic energy increases and it puts out energy by work on the air, energy by heat in the exhaust, and a tiny bit of energy by mechanical waves in sound.

$$\boxed{\Delta K + \Delta U + \Delta E_{int} = W + Q + T_{MW} + T_{MT}}$$

(c) You take in energy by matter transfer. Your fund of chemical potential energy increases. You are always putting out energy by heat into the surrounding air.

$$\boxed{\Delta U = Q + T_{MT}}$$

(d) Your house is in steady state, keeping constant energy as it takes in energy by electrical transmission to run the clocks and, we assume, an air conditioner. It absorbs sunlight, taking in energy by electromagnetic radiation. Energy enters the house by matter transfer in the form of natural gas being piped into the home for clothes dryers, water heaters, and stoves. Matter transfer also occurs by means of leaks of air through doors and windows.

$$\boxed{0 = Q + T_{MT} + T_{ET} + T_{ER}}$$

P8.2 (a) The system of the ball and the Earth is isolated. The gravitational energy of the system decreases as the kinetic energy increases.

$$\boxed{\Delta K + \Delta U = 0}$$

$$\left(\frac{1}{2}mv^2 - 0\right) + (-mgh - 0) = 0 \rightarrow \frac{1}{2}mv^2 = mgy$$

$$\boxed{v = \sqrt{2gh}}$$

(b) The gravity force does positive work on the ball as the ball moves downward. The Earth is assumed to remain stationary, so no work is done on it.

$$\Delta K = W$$

$$\left(\frac{1}{2}mv^2 - 0\right) = mgh \rightarrow \frac{1}{2}mv^2 = mgy$$

$$\boxed{v = \sqrt{2gh}}$$

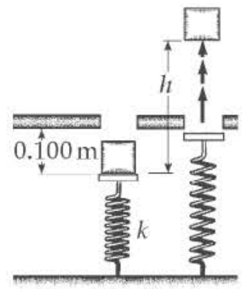
Section 8.2 Analysis Model: Isolated system (Energy)

P8.3 From conservation of energy for the block-spring-Earth system,

$$U_{gf} = U_{si}$$

or

$$(0.250\ \text{kg})(9.80\ \text{m/s}^2)h = \left(\frac{1}{2}\right)(5\,000\ \text{N/m})(0.100\ \text{m})^2$$



ANS. FIG. P8.3

This gives a maximum height, $h = \boxed{10.2\ \text{m}}$.

P8.4 (a) $\Delta K + \Delta U = 0 \rightarrow \Delta K = -\Delta U$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -(mgy_f - mgy_i)$$

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + mgy_f$$

We use the Pythagorean theorem to express the original kinetic energy in terms of the velocity components (kinetic energy itself does not have components):

$$\left(\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2\right) = \left(\frac{1}{2}mv_{xf}^2 + 0\right) + mgy_f$$

$$\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2 = \frac{1}{2}mv_{xf}^2 + mgy_f$$

Because $v_{xi} = v_{xf}$, we have

$$\frac{1}{2}mv_{yi}^2 = mgy_f \rightarrow y_f = \frac{v_{yi}^2}{2g}$$

so for the first ball:

$$y_f = \frac{v_{yi}^2}{2g} = \frac{[(1\,000\ \text{m/s})\sin 37.0°]^2}{2(9.80\ \text{m/s}^2)} = \boxed{1.85 \times 10^4\ \text{m}}$$

and for the second,

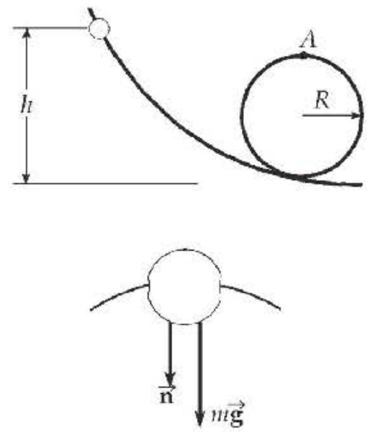
$$y_f = \frac{(1\,000\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{5.10 \times 10^4\ \text{m}}$$

(b) The total energy of each ball-Earth system is constant with value

$$E_{mech} = K_i + U_i = K_i + 0$$

$$E_{mech} = \frac{1}{2}(20.0\text{ kg})(1\,000\text{ m/s})^2 = \boxed{1.00\times10^7\text{ J}}$$

P8.5 The speed at the top can be found from the conservation of energy for the bead-track-Earth system, and the normal force can be found from Newton's second law.



ANS. FIG. P8.5

(a) We define the bottom of the loop as the zero level for the gravitational potential energy.

Since $v_i = 0$,

$$E_i = K_i + U_i = 0 + mgh = mg(3.50R)$$

The total energy of the bead at point Ⓐ can be written as

$$E_A = K_A + U_A = \frac{1}{2}mv_A^2 + mg(2R)$$

Since mechanical energy is conserved, $E_i = E_A$, we get

$$mg(3.50R) = \frac{1}{2}mv_A^2 + mg(2R)$$

simplifying,

$$v_A^2 = 3.00\text{ gR}$$

$$\boxed{v_A = \sqrt{3.00gR}}$$

(b) To find the normal force at the top, we construct a force diagram as shown, where we assume that n is downward, like mg. Newton's second law gives $\sum F = ma_c$, where a_c is the centripetal acceleration.

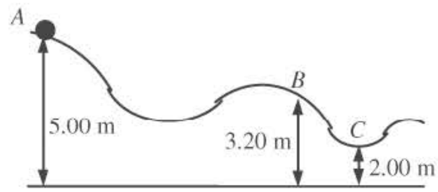
$$\sum F_y = ma_y: \quad n + mg = \frac{mv^2}{r}$$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00gR}{R} - g\right] = 2.00mg$$

$$n = 2.00(5.00\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)$$

$$= \boxed{0.098\,0\text{ N downward}}$$

P8.6 (a) Define the system as the block and the Earth.



ANS. FIG. P8.6

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv_B^2 - 0\right) + \left(mgh_B - mgh_A\right) = 0$$

$$\frac{1}{2}mv_B^2 = mg\left(h_A - h_B\right)$$

$$v_B = \sqrt{2g\left(h_A - h_B\right)}$$

$$v_B = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(5.00\text{ m} - 3.20\text{ m}\right)} = \boxed{5.94\text{ m/s}}$$

Similarly,

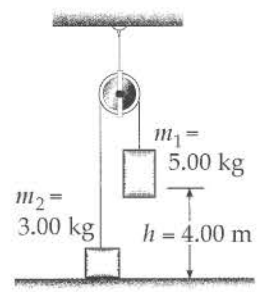
$$v_C = \sqrt{2g\left(h_A - h_C\right)}$$

$$v_C = \sqrt{2g\left(5.00 - 2.00\right)} = \boxed{7.67\text{ m/s}}$$

(b) Treating the block as the system,

$$W_g\Big|_{A\to C} = \Delta K = \frac{1}{2}mv_C^2 - 0 = \frac{1}{2}(5.00\text{ kg})(7.67\text{ m/s})^2 = \boxed{147\text{ J}}$$

P8.7 We assign height $y = 0$ to the table top. Using conservation of energy for the system of the Earth and the two objects:



ANS. FIG. P8.7

(a) Choose the initial point before release and the final point, which we code with the subscript *fa*, just before the larger object hits the floor. No external forces do work on the system and no friction acts within the system. Then total mechanical energy of the system remains constant and the energy version of the isolated system model gives

$$(K_A + K_B + U_g)_i = (K_A + K_B + U_g)_{fa}$$

At the initial point, K_{Ai} and K_{Bi} are zero and we define the gravitational potential energy of the system as zero. Thus the total initial energy is zero, and we have

$$0 = \frac{1}{2}(m_1 + m_2)v_{fa}^2 + m_2gh + m_1g(-h)$$

Here we have used the fact that because the cord does not stretch, the two blocks have the same speed. The heavier mass moves down, losing gravitational potential energy, as the lighter mass moves up, gaining gravitational potential energy. Simplifying,

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)v_{fa}^2$$

$$v_{fa} = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)}} = \sqrt{\frac{2(5.00 \text{ kg} - 3.00 \text{ kg})g(4.00 \text{ m})}{(5.00 \text{ kg} + 3.00 \text{ kg})}}$$

$$= \sqrt{19.6} \text{ m/s} = \boxed{4.43 \text{ m/s}}$$

(b) Now we apply conservation of energy for the system of the 3.00-kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00-kg object reaches its highest position in its free fall.

$$\Delta K + \Delta U = 0 \quad \rightarrow \quad \Delta K = -\Delta U$$

$$0 - \frac{1}{2}m_2v^2 = -m_2g\Delta y \rightarrow \Delta y = \frac{v^2}{2g}$$

$$\Delta y = 1.00 \text{ m}$$

$$y_{max} = 4.00 \text{ m} + \Delta y = \boxed{5.00 \text{ m}}$$

P8.8 We assume $m_1 > m_2$. We assign height $y = 0$ to the table top.

(a) $\Delta K + \Delta U = 0$

$$\Delta K_1 + \Delta K_2 + \Delta U_1 + \Delta U_2 = 0$$

$$\left[\frac{1}{2}m_1v^2 - 0\right] + \left[\frac{1}{2}m_2v^2 - 0\right] + (0 - m_1gh) + (m_2gh - 0) = 0$$

$$\frac{1}{2}(m_1 + m_2)v^2 = m_1gh - m_2gh = (m_1 - m_2)gh$$

$$v = \boxed{\sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2}}}$$

(b) We apply conservation of energy for the system of mass m_2 and the Earth during the time interval between the instant when the string goes slack and the instant mass m_2 reaches its highest position in its free fall.

$$\Delta K + \Delta U = 0 \quad \rightarrow \quad \Delta K = -\Delta U$$

$$0 - \frac{1}{2}m_2v^2 = -m_2g\,\Delta y \rightarrow \Delta y = \frac{v^2}{2g}$$

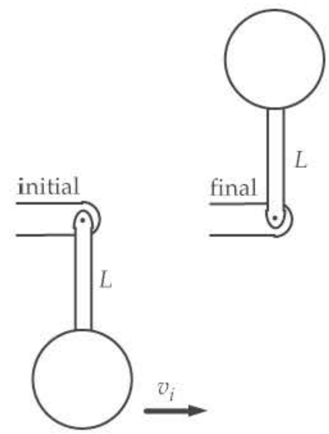
The maximum height of the block is then

$$y_{max} = h + \Delta y = h + \frac{2(m_1 - m_2)gh}{2g(m_1 + m_2)} = h + \frac{(m_1 - m_2)h}{m_1 + m_2}$$

$$y_{max} = \frac{(m_1 + m_2)h}{m_1 + m_2} + \frac{(m_1 - m_2)h}{m_1 + m_2}$$

$$y_{max} = \boxed{\frac{2m_1 h}{m_1 + m_2}}$$

P8.9 The force of tension and subsequent force of compression in the rod do no work on the ball, since they are perpendicular to each step of displacement. Consider energy conservation of the ball-Earth system between the instant just after you strike the ball and the instant when it reaches the top. The speed at the top is zero if you hit it just hard enough to get it there. We ignore the mass of the "light" rod.



ANS. FIG. P8.9

$\Delta K + \Delta U = 0$:

$$\left(0 - \frac{1}{2}mv_i^2\right) + \left[mg(2L) - 0\right] = 0$$

$$v_i = \sqrt{4gL} = \sqrt{4(9.80 \text{ m/s}^2)(0.770 \text{ m})}$$

$$v_i = \boxed{5.49 \text{ m/s}}$$

P8.10 (a) One child in one jump converts chemical energy into mechanical energy in the amount that the child-Earth system has as gravitational energy when she is at the top of her jump:

$$mgy = (36 \text{ kg})(9.80 \text{ m/s}^2)(0.25 \text{ m}) = 88.2 \text{ J}$$

For all of the jumps of the children the energy is

$$12(1.05 \times 10^6)(88.2 \text{ J}) = \boxed{1.11 \times 10^9 \text{ J}}$$

(b) The seismic energy is modeled as

$$E = \left(\frac{0.01}{100}\right)(1.11 \times 10^9 \text{ J}) = 1.11 \times 10^5 \text{ J}$$

making the Richter magnitude

$$\frac{\log E - 4.8}{1.5} = \frac{\log(1.11 \times 10^5) - 4.8}{1.5} = \frac{5.05 - 4.8}{1.5} = \boxed{0.2}$$

P8.11 When block B moves up by 1 cm, block A moves down by 2 cm and the separation becomes 3 cm. We then choose the final point to be when B has moved up by $\frac{h}{3}$ and has speed $\frac{v_A}{2}$. Then A has moved down $\frac{2h}{3}$ and has speed v_A:

$$\Delta K + \Delta U = 0$$

$$\left(K_A + K_B + U_g\right)_f - \left(K_A + K_B + U_g\right)_i = 0$$

$$\left(K_A + K_B + U_g\right)_i = \left(K_A + K_B + U_g\right)_f$$

$$0 + 0 + 0 = \frac{1}{2}mv_A^2 + \frac{1}{2}m\left(\frac{v_A}{2}\right)^2 + \frac{mgh}{3} - \frac{mg2h}{3}$$

$$\frac{mgh}{3} = \frac{5}{8}mv_A^2$$

$$\boxed{v_A = \sqrt{\frac{8gh}{15}}}$$

Section 8.3 Situations Involving Kinetic Friction

P8.12 We could solve this problem using Newton's second law, but we will use the nonisolated system energy model, here written as $-f_k d = K_f - K_i$, where the kinetic energy change of the sled after the kick results only from the friction between the sled and ice. The weight and normal force both act at 90° to the motion, and therefore do no work on the sled. The friction force is

$$f_k = \mu_k n = \mu_k mg$$

Since the final kinetic energy is zero, we have

$$-f_k d = -K_i$$

or $$\frac{1}{2}mv_i^2 = \mu_k mgd$$

Thus,

$$d = \frac{mv_i^2}{2f_k} = \frac{mv_i^2}{2\mu_k mg} = \frac{v_i^2}{2\mu_k g} = \frac{(2.00 \text{ m/s})^2}{2(0.100)\left(9.80 \text{ m/s}^2\right)} = \boxed{2.04 \text{ m}}$$

P8.13 We use the nonisolated system energy model, here written as $-f_k d = K_f - K_i$, where the kinetic energy change of the sled after the kick results only from the friction between the sled and ice.

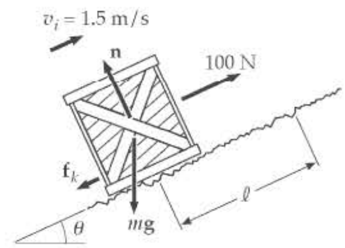
$$\Delta K + \Delta U = -f_k d:$$

$$0 - \frac{1}{2}mv^2 = -f_k d$$

$$\frac{1}{2}mv^2 = \mu_k mgd$$

which gives $\boxed{d = \dfrac{v^2}{2\mu_k g}}$

P8.14 (a) The force of gravitation is



$$(10.0\text{ kg})(9.80\text{ m/s}^2) = 98.0\text{ N}$$

straight down, at an angle of

$$(90.0° + 20.0°) = 110.0°$$

with the motion. The work done by the gravitational force on the crate is

$$W_g = \vec{F}\cdot\Delta\vec{r} = mg\ell\cos(90.0° + \theta)$$
$$= (98.0\text{ N})(5.00\text{ m})\cos 110.0° = \boxed{-168\text{ J}}$$

(b) We set the x and y axes parallel and perpendicular to the incline, respectively.

From $\sum F_y = ma_y$, we have

$$n - (98.0\text{ N})\cos 20.0° = 0$$

so $n = 92.1\text{ N}$

and

$$f_k = \mu_k n = 0.400\,(92.1\text{ N}) = 36.8\text{ N}$$

Therefore,

$$\Delta E_{int} = f_k d = (36.8\text{ N})(5.00\text{ m}) = \boxed{184\text{ J}}$$

(c) $W_F = F\ell = (100\text{ N})(5.00\text{ m}) = \boxed{500\text{ J}}$

(d) We use the energy version of the nonisolated system model.

$$\Delta K = -f_k d + \sum W_{\text{other forces}}$$

$$\Delta K = -f_k d + W_g + W_{\text{applied force}} + W_n$$

The normal force does zero work, because it is at 90° to the motion.

$$\Delta K = -184\text{ J} - 168\text{ J} + 500\text{ J} + 0 = \boxed{148\text{ J}}$$

(e) Again, $K_f - K_i = -f_k d + \sum W_{\text{other forces}}$, so

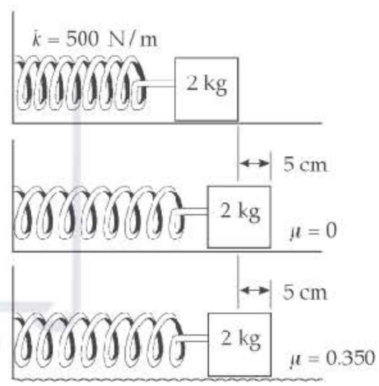
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \sum W_{\text{other forces}} - f_k d$$

$$v_f = \sqrt{\frac{2}{m}\left[\Delta K + \frac{1}{2}mv_i^2\right]}$$

$$= \sqrt{\left(\frac{2}{10.0\text{ kg}}\right)[148\text{ J } + \frac{1}{2}(10.0\text{ kg})(1.50\text{ m/s})^2]}$$

$$v_f = \sqrt{\frac{2(159\text{ kg}\cdot\text{m}^2/\text{s}^2)}{10.0\text{ kg}}} = \boxed{5.65\text{ m/s}}$$

P8.15 (a) The spring does positive work on the block:



ANS. FIG. P8.15

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$W_s = \frac{1}{2}(500\text{ N/m})(5.00\times 10^{-2}\text{ m})^2 - 0$$

$$= 0.625\text{ J}$$

Applying $\Delta K = W_s$:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_s \rightarrow \frac{1}{2}mv_f^2 - 0 = W_s$$

so

$$v_f = \sqrt{\frac{2(W_s)}{m}}$$

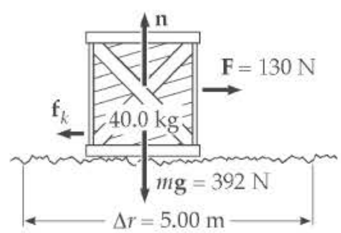
$$= \sqrt{\frac{2(0.625)}{2.00}}\text{ m/s} = \boxed{0.791\text{ m/s}}$$

(b) Now friction results in an increase in internal energy $f_k d$ of the block-surface system. From conservation of energy for a nonisolated system,

$$W_s = \Delta K + \Delta E_{int}$$
$$\Delta K = W_s - f_k d$$
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_s - f_k d = W_s - \mu_s mgd$$
$$\frac{1}{2}mv_f^2 = 0.625\text{ J} - (0.350)(2.00\text{ kg})(9.80\text{ m/s}^2)(0.050\,0\text{ m})$$
$$\frac{1}{2}(2.00\text{ kg})v_f^2 = 0.625\text{ J} - 0.343\text{ J} = 0.282\text{ J}$$
$$v_f = \sqrt{\frac{2(0.282)}{2.00}}\text{ m/s} = \boxed{0.531\text{ m/s}}$$

P8.16 $\sum F_y = ma_y$: $n - 392\text{ N} = 0$

$$n = 392\text{ N}$$
$$f_k = \mu_k n = (0.300)(392\text{ N}) = 118\text{ N}$$



ANS. FIG. P8.16

(a) The applied force and the motion are both horizontal.

$$W_F = Fd\cos\theta$$
$$= (130\text{ N})(5.00\text{ m})\cos 0°$$
$$= \boxed{650\text{ J}}$$

(b) $\Delta E_{int} = f_k d = (118\text{ N})(5.00\text{ m}) = \boxed{588\text{ J}}$

(c) Since the normal force is perpendicular to the motion,

$$W_n = nd\cos\theta = (392\text{ N})(5.00\text{ m})\cos 90° = \boxed{0}$$

(d) The gravitational force is also perpendicular to the motion, so

$$W_g = mgd\cos\theta = (392\text{ N})(5.00\text{ m})\cos(-90°) = \boxed{0}$$

(e) We write the energy version of the nonisolated system model as

$$\Delta K = K_f - K_i = \sum W_{other} - \Delta E_{int}$$
$$\frac{1}{2}mv_f^2 - 0 = 650\text{ J} - 588\text{ J} + 0 + 0 = \boxed{62.0\text{ J}}$$

(f) $v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(62.0\text{ J})}{40.0\text{ kg}}} = \boxed{1.76\text{ m/s}}$

P8.17 (a) $\Delta E_{int} = -\Delta K = -\frac{1}{2}m\left(v_f^2 - v_i^2\right)$:

$$\Delta E_{int} = -\frac{1}{2}(0.400\text{ kg})\left[(6.00)^2 - (8.00)^2\right](\text{m/s})^2 = \boxed{5.60\text{ J}}$$

(b) After N revolutions, the object comes to rest and $K_f = 0$.

Thus,

$$\Delta E_{int} = -\Delta K$$

$$f_k d = -(0 - K_i) = \frac{1}{2}mv_i^2$$

or

$$\mu_k mg\left[N(2\pi r)\right] = \frac{1}{2}mv_i^2$$

This gives

$$N = \frac{\frac{1}{2}mv_i^2}{\mu_k mg(2\pi r)} = \frac{\frac{1}{2}(8.00\text{ m/s})^2}{(0.152)\left(9.80\text{ m/s}^2\right)2\pi(1.50\text{ m})}$$

$$= \boxed{2.28\text{ rev}}$$

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

P8.18 (a) If only conservative forces act, then the total mechanical energy does not change.

$$\Delta K + \Delta U = 0 \quad \text{or} \quad U_f = K_i - K_f + U_i$$

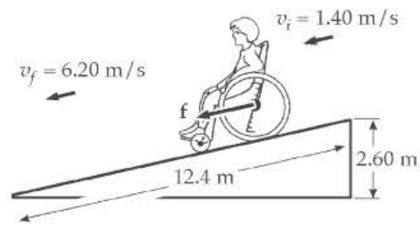
$$U_f = 30.0\text{ J} - 18.0\text{ J} + 10.0\text{ J} = \boxed{22.0\text{ J}}$$

$$E = K + U = 30.0\text{ J} + 10.0\text{ J} = \boxed{40.0\text{ J}}$$

(b) $\boxed{\text{Yes}}$, if the potential energy is less than 22.0 J.

(c) If the potential energy is 5.00 J, the total mechanical energy is $E = K + U = 18.0\text{ J} + 5.00\text{ J} = 23.0\text{ J}$, less than the original 40.0 J. The total mechanical energy has decreased, so a nonconservative force must have acted.

P8.19 The boy converts some chemical energy in his body into mechanical energy of the boy-chair-Earth system. During this conversion, the energy can be measured as the work his hands do on the wheels.



ANS. FIG. P8.19

$$\Delta K + \Delta U + \Delta U_{body} = -f_k d$$

$$(K_f - K_i) + (U_f - U_i) + \Delta U_{body} = -f_k d$$

$$K_i + U_i + W_{hands\text{-}on\text{-}wheels} - f_k d = K_f$$

Rearranging and renaming, we have

$$\frac{1}{2}mv_i^2 + mgy_i + W_{by\ boy} - f_k d = \frac{1}{2}mv_f^2$$

$$W_{by\ boy} = \frac{1}{2}m(v_f^2 - v_i^2) - mgy_i + f_k d$$

$$W_{by\ boy} = \frac{1}{2}(47.0\text{ kg})\left[(6.20\text{ m/s})^2 - (1.40\text{ m/s})^2\right]$$
$$-(47.0\text{ kg})(9.80\text{ m/s}^2)(2.60\text{ m})$$
$$+(41.0\text{ N})(12.4\text{ m})$$

$$W_{by\ boy} = \boxed{168\text{ J}}$$

P8.20 (a) Apply conservation of energy to the bead-string-Earth system to find the speed of the bead at Ⓑ. Friction transforms mechanical energy of the system into internal energy $\Delta E_{int} = f_k d$.

$$\Delta K + \Delta U + \Delta E_{int} = 0$$

$$\left[\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2\right] + (mgy_B - mgy_A) + f_k d = 0$$

$$\left[\frac{1}{2}mv_B^2 - 0\right] + (0 - mgy_A) + f_k d = 0 \rightarrow \frac{1}{2}mv_B^2 = mgy_A - f_k d$$

$$v_B = \sqrt{2gy_A - \frac{2f_k d}{m}}$$

For $y_A = 0.200$ m, $f_k = 0.025$ N, $d = 0.600$ m, and $m = 25.0 \times 10^{-3}$ kg:

$$v_B = \sqrt{2(9.80\text{ m/s}^2)(0.200\text{ m}) - \frac{2(0.025\text{ N})(0.600\text{ m})}{25.0\times10^{-3}\text{ kg}}}$$
$$= \sqrt{2.72}\text{ m/s}$$

$$\boxed{v_B = 1.65\text{ m/s}}$$

(b) The red bead slides a greater distance along the curved path, so friction transforms more of the mechanical energy of the system into internal energy. There is less of the system's original potential energy in the form of kinetic energy when the bead arrives at point Ⓑ. The result is that the green bead arrives at point Ⓑ first and at higher speed.

P8.21 Use Equation 8.16: $\Delta E_{mech} = \Delta K + \Delta U = -f_k d$

$$(K_f - K_i) + (U_f - U_i) = -f_k d$$
$$K_i + U_i - f_k d = K_f + U_f$$

(a) $K_i + U_i - f_k d = K_f + U_f$

$$0 + \frac{1}{2}kx^2 - f\Delta x = \frac{1}{2}mv^2 + 0$$

$$\frac{1}{2}(8.00\text{ N/m})(5.00\times10^{-2}\text{ m})^2 - (3.20\times10^{-2}\text{ N})(0.150\text{ m}) = \frac{1}{2}(5.30\times10^{-3}\text{ kg})v^2$$

$$v = \sqrt{\frac{2(5.20\times10^{-3}\text{ J})}{5.30\times10^{-3}\text{ kg}}} = \boxed{1.40\text{ m/s}}$$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $|\vec{F}_s| = kx$; the spring is compressed by

$$\frac{3.20\times10^{-2}\text{ N}}{8.00\text{ N/m}} = 0.400\text{ cm}$$

and the ball has moved

$$5.00\text{ cm} - 0.400\text{ cm} = \boxed{4.60\text{ cm from the start}}$$

(c) Between start and maximum speed points,

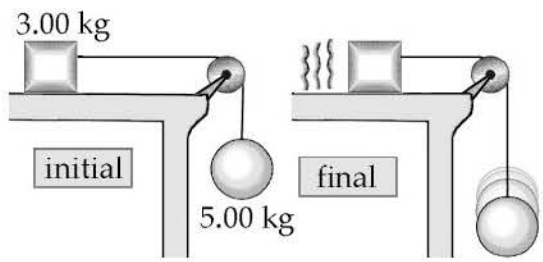
$$\frac{1}{2}kx_i^2 - f\Delta x = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}(8.00\text{ N/m})(5.00\times10^{-2}\text{ m})^2 - (3.20\times10^{-2}\text{ N})(4.60\times10^{-2}\text{ m})$$
$$= \frac{1}{2}(5.30\times10^{-3}\text{ kg})v^2 + \frac{1}{2}(8.00\text{ N/m})(4.00\times10^{-3}\text{ m})^2$$

$$v = \boxed{1.79\text{ m/s}}$$

P8.22 For the Earth plus objects 1 (block) and 2 (ball), we write the energy model equation as

$$(K_1 + K_2 + U_1 + U_2)_f - (K_1 + K_2 + U_1 + U_2)_i = \sum W_{\text{other forces}} - f_k d$$



ANS. FIG. P8.22

Choose the initial point before release and the final point after each block has moved 1.50 m. Choose $U = 0$ with the 3.00-kg block on the tabletop and the 5.00-kg block in its final position.

So $\qquad K_{1i} = K_{2i} = U_{1i} = U_{1f} = U_{2f} = 0$

We have chosen to include the Earth in our system, so gravitation is an internal force. Because the only external forces are friction and normal forces exerted by the table and the pulley at right angles to the motion,

$$\sum W_{\text{other forces}} = 0$$

We now have

$$\frac{1}{2}m_1 v_f^2 + \frac{1}{2}m_2 v_f^2 + 0 + 0 - 0 - 0 - 0 - m_2 g y_{2i} = 0 - f_k d$$

where the friction force is

$$f_k = \mu_k n = \mu_k m_A g$$

The friction force causes a negative change in mechanical energy because the force opposes the motion. Since all of the variables are known except for v_f, we can substitute and solve for the final speed.

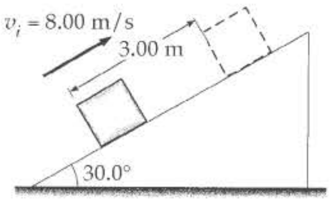
$$\frac{1}{2}m_1 v_f^2 + \frac{1}{2}m_2 v_f^2 - m_2 g y_{2i} = -f_k d$$

$$v^2 = \frac{2gh(m_2 - \mu_k m_1)}{m_1 + m_2}$$

$$v = \sqrt{\frac{2(9.80 \text{ m/s}^2)(1.50 \text{ m})[5.00 \text{ kg} - 0.400(3.00 \text{ kg})]}{8.00 \text{ kg}}}$$

$$= \boxed{3.74 \text{ m/s}}$$

P8.23 We consider the block-plane-planet system between an initial point just after the block has been given its shove and a final point when the block comes to rest.



ANS. FIG. P8.23

(a) The change in kinetic energy is

$$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= 0 - \frac{1}{2}(5.00\text{ kg})(8.00\text{ m/s})^2 = \boxed{-160\text{ J}}$$

(b) The change in gravitational potential energy is

$$\Delta U = U_f - U_i = mgh$$

$$= (5.00\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m})\sin 30.0° = \boxed{73.5\text{ J}}$$

(c) The nonisolated system energy model we write as

$$\Delta K + \Delta U = \sum W_{\text{other forces}} - f_k d = 0 - f_k d$$

The force of friction is the only unknown, so we may find it from

$$f_k = \frac{\Delta K - \Delta U}{d} = \frac{+160\text{ J} - 73.5\text{ J}}{3.00\text{ m}} = \boxed{28.8\text{ N}}$$

(d) The forces perpendicular to the incline must add to zero.

$$\sum F_y = 0: \quad +n - mg\cos 30.0° = 0$$

Evaluating,

$$n = mg\cos 30.0° = (5.00\text{ kg})(9.80\text{ m/s}^2)\cos 30.0° = 42.4\text{ N}$$

Now $f_k = \mu_k n$ gives

$$\mu_k = \frac{f_k}{n} = \frac{28.8\text{ N}}{42.4\text{ N}} = \boxed{0.679}$$

P8.24 (a) The object drops distance $d = 1.20$ m until it hits the spring, then it continues until the spring is compressed a distance x.

$$\Delta K + \Delta U = 0$$

$$K_f - K_i + U_f - U_i = 0$$

$$0 - 0 + \left(\frac{1}{2}kx^2 - 0\right) + \left[mg(-x) - mgd\right] = 0$$

$$\frac{1}{2}kx^2 - mg(x + d) = 0$$

$$\frac{1}{2}(320\text{ N/m})x^2-(1.50\text{ kg})(9.80\text{ m/s}^2)(x+1.20\text{ m})=0$$

Dropping units, we have

$$160x^2-(14.7)x-17.6=0$$

$$x=\frac{14.7\pm\sqrt{(-14.7)^2-4(160)(-17.6)}}{2(160)}$$

$$x=\frac{14.7\pm 107}{320}$$

The negative root does not apply because x is a distance. We have

$$x=\boxed{0.381\text{ m}}$$

(b) This time, friction acts through distance $(x + d)$ in the object-spring-Earth system:

$$\Delta K+\Delta U=-f_k(x+d)$$

$$0-0+\left(\frac{1}{2}kx^2-0\right)+\left[mg(-x)-mgd\right]=-f_k(x+d)$$

$$\frac{1}{2}kx^2-(mg-f_k)x-(mg-f_k)d=0$$

where $mg-f_k=14.0$ N. Suppressing units, we have

$$160x^2-14.0x-16.8=0$$

$$160x^2-14.0x-16.8=0$$

$$x=\frac{14.0\pm\sqrt{(-14.0)^2-4(160)(-16.8)}}{2(160)}$$

$$x=\frac{14.0\pm 105}{320}$$

The positive root is $x=\boxed{0.371\text{ m.}}$

(c) On the Moon, we have

$$\frac{1}{2}kx^2-mg(x+d)=0$$

$$\frac{1}{2}(320\text{ N/m})x^2-(1.50\text{ kg})(1.63\text{ m/s}^2)(x+1.20\text{ m})=0$$

Suppressing units,

$$160x^2-2.45x-2.93=0$$

$$x = \frac{2.45 \pm \sqrt{(-2.45)^2 - 4(160)(-2.93)}}{2(160)}$$

$$x = \frac{2.45 \pm 43.3}{320}$$

$$x = \boxed{0.143\text{ m}}$$

P8.25 The spring is initially compressed by $x_i = 0.100$ m. The block travels up the ramp distance d.

The spring does work $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}kx_i^2 - 0 = \frac{1}{2}kx_i^2$ on the block.

Gravity does work $W_g = mgd\cos(90° + 60.0°) = mgd\sin(60.0°)$ on the block. There is no friction.

(a) $\sum W = \Delta K$: $\quad W_s + W_g = 0$

$$\frac{1}{2}kx_i^2 - mgd\ \sin(60.0°) = 0$$

$$\frac{1}{2}(1.40\times 10^3\text{ N/m})(0.100\text{ m})^2 - (0.200\text{ kg})(9.80\text{ m/s}^2)d(\sin 60.0°) = 0$$

$$d = \boxed{4.12\text{ m}}$$

(b) Within the system, friction transforms kinetic energy into internal energy:

$$\Delta E_{int} = f_k d = (\mu_k n)d = \mu_k(mg\ \cos 60.0°)d$$

$$\sum W = \Delta K + \Delta E_{int}: \quad W_s + W_g - \Delta E_{int} = 0$$

$$\frac{1}{2}kx_i^2 - mgd\ \sin 60.0° - \mu_k mg\ \cos 60.0° d = 0$$

$$\frac{1}{2}(1.40\times 10^3\text{ N/m})(0.100\text{ m})^2 - (0.200\text{ kg})(9.80\text{ m/s}^2)d(\sin 60.0°) - (0.400)(0.200\text{ kg})(9.80\text{ m/s}^2)(\cos 60.0°)d = 0$$

$$d = \boxed{3.35\text{ m}}$$

P8.26 Air resistance acts like friction. Consider the whole motion:

$$\Delta K + \Delta U = -f_{air}d \rightarrow K_i + U_i - f_{air}d = K_f + U_f$$

(a) $0 + mgy_i - f_1d_1 - f_2d_2 = \frac{1}{2}mv_f^2 + 0$

$$(80.0\text{ kg})(9.80\text{ m/s}^2)1\ 000\text{ m} - (50.0\text{ N})(800\text{ m}) - (3\ 600\text{ N})(200\text{ m}) = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$784\ 000\text{ J} - 40\ 000\text{ J} - 720\ 000\text{ J} = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(24\ 000\text{ J})}{80.0\text{ kg}}} = \boxed{24.5\text{ m/s}}$$

(b) Yes. This is too fast for safety.

(c) Now in the same energy equation as in part (a), d_2 is unknown, and $d_1 = 1\ 000\text{ m} - d_2$:

$$784\ 000\text{ J} - (50.0\text{ N})(1\ 000\text{ m} - d_2) - (3\ 600\text{ N})d_2 = \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2$$

$$784\ 000\text{ J} - 50\ 000\text{ J} - (3\ 550\text{ N})d_2 = 1\ 000\text{ J}$$

$$d_2 = \frac{733\ 000\text{ J}}{3\ 550\text{ N}} = \boxed{206\text{ m}}$$

(d) The air drag is proportional to the square of the skydiver's speed, so it will change quite a bit, It will be larger than her 784-N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down whenever she moves near terminal speed.

P8.27 (a) Yes, the child-Earth system is isolated because the only force that can do work on the child is her weight. The normal force from the slide can do no work because it is always perpendicular to her displacement. The slide is frictionless, and we ignore air resistance.

(b) No, because there is no friction.

(c) At the top of the water slide,

$$U_g = mgh \quad \text{and} \quad K = 0: \quad E = 0 + mgh \rightarrow \boxed{E = mgh}$$

(d) At the launch point, her speed is v_i, and height $h = h/5$:

$$E = K + U_g$$

$$E = \boxed{\frac{1}{2}mv_i^2 + \frac{mgh}{5}}$$

(e) At her maximum airborne height, $h = y_{\max}$:

$$E = \frac{1}{2}mv^2 + mgh = \frac{1}{2}m(v_{xi}{}^2 + v_{yi}{}^2) + mgy_{\max}$$

$$E = \frac{1}{2}m(v_{xi}{}^2 + 0) + mgy_{\max} \rightarrow E = \boxed{\frac{1}{2}mv_{xi}^2 + mgy_{\max}}$$

(f) $E = mgh = \dfrac{1}{2}mv_i{}^2 + mgh/5 \rightarrow \boxed{v_i = \sqrt{\dfrac{8gh}{5}}}$

(g) At the launch point, her velocity has components $v_{xi} = v_i \cos\theta$ and $v_{yi} = v_i \sin\theta$:

$$E = \frac{1}{2}mv_i^2 + \frac{mgh}{5} = \frac{1}{2}mv_{xi}^2 + mgy_{\max}$$

$$\rightarrow \frac{1}{2}\cancel{m}v_i^2 + \frac{\cancel{m}gh}{5} = \frac{1}{2}\cancel{m}(v_i\cos\theta)^2 + \cancel{m}gy_{\max}$$

$$\rightarrow \frac{1}{2}v_i^2\left(1 - \cos^2\theta\right) + \frac{gh}{5} = gh_{\max}$$

$$\rightarrow h_{\max} = \frac{1}{2\cancel{g}}\left(\frac{8\cancel{g}h}{5}\right)\left(1 - \cos^2\theta\right) + \frac{\cancel{g}h}{5\cancel{g}}$$

$$\rightarrow h_{\max} = \left(\frac{4h}{5}\right)\left(1 - \cos^2\theta\right) + \frac{h}{5} \rightarrow \boxed{h_{\max} = h\left(1 - \frac{4}{5}\cos^2\theta\right)}$$

(h) No. If friction is present, mechanical energy of the system would not be conserved, so her kinetic energy at all points after leaving the top of the waterslide would be reduced when compared with the frictionless case. Consequently, her launch speed, maximum height reached, and final speed would be reduced as well.

Section 8.5 Power

P8.28 (a) The moving sewage possesses kinetic energy in the same amount as it enters and leaves the pump. The work of the pump increases the gravitational energy of the sewage-Earth system. We take the equation $K_i + U_{gi} + W_{\text{pump}} = K_f + U_{gf}$, subtract out the K terms, and choose $U_{gi} = 0$ at the bottom of the pump, to obtain $W_{\text{pump}} = mgy_f$. Now we differentiate through with respect to time:

$$
\begin{aligned}
P_{\text{pump}} &= \frac{\Delta m}{\Delta t} g y_f = \rho \frac{\Delta V}{\Delta t} g y_f \\
&= \left(1\,050\ \text{kg/m}^3\right)\left(1.89 \times 10^6\ \text{L/d}\right) \\
&\qquad \times \left(\frac{1\ \text{m}^3}{1\,000\ \text{L}}\right)\left(\frac{1\ \text{d}}{86\,400\ \text{s}}\right)\left(\frac{9.80\ \text{m}}{\text{s}^2}\right)(5.49\ \text{m}) \\
&= \boxed{1.24 \times 10^3\ \text{W}}
\end{aligned}
$$

(b)
$$
\begin{aligned}
\text{efficiency} &= \frac{\text{useful output work}}{\text{total input work}} = \frac{\text{useful output work}/\Delta t}{\text{useful input work}/\Delta t} \\
&= \frac{\text{mechanical output power}}{\text{input electric power}} = \frac{1.24\ \text{kW}}{5.90\ \text{kW}} \\
&= \boxed{0.209} = 20.9\%
\end{aligned}
$$

The remaining power, 5.90 – 1.24 kW = 4.66 kW, is the rate at which internal energy is injected into the sewage and the surroundings of the pump.

P8.29 The Marine must exert an 820-N upward force, opposite the gravitational force, to lift his body at constant speed. The Marine's power output is the work he does divided by the time interval:

$$\text{Power} = \frac{W}{t}$$

$$P = \frac{mgh}{t} = \frac{(820\ \text{N})(12.0\ \text{m})}{8.00\ \text{s}} = 1\,230\ \text{W} = \boxed{1.23\ \text{kW}}$$

P8.30 (a) $P_{\text{av}} = \dfrac{W}{\Delta t} = \dfrac{K_f}{\Delta t} = \dfrac{mv^2}{2\Delta t} = \dfrac{(0.875\ \text{kg})(0.620\ \text{m/s})^2}{2\left(21 \times 10^{-3}\ \text{s}\right)} = \boxed{8.01\ \text{W}}$

(b) $\boxed{\text{Some of the energy transferring into the system of the train goes into internal energy in warmer track and moving parts and some leaves the system by sound. To account for this as well as the stated increase in kinetic energy, energy must be transferred at a rate higher than 8.01 W.}}$

P8.31 When the car moves at constant speed on a level roadway, the power used to overcome the total friction force equals the power input from the engine, or $P_{\text{output}} = f_{\text{total}}\, v = P_{\text{input}}$. This gives

$$f_{\text{total}} = \frac{P_{\text{input}}}{v} = \frac{175\ \text{hp}}{29\ \text{m/s}}\left(\frac{746\ \text{W}}{1\ \text{hp}}\right)$$

$$= 4.5\times10^{5}\ \text{N or about } \boxed{5\times10^{5}\ \text{N.}}$$

P8.32 Neglecting any variation of gravity with altitude, the work required to lift a 3.20×10^{7} kg load at constant speed to an altitude of $\Delta y = 1.75$ km is

$$\begin{aligned} W = \Delta PE_g &= mg(\Delta y) \\ &= (3.20\times10^{7}\ \text{kg})(9.80\ \text{m/s}^2)(1.75\times10^{3}\ \text{m}) \\ &= 5.49\times10^{11}\ \text{J} \end{aligned}$$

The time required to do this work using a $P = 2.70$ kW $= 2.70\times10^{3}$ J/s pump is

$$\begin{aligned} \Delta t = \frac{W}{P} &= \frac{5.49\times10^{11}\ \text{J}}{2.70\times10^{3}\ \text{J/s}} = \boxed{2.03\times10^{8}\ \text{s}} \\ &= (2.03\times10^{8}\ \text{s})\left(\frac{1\ \text{h}}{3\ 600\ \text{s}}\right) \\ &= \boxed{5.64\times10^{4}\ \text{h}} = 6.44\ \text{yr} \end{aligned}$$

P8.33 energy = power × time

For the 28.0-W bulb:

Energy used = $(28.0\ \text{W})(1.00\times10^{4}\ \text{h}) = 280$ kWh

total cost = \$4.50 + (280 kWh)(\$0.200/kWh) = \$60.50

For the 100-W bulb:

Energy used = $(100\ \text{W})(1.00\times10^{4}\ \text{h}) = 1.00\times10^{3}$ kWh

$$\#\text{ of bulbs used} = \frac{1.00\times10^{4}\ \text{h}}{750\ \text{h/bulb}} = 13.3 = 13\ \text{bulbs}$$

$$\text{total cost} = 13(\$0.420) + (1.00 \times 10^3 \text{ kWh})(\$0.200/\text{kWh}) = \$205.46$$

Savings with energy-efficient bulb:

$$\$205.46 - \$60.50 = \$144.96 = \boxed{\$\,145}$$

P8.34 The useful output energy is

$$120 \text{ Wh}(1 - 0.60) = mg(y_f - y_i) = F\,\Delta y$$

$$\Delta y = \frac{120 \text{ W}(3\ 600 \text{ s})0.40}{890 \text{ N}}\left(\frac{\text{J}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right) = \boxed{194 \text{ m}}$$

P8.35 A 1 300-kg car speeds up from rest to 55.0 mi/h = 24.6 m/s in 15.0 s. The output work of the engine is equal to its final kinetic energy,

$$\frac{1}{2}(1\ 300 \text{ kg})(24.6 \text{ m/s})^2 = 390 \text{ kJ}$$

with power $P = \dfrac{390\ 000 \text{ J}}{15.0 \text{ s}}\boxed{\sim 10^4 \text{ W,}}$ around 30 horsepower.

P8.36 $P = \dfrac{W}{\Delta t}$

older-model: $W = \dfrac{1}{2}mv^2$

newer-model: $W = \dfrac{1}{2}m(2v)^2 = \dfrac{1}{2}(4mv^2) \quad \rightarrow \quad P_{\text{newer}} = \dfrac{4mv^2}{2\Delta t} = 4\dfrac{mv^2}{2\Delta t}$

$\boxed{\text{The power of the sports car is four times that of the older-model car.}}$

***P8.37** (a) The fuel economy for walking is

$$\frac{1 \text{ h}}{220 \text{ kcal}}\left(\frac{3 \text{ mi}}{\text{h}}\right)\left(\frac{1 \text{ kcal}}{4\,186 \text{ J}}\right)\left(\frac{1.30\times10^8 \text{ J}}{1 \text{ gal}}\right) = \boxed{423 \text{ mi/gal}}$$

(b) For bicycling:

$$\frac{1 \text{ h}}{400 \text{ kcal}}\left(\frac{10 \text{ mi}}{\text{h}}\right)\left(\frac{1 \text{ kcal}}{4\,186 \text{ J}}\right)\left(\frac{1.30\times10^8 \text{ J}}{1 \text{ gal}}\right) = \boxed{776 \text{ mi/gal}}$$

P8.38 (a) The distance moved upward in the first 3.00 s is

$$\Delta y = \bar{v}\Delta t = \left[\frac{0 + 1.75 \text{ m/s}}{2}\right](3.00 \text{ s}) = 2.63 \text{ m}$$

The motor and the Earth's gravity do work on the elevator car:

$$W_{\text{motor}} + W_{\text{gravity}} = \Delta K$$

$$W_{\text{motor}} + (mg\Delta y)\cos 180° = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{\text{motor}} - (mg\Delta y) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{\text{motor}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mg\Delta y$$

$$W_{\text{motor}} = \frac{1}{2}(650\text{ kg})(1.75\text{ m/s})^2 - 0 + (650\text{ kg})g(2.63\text{ m})$$

$$= 1.77 \times 10^4\text{ J}$$

Also, $W = \bar{P}\Delta t$ so $\bar{P} = \dfrac{W}{\Delta t} = \dfrac{1.77\times 10^4\text{ J}}{3.00\text{ s}} = \boxed{5.91\times 10^3\text{ W}} = 7.92\text{ hp}$.

(b) When moving upward at constant speed (v = 1.75 m/s), the applied force equals the weight = (650 kg)(9.80 m/s^2) = 6.37×10^3 N. Therefore,

$$P = Fv = (6.37\times 10^3\text{ N})(1.75\text{ m/s}) = \boxed{1.11\times 10^4\text{ W}} = 14.9\text{ hp}$$

P8.39 As the piano is lifted at constant speed up to the apartment, the total work that must be done on it is

$$W_{nc} = \Delta K + \Delta U_g = 0 + mg(y_f - y_i)$$

$$= (3.50\times 10^3\text{ N})(25.0\text{ m})$$

$$= 8.75\times 10^4\text{ J}$$

The three workmen (using a pulley system with an efficiency of 0.750) do work on the piano at a rate of

$$P_{\text{net}} = 0.750\left(3P_{\substack{\text{single}\\\text{worker}}}\right) = 0.750[3(165\text{ W})] = 371\text{ W} = 371\text{ J/s}$$

so the time required to do the necessary work on the piano is

$$\Delta t = \frac{W_{nc}}{P_{\text{net}}} = \frac{8.75\times 10^4\text{ J}}{371\text{ J/s}} = \boxed{236\text{ s}} = (236\text{ s})\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{3.93\text{ min}}$$

P8.40 (a) Burning 1 kg of fat releases energy

$$1\text{ kg}\left(\frac{1\,000\text{ g}}{1\text{ kg}}\right)\left(\frac{9\text{ kcal}}{1\text{ g}}\right)\left(\frac{4\,186\text{ J}}{1\text{ kcal}}\right) = 3.77\times 10^7\text{ J}$$

The mechanical energy output is

$$\left(3.77\times10^{7}\text{ J}\right)(0.20)=nFd\cos\theta$$

where n is the number of flights of stairs. Then

$$7.53\times10^{6}\text{ J}=nmg\Delta y\cos0°$$

$$7.53\times10^{6}\text{ J}=n(75\text{ kg})\left(9.8\text{ m/s}^2\right)(80\text{ steps})(0.150\text{ m})$$

$$7.53\times10^{6}\text{ J}=n\left(8.82\times10^{3}\text{ J}\right)$$

where the number of times she must climb the stairs is

$$n=\frac{7.53\times10^{6}\text{ J}}{8.82\times10^{3}\text{ J}}=\boxed{854}$$

(b) Her mechanical power output is

$$P=\frac{W}{t}=\frac{8.82\times10^{3}\text{ J}}{65\text{ s}}=\boxed{136\text{ W}}=(136\text{ W})\left(\frac{1\text{ hp}}{746\text{ W}}\right)$$

$$=\boxed{0.182\text{ hp}}$$

(c) $\boxed{\text{This method is impractical compared to limiting food intake.}}$

P8.41 The energy of the car-Earth system is $E=\frac{1}{2}mv^2+mgy$:

$$E=\frac{1}{2}mv^2+mgd\sin\theta$$

where d is the distance the car has moved along the track.

$$P=\frac{dE}{dt}=mv\frac{dv}{dt}+mgv\sin\theta$$

(a) When speed is constant,

$$P=mgv\sin\theta=(950\text{ kg})\left(9.80\text{ m/s}^2\right)(2.20\text{ m/s})\sin30.0°$$

$$=\boxed{1.02\times10^{4}\text{ W}}$$

(b) $\frac{dv}{dt}=a=\frac{2.20\text{ m/s}-0}{12\text{ s}}=0.183\text{ m/s}^2$

Maximum power is injected just before maximum speed is attained:

$$P=mva+mgv\sin\theta$$

$$=(950\text{ kg})(2.20\text{ m/s})\left(0.183\text{ m/s}^2\right)+1.02\times10^{4}\text{ W}$$

$$=\boxed{1.06\times10^{4}\text{ W}}$$

(c) At the top end,

$$\frac{1}{2}mv^2 + mgd\sin\theta$$
$$= 950\text{ kg}\left(\frac{1}{2}(2.20\text{ m/s})^2 + (9.80\text{ m/s}^2)(1\,250\text{ m})\sin 30°\right)$$
$$= \boxed{5.82\times 10^6\text{ J}}$$

Additional Problems

***P8.42** At a pace I could keep up for a half-hour exercise period, I climb two stories up, traversing forty steps each 18 cm high, in 20 s. My output work becomes the final gravitational energy of the system of the Earth and me,

$$mgy = (85\text{ kg})(9.80\text{ m/s}^2)(40\times 0.18\text{ m}) = 6\,000\text{ J}$$

making my sustainable power $\dfrac{6\,000\text{ J}}{20\text{ s}} = \boxed{\sim 10^2\text{ W}}$.

P8.43 (a) $U_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m}) = \boxed{0.588\text{ J}}$

(b) $K_A + U_A = K_B + U_B$

$K_B = K_A + U_A - U_B = mgR = \boxed{0.588\text{ J}}$

(c) $v_B = \sqrt{\dfrac{2K_B}{m}} = \sqrt{\dfrac{2(0.588\text{ J})}{0.200\text{ kg}}} = \boxed{2.42\text{ m/s}}$

(d) $U_C = mgh_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) = \boxed{0.392\text{ J}}$

$K_C = K_A + U_A - U_C = mg(h_A - h_C)$
$K_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300 - 0.200)\text{ m} = \boxed{0.196\text{ J}}$

P8.44 (a) Let us take $U = 0$ for the particle-bowl-Earth system when the particle is at Ⓑ. Since $v_B = 1.50$ m/s and $m = 200$ g,

$$K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.200\text{ kg})(1.50\text{ m/s})^2 = \boxed{0.225\text{ J}}$$

(b) At Ⓐ, $v_i = 0$, $K_A = 0$, and the whole energy at Ⓐ is $U_A = mgR$:

$$E_i = K_A + U_A = 0 + mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m})$$
$$= 0.588\text{ J}$$

At Ⓑ,

$$E_f = K_B + U_B = 0.225 \text{ J} + 0$$

The decrease in mechanical energy is equal to the increase in internal energy.

$$E_{\text{mech, i}} + \Delta E_{\text{int}} = E_{\text{mech, f}}$$

The energy transformed is

$$\Delta E_{\text{int}} = -\Delta E_{\text{mech}} = E_{\text{mech, i}} - E_{\text{mech, f}} = 0.588 \text{ J} - 0.225 \text{ J} = \boxed{0.363 \text{ J}}$$

(c) No.

(d) It is possible to find an effective coefficient of friction, but not the actual value of μ since n and f vary with position.

P8.45 Taking $y = 0$ at ground level, and using conservation of energy from when the boy starts from rest ($v_i = 0$) at the top of the slide ($y_i = H$) to the instant he leaves the lower end ($y_f = h$) of the frictionless slide at speed v, where his velocity is horizontal ($v_{xf} = v$, $v_{yf} = 0$), we have

$$E_0 = E_{\text{top}} \rightarrow \frac{1}{2}mv^2 + mgh = 0 + mgH$$

or $$v^2 = 2g(H - h) \qquad \textbf{[1]}$$

Considering his flight as a projectile after leaving the end of the slide,

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$$

gives the time to drop distance h to the ground as

$$-h = 0 + \frac{1}{2}(-g)t^2 \qquad \text{or} \qquad t = \sqrt{\frac{2h}{g}}$$

The horizontal distance traveled (at constant horizontal velocity) during this time is d, so

$$d = vt = v\sqrt{\frac{2h}{g}} \qquad \text{and} \qquad v = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting this expression for v into equation [1] above gives

$$\frac{gd^2}{2h} = 2g(H - h) \qquad \text{or} \qquad \boxed{H = h + \frac{d^2}{4h}}$$

P8.46 (a) Mechanical energy is conserved in the two blocks-Earth system:

$$m_2 gy = \frac{1}{2}(m_1 + m_2)v^2$$

$$v = \left[\frac{2m_2 gy}{m_1 + m_2}\right]^{1/2} = \left[\frac{2(1.90\text{ kg})(9.80\text{ m/s}^2)(0.900\text{ m})}{5.40\text{ kg}}\right]^{1/2}$$

$$= \boxed{2.49\text{ m/s}}$$

(b) For the 3.50-kg block from when the string goes slack until just before the block hits the floor, conservation of energy gives

$$\frac{1}{2}(m_2)v^2 + m_2 gy = \frac{1}{2}(m_2)v_d^2$$

$$v_d = \left[2gy + v^2\right]^{1/2} = \left[2(9.80\text{ m/s}^2)(1.20\text{ m}) + (2.49\text{ m/s})^2\right]^{1/2}$$

$$= \boxed{5.45\text{ m/s}}$$

(c) The 3.50-kg block takes this time in flight to the floor: from $y = (1/2)gt^2$ we have $t = [2(1.2)/9.8]^{1/2} = 0.495$ s. Its horizontal component of displacement at impact is then

$$x = v_d t = (2.49\text{ m/s})(0.495\text{ s}) = \boxed{1.23\text{ m}}$$

(d) $\boxed{\text{No.}}$

(e) $\boxed{\text{Some of the kinetic energy of } m_2 \text{ is transferred away as sound and some is transformed to internal energy in } m_1 \text{ and the floor.}}$

P8.47 (a) Given $m = 4.00$ kg and $x = t + 2.0t^3$, we find the velocity by differentiating:

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(t + 2t^3\right) = 1 + 6t^2$$

Then the kinetic energy from its definition is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(4.00)\left(1 + 6t^2\right)^2 = \boxed{2 + 24t^2 + 72t^4}$$

where K is in J and t is in s.

(b) Acceleration is the measure of how fast velocity is changing:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(1 + 6t^2\right) = \boxed{12t}$$

where a is in m/s^2 and t is in s.

Newton's second law gives the total force exerted on the particle

by the rest of the universe:

$$\sum F = ma = (4.00\text{ kg})(12t) = \boxed{48t}$$

where F is in N and t is in s.

(c) Power is how fast work is done to increase the object's kinetic energy:

$$P = \frac{dW}{dt} = \frac{dK}{dt} = \frac{d}{dt}\left(2.00 + 24t^2 + 72t^4\right) = \boxed{48t + 288t^3}$$

where P is in W [watts] and t is in s.

Alternatively, we could use $P = Fv = 48t(1.00 + 6.0t^2)$.

(d) The work-kinetic energy theorem $\Delta K = \sum W$ lets us find the work done on the object between $t_i = 0$ and $t_f = 2.00$ s. At $t_i = 0$ we have $K_i = 2.00$ J. At $t_f = 2.00$ s, suppressing units,

$$K_f = [2 + 24(2.00\text{ s})^2 + 72(2.00\text{ s})^4] = 1250\text{ J}$$

Therefore the work input is

$$W = K_f - K_i = 1\,248\text{ J} = \boxed{1.25 \times 10^3\text{ J}}$$

Alternatively, we could start from

$$W = \int_{t_i}^{t_f} P\,dt = \int_0^{2\text{ s}} \left(48t + 288t^3\right)dt$$

P8.48 The distance traveled by the ball from the top of the arc to the bottom is πR. The change in internal energy of the system due to the nonconservative force, the force exerted by the pitcher, is

$$\Delta E = Fd\cos 0° = F(\pi R)$$

We shall assign the gravitational energy of the ball-Earth system to be zero with the ball at the bottom of the arc.

Then

$$\Delta E_{\text{mech}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i$$

becomes

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgy_i + F(\pi R) = \frac{1}{2}mv_i^2 + mg2R + F(\pi R)$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + (2mg + \pi F)R$$

Solve for R, which is the length of her arms.

$$R = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{2mg + \pi F} = m\frac{v_f^2 - v_i^2}{4mg + 2\pi F}$$

$$R = (0.180\text{ kg})\frac{(25.0\text{ m/s})^2 - 0}{4(0.180\text{ kg})g + 2\pi(12.0\text{ N})} = 1.36\text{ m}$$

We find that her arms would need to be 1.36 m long to perform this task. This is significantly longer than the human arm.

P8.49 (a) $(K + U_g)_A = (K + U_g)_B$

$$0 + mgy_A = \frac{1}{2}mv_B^2 + 0$$

$$v_B = \sqrt{2gy_A} = \sqrt{2(9.80\text{ m/s}^2)6.30\text{ m}} = \boxed{11.1\text{ m/s}}$$

(b) $(K + U_g + U_{\text{chemical}})_B = (K + U_g)_D$

$$\frac{1}{2}mv_B^2 + U_{\text{chemical}} = \frac{1}{2}mv_D^2 + mg(y_D - y_B)$$

$$U_{\text{chemical}} = \frac{1}{2}mv_D^2 - \frac{1}{2}mv_B^2 + mg(y_D - y_B)$$

$$= \frac{1}{2}m(v_D^2 - v_B^2) + mg(y_D - y_B)$$

$$U_{\text{chemical}} = \frac{1}{2}(76.0\text{ kg})\left[(5.14\text{ m/s})^2 - (11.1\text{ m/s})^2\right] + (76.0\text{ kg})(9.80\text{ m/s}^2)(6.30\text{ m})$$

$$U_{\text{chemical}} = \boxed{1.00\times10^3\text{ J}}$$

(c) $(K + U_g)_D = (K + U_g)_E$ where E is the apex of his motion:

$$\frac{1}{2}mv_D^2 + 0 = 0 + mg(y_E - y_D)$$

$$y_E - y_D = \frac{v_D^2}{2g} = \frac{(5.14\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{1.35\text{ m}}$$

P8.50 (a) Simplified, the equation is

$$0 = (9700\text{ N/m})x^2 - (450.8\text{ N})x - 1395\text{ N}\cdot\text{m}$$

Then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{450.8\text{ N} \pm \sqrt{(450.8\text{ N})^2 - 4(9700\text{ N/m})(-1395\text{ N}\cdot\text{m})}}{2(9700\text{ N/m})}$$

$$= \frac{450.8\text{ N} \pm 7370\text{ N}}{19\,400\text{ N/m}} = \boxed{0.403\text{ m or } -0.357\text{ m}}$$

(b) From a perch at a height of 2.80 m above the top of a pile of mattresses, a 46.0-kg child jumps upward at 2.40 m/s. The mattresses behave as a linear spring with force constant 19.4 kN/m. Find the maximum amount by which they are compressed when the child lands on them.

(c) $\boxed{0.023\,2\text{ m.}}$

(d) This result is the distance by which the mattresses compress if the child just stands on them. It is the location of the equilibrium position of the oscillator.

P8.51 (a) The total external work done on the system of Jonathan-bicycle is

$$W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \frac{1}{2}(85.0\text{ kg})\left[(1.00\text{ m/s})^2 - (6.00\text{ m/s})^2\right]$$

$$= \boxed{-1\,490\text{ J}}$$

(b) Gravity does work on the Jonathan-bicycle system, and the potential (chemical) energy stored in Jonathan's body is transformed into kinetic energy:

$$\Delta K + \Delta U_{\text{chem}} = W_g$$

$$\Delta U_{\text{chem}} = W_g - \Delta K = -mgh - \Delta K$$

$$\Delta U_{\text{chem}} = -(85.0\text{ kg})g(7.30\text{ m}) - \Delta K = -6\,080 - 1\,490$$

$$= \boxed{-7\,570\text{ J}}$$

(c) Jonathan does work on the bicycle (and his mass). Treat his work as coming from outside the bicycle-Jonathan's mass system:

$$\Delta K + \Delta U_g = W_j$$

$$W_f = \Delta K + mgh = -1\ 490\ \text{J} + 6\ 080\ \text{J} = \boxed{4\ 590\ \text{J}}$$

P8.52 (a) The total external work done on the system of Jonathan-bicycle is

$$W = \Delta K = \boxed{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$$

(b) Gravity does work on the Jonathan-bicycle system, and the potential (chemical) energy stored in Jonathan's body is transformed into kinetic energy:

$$\Delta K + \Delta U_{\text{chem}} = W_g$$

$$\Delta U_{\text{chem}} = W_g - \Delta K = \boxed{-mgh - \left(\frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2}\right)}$$

(c) Jonathan does work on the bicycle (and his mass). Treat his work as coming from outside the bicycle-Jonathan's mass system:

$$\Delta K + \Delta U_g = W_j$$

$$W_j = \Delta K + mgh = \boxed{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh}$$

P8.53 (a) The block-spring-surface system is isolated with a nonconservative force acting. Therefore, Equation 8.2 becomes

$$\Delta K + \Delta U + \Delta E_{\text{int}} = 0$$

$$\left(\tfrac{1}{2}mv^2 - 0\right) + \left(\tfrac{1}{2}kx^2 - \tfrac{1}{2}kx_i^{\,2}\right) + f_k\left(x_i - x\right) = 0$$

To find the maximum speed, differentiate the equation with respect to x:

$$mv\frac{dv}{dx} + kx - f_k = 0$$

Now set $dv/dx = 0$:

$$kx - f_k = 0 \rightarrow \quad x = \frac{f_k}{k} = \frac{4.0\ \text{N}}{1.0 \times 10^3\ \text{N/m}} = 4.0 \times 10^{-3}\ \text{m}$$

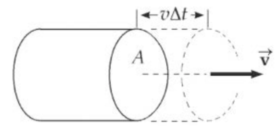
This is the compression distance of the spring, so the position of the block relative to $x = 0$ is $\boxed{x = -4.0 \times 10^{-3}\ \text{m.}}$

(b) By the same approach,

$$kx - f_k = 0 \rightarrow \quad x = \frac{f_k}{k} = \frac{10.0\text{ N}}{1.0 \times 10^3\text{ N/m}} = 1.0 \times 10^{-2}\text{ m}$$

so the position of the block is $\boxed{x = -1.0 \times 10^{-2}\text{ m.}}$

P8.54 $P\Delta t = W = \Delta K = \dfrac{(\Delta m)v^2}{2}$



ANS. FIG. P8.54

The density is $\rho = \dfrac{\Delta m}{\text{volume}} = \dfrac{\Delta m}{A\Delta x}$

Substituting this into the first equation and solving for *P*, since $\dfrac{\Delta x}{\Delta t} = v$ for a constant speed, we get

$$P = \boxed{\frac{\rho A v^3}{2}}$$

Also, since $P = Fv$,

$$\boxed{F = \frac{\rho A v^2}{2}}$$

Our model predicts the same proportionalities as the empirical equation, and gives $D = 1$ for the drag coefficient. Air actually slips around the moving object, instead of accumulating in front of it. For this reason, the drag coefficient is not necessarily unity. It is typically less than one for a streamlined object and can be greater than one if the airflow around the object is complicated.

P8.55 $P = \dfrac{1}{2}D\rho\pi r^2 v^3$

(a) We use 1.20 kg/m^3 for the density of air, and calculate

$$P_a = \frac{1}{2}(1)\left(1.20\text{ kg/m}^3\right)\pi(1.50\text{ m})^2(8.00\text{ m/s})^3$$
$$= \boxed{2.17 \times 10^3\text{ W}}$$

(b) We solve part (b) by proportion:

$$\frac{P_b}{P_a} = \frac{v_b^3}{v_a^3} = \left(\frac{24\text{ m/s}}{8\text{ m/s}}\right)^3 = 3^3 = 27$$

$$P_b = 27\left(2.17 \times 10^3\text{ W}\right) = 5.86 \times 10^4\text{ W} = \boxed{58.6\text{ kW}}$$

P8.56 (a) In Example 8.3, $m = 35.0\ g$, $y_A = -0.120$ m, $y_B = 0$, and $k = 958$ N/m. Friction $f_k = 2.00$ N acts over distance $d = 0.600$ m. For the ball-spring-Earth system, $K_i = 0$, $U_{gi} = mgy_A$, $U_{si} = \frac{1}{2}kx^2$, where $x = |y_A|$; $K_f = 0$, $U_{gf} = mgy_C$, and $U_{sf} = 0$.

$$\Delta K + \Delta U = -f_k d$$

$$0 + (mgy_C - mgy_A) + \left(0 - \frac{1}{2}kx^2\right) = -f_k d$$

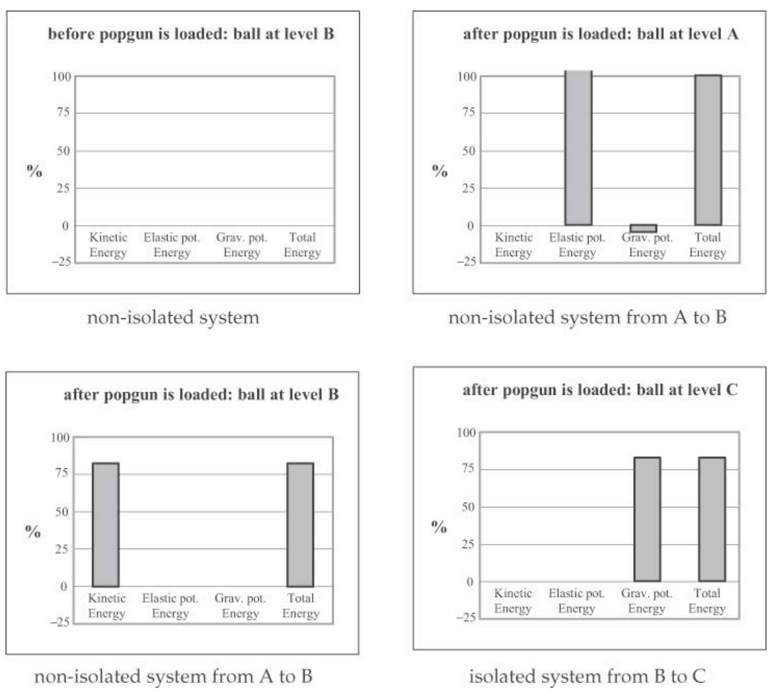
$$mgy_C = mgy_A + \frac{1}{2}kx^2 - f_k d$$

$$y_C = y_A + \frac{\frac{1}{2}kx^2 - f_k d}{mg}$$

$$= -0.120 + \frac{\frac{1}{2}(958\ \text{N/m})(0.120\ \text{m})^2 - (2.00\ \text{N})(0.600\ \text{m})}{(0.035\ \text{kg})g}$$

$$= \boxed{16.5\ \text{m}}$$

(b) The ball-spring-Earth system is not isolated as the popgun is loaded. In addition, as the ball travels up the barrel, a nonconservative force acts within the system. The system is isolated after the ball leaves the barrel.



ANS. FIG. P8.56

P8.57 (a) To calculate the change in kinetic energy, we integrate the expression for *a* as a function of time to obtain the car's velocity:

$$v = \int_0^t a\,dt = \int_0^t \left(1.16t - 0.210t^2 + 0.240t^3\right)dt$$

$$= 1.16\frac{t^2}{2} - 0.210\frac{t^3}{3} + 0.240\frac{t^4}{4}\bigg|_0^t = 0.580t^2 - 0.070t^3 + 0.060t^4$$

At $t = 0$, $v_i = 0$. At $t = 2.5$ s,

$$v_f = \left(0.580\text{ m/s}^3\right)\left(2.50\text{ s}\right)^2 - \left(0.070\text{ m/s}^4\right)\left(2.50\text{ s}\right)^3$$
$$+ \left(0.060\text{ m/s}^5\right)\left(2.50\text{ s}\right)^4 = 4.88\text{ m/s}$$

The change in kinetic energy during this interval is then

$$K_i + W = K_f$$

$$0 + W = \frac{1}{2}mv_f^2 = \frac{1}{2}\left(1\,160\text{ kg}\right)\left(4.88\text{ m/s}\right)^2 = \boxed{1.38\times10^4\text{ J}}$$

(b) The road does work on the car when the engine turns the wheels and the car moves. The engine and the road together transform chemical potential energy in the gasoline into kinetic energy of the car.

$$P = \frac{W}{\Delta t} = \frac{1.38\times10^4\text{ J}}{2.50\text{ s}}$$

$$P = \boxed{5.52\times10^3\text{ W}}$$

(c) The value in (b) represents only energy that leaves the engine and is transformed to kinetic energy of the car. Additional energy leaves the engine by sound and heat. More energy leaves the engine to do work against friction forces and air resistance.

P8.58 At the bottom of the circle, the initial speed of the coaster is 22.0 m/s. As the coaster travels up the circle, it will slow down. At the top of the track, the centripetal acceleration must be at least that of gravity, *g*, to remain on the track. Apply conservation of energy to the roller coaster-Earth system to find the speed of the coaster at the top of the circle so that we may find the centripetal acceleration of the coaster.

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv_{\text{top}}^2 - \frac{1}{2}mv_{\text{bottom}}^2\right) + \left(mgy_{\text{top}} - mgy_{\text{bottom}}\right) = 0$$

$$\left(\frac{1}{2}mv_{\text{top}}^2 - \frac{1}{2}mv_{\text{bottom}}^2\right) + (mg2R - 0) = 0 \rightarrow v_{\text{top}}^2 = v_{\text{bottom}}^2 - 4gR$$

$$v_{\text{top}}^2 = (22.0\ \text{m/s})^2 - 4g(12.0\ \text{m}) = 13.6\ \text{m}^2/\text{s}^2$$

For this speed, the centripetal acceleration is

$$a_c = \frac{v_{\text{top}}^2}{R} = \frac{13.6\ \text{m}^2/\text{s}^2}{12.0\ \text{m}} = 1.13\ \text{m/s}^2$$

> The centripetal acceleration of each passenger as the coaster passes over the top of the circle is $1.13\ \text{m/s}^2$. Since this is less than the acceleration due to gravity, the unrestrained passengers will fall out of the cars!

P8.59 (a) The energy stored in the spring is the elastic potential energy, $U = \frac{1}{2}kx^2$, where $k = 850$ N/m. At $x = 6.00$ cm,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(850\ \text{N/m})(0.0600\ \text{m})^2 = \boxed{1.53\ \text{J}}$$

At the equilibrium position, $x = 0$, $U = \boxed{0\ \text{J}}$.

(b) Applying energy conservation to the block-spring system:

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + (U_f - U_i) = 0 \rightarrow \left(\frac{1}{2}mv_f^2 - 0\right) = -(U_f - U_i)$$

$$\frac{1}{2}mv_f^2 = U_i - U_f$$

because the block is released from rest. For $x_f = 0$, $U = 0$, and

$$\frac{1}{2}mv_f^2 = U_i - U_f \rightarrow v_f = \sqrt{\frac{2(U_i - U_f)}{m}}$$

$$v_f = \sqrt{\frac{2(1.53\ \text{J})}{1.00\ \text{kg}}}$$

$$\boxed{v_f = 1.75\ \text{m/s}}$$

(c) From (b) above, for $x_f = x_i/2 = 3.00$ cm,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(850\ \text{N/m})(0.0300\ \text{m})^2 = 0.383\ \text{J}$$

and

$$\frac{1}{2}mv_f^2 = U_i - U_f \rightarrow v_f = \sqrt{\frac{2(U_i - U_f)}{m}}$$

$$v_f = \sqrt{\frac{2(1.53\text{ J} - 0.383\text{ J})}{1.00\text{ kg}}} = \sqrt{\frac{2(1.15\text{ J})}{1.00\text{ kg}}}$$

$$\boxed{v_f = 1.51\text{ m/s}}$$

P8.60 (a) The suggested equation $P\Delta t = bwd$ implies all of the following cases:

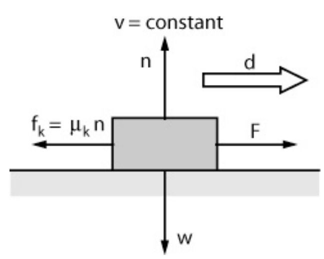
(1) $P\Delta t = b\left(\frac{w}{2}\right)(2d)$

(2) $P\left(\frac{\Delta t}{2}\right) = b\left(\frac{w}{2}\right)d$

(3) $P\left(\frac{\Delta t}{2}\right) = bw\left(\frac{d}{2}\right)$ and

(4) $\left(\frac{P}{2}\right)\Delta t = b\left(\frac{w}{2}\right)d$

These are all of the proportionalities Aristotle lists.



ANS FIG. P8.60

(b) For one example, consider a horizontal force F pushing an object of weight w at constant velocity across a horizontal floor with which the object has coefficient of friction μ_k.

$\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ implies that

$$+n - w = 0 \text{ and } F - \mu_k n = 0$$

so that $F = \mu_k w$.

As the object moves a distance d, the agent exerting the force does work

$$W = Fd\cos\theta = Fd\cos 0° = \mu_k wd$$

and puts out power $P = \dfrac{W}{\Delta t}$

This yields the equation $P\Delta t = \mu_k wd$ which represents Aristotle's theory with $b = \mu_k$.

Our theory is more general than Aristotle's. Ours can also describe accelerated motion.

P8.61 $k = 2.50 \times 10^4$ N/m, $m = 25.0$ kg

$x_A = -0.100$ m, $U_g\big|_{x=0} = U_s\big|_{x=0} = 0$

(a) At point A, the total energy of the child-pogo-stick-Earth system is given by

$$E_{\text{mech}} = K_A + U_{gA} + U_s \quad \rightarrow \quad E_{\text{mech}} = 0 + mgx_A + \frac{1}{2}kx_A^2$$

$$E_{\text{mech}} = (25.0\text{ kg})(9.80\text{ m/s}^2)(-0.100\text{ m}) + \frac{1}{2}(2.50\times10^4\text{ N/m})(-0.100\text{ m})^2$$

$$E_{\text{mech}} = -24.5\text{ J} + 125\text{ J} = \boxed{100\text{ J}}$$

(b) Since only conservative forces are involved, the total energy of the child-pogo-stick-Earth system at point *C* is the same as that at point *A*.

$$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$$

$$0 + (25.0\text{ kg})(9.80\text{ m/s}^2)x_C + 0 = 0 - 24.5\text{ J} + 125\text{ J}$$

$$x_C = \boxed{0.410\text{ m}}$$

(c) $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$

$$\frac{1}{2}(25.0\text{ kg})v_B^2 + 0 + 0 = 0 + (-24.5\text{ J}) + 125\text{ J}$$

$$v_B = \boxed{2.84\text{ m/s}}$$

(d) The energy of the system for configurations in which the spring is compressed is

$$E = K + \frac{1}{2}kx^2 - mgx$$

where x is the compression distance of the spring.

To find the position x for which the kinetic energy is a maximum, solve this expression for K, differentiate with respect to x, and set the result equal to zero:

$$K = E - \frac{1}{2}kx^2 + mgx$$

$$\frac{dK}{dx} = 0 - kx + mg = 0 \quad \rightarrow \quad x = \frac{mg}{k}$$

Substitute numerical values:

$$x = \frac{(25.0 \text{ kg})(9.80 \text{ m/s}^2)}{2.50 \times 10^4 \text{ N/m}} = 0.009\ 8 \text{ m} = 0.98 \text{ cm}$$

Because this is the value for the compression distance of the spring, this position is 0.98 cm below $x = 0$.

$$K = K_{max} \text{ at } x = \boxed{-9.80 \text{ mm}}$$

(e) $$K_{max} = K_A + \left(U_{gA} - U_g\big|_{x=-9.80 \text{ mm}}\right) + \left(U_{sA} - U_s\big|_{x=-9.80 \text{ mm}}\right)$$

or

$$\frac{1}{2}(25.0 \text{ kg})v_{max}^2$$
$$= (25.0 \text{ kg})(9.80 \text{ m/s}^2)\left[(-0.100 \text{ m}) - (-0.009\ 8 \text{ m})\right]$$
$$+\frac{1}{2}(2.50 \times 10^4 \text{ N/m})\left[(-0.100 \text{ m})^2 - (-0.009\ 8 \text{ m})^2\right]$$

yielding $v_{max} = \boxed{2.85 \text{ m/s}}$

P8.62 (a) Between the second and the third picture, $\Delta E_{mech} = \Delta K + \Delta U$:

$$-\mu mgd = -\frac{1}{2}mv_i^2 + \frac{1}{2}kd^2$$

$$\frac{1}{2}(50.0 \text{ N/m})d^2 + 0.250(1.00 \text{ kg})(9.80 \text{ m/s}^2)d$$
$$-\frac{1}{2}(1.00 \text{ kg})(3.00 \text{ m/s})^2 = 0$$

$$d = \frac{[-2.45 \pm 21.35] \text{ N}}{50.0 \text{ N/m}} = \boxed{0.378 \text{ m}}$$

(b) Between picture two and picture four, $\Delta E_{mech} = \Delta K + \Delta U$:

$$-\mu mg(2d) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

which gives

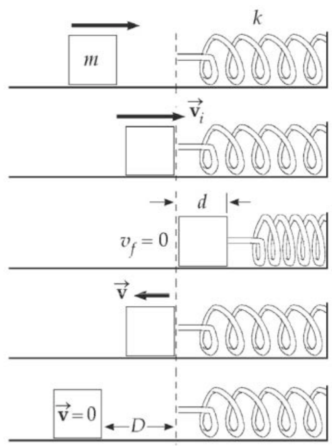
$$v_f = \sqrt{(3.00\text{ m/s})^2 - \frac{2}{(1.00\text{ kg})}(2.45\text{ N})(2)(0.378\text{ m})}$$

$$= \boxed{2.30\text{ m/s}}$$

(c) For the motion from picture two to picture five in the figure below, $\Delta E_{mech} = \Delta K + \Delta U$:

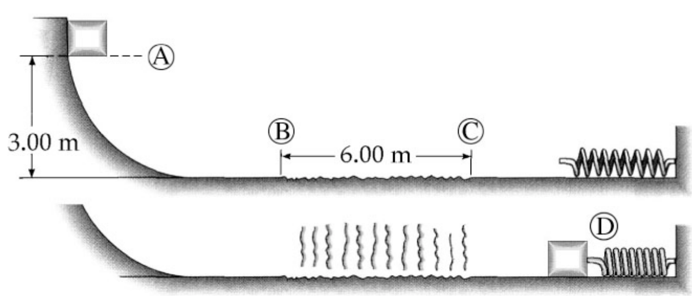
$$-\mu mg(D + 2d) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$D = \frac{(1.00\text{ kg})(3.00\text{ m/s})^2}{2(0.250)(1.00\text{ kg})(9.80\text{ m/s}^2)} - 2(0.378\text{ m}) = \boxed{1.08\text{ m}}$$



ANS. FIG P8.62

P8.63 The easiest way to solve this problem about a chain-reaction process is by considering the energy changes experienced by the block between the point of release (initial) and the point of full compression of the spring (final). Recall that the change in potential energy (gravitational and elastic) plus the change in kinetic energy must equal the work done on the block by non-conservative forces. We choose the gravitational potential energy to be zero along the flat portion of the track.



ANS. FIG. P8.63

There is zero spring potential energy in situation Ⓐ and zero gravitational potential energy in situation Ⓓ. Putting the energy equation into symbols:

$$K_D - K_A - U_{gA} + U_{sD} = -f_k d_{BC}$$

Expanding into specific variables:

$$0 - 0 - mgy_A + \frac{1}{2}kx_s^2 = -f_k d_{BC}$$

The friction force is $f_k = \mu_k mg$, so

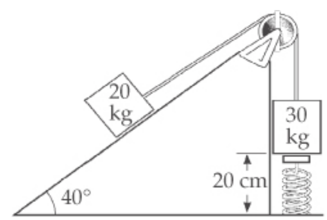
$$mgy_A - \frac{1}{2}kx^2 = \mu_k mgd$$

Solving for the unknown variable μ_k gives

$$\mu_k = \frac{y_A}{d} - \frac{kx^2}{2mgd}$$

$$= \frac{3.00\text{ m}}{6.00\text{ m}} - \frac{(2\,250\text{ N/m})(0.300\text{ m})^2}{2(10.0\text{ kg})(9.80\text{ m/s}^2)(6.00\text{ m})} = \boxed{0.328}$$

P8.64 We choose the zero configuration of potential energy for the 30.0-kg block to be at the unstretched position of the spring, and for the 20.0-kg block to be at its lowest point on the incline, just before the system is released from rest. From conservation of energy, we have



ANS. FIG. P8.64

$$(K + U)_i = (K + U)_f$$

$$0 + (30.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) + \frac{1}{2}(250\text{ N/m})(0.200\text{ m})^2$$
$$= \frac{1}{2}(20.0\text{ kg} + 30.0\text{ kg})v^2$$
$$+ (20.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})\sin 40.0°$$

$$58.8\text{ J} + 5.00\text{ J} = (25.0\text{ kg})v^2 + 25.2\text{ J}$$

$$\boxed{v = 1.24\text{ m/s}}$$

P8.65 (a) For the isolated spring-block system,

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv^2 - 0\right) + \left(0 - \frac{1}{2}kx^2\right) = 0$$

$$x = \sqrt{\frac{m}{k}}v = \sqrt{\frac{0.500 \text{ kg}}{450 \text{ N/m}}}(12.0 \text{ m/s})$$

$$x = \boxed{0.400 \text{ m}}$$

(b) $\Delta K + \Delta U + \Delta E_{int} = 0$

$$\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + (2mgR - 0) + f_k(\pi R) = 0$$

$$v_f = \sqrt{v_i^2 - 4gR - \frac{2\pi f_k R}{m}}$$

$$= \sqrt{(12.0 \text{ m/s})^2 - 4(9.80 \text{ m/s}^2)(1.00 \text{ m}) - \frac{2\pi(7.00 \text{ N})(1.00 \text{ m})}{0.500 \text{ kg}}}$$

$$v_f = \boxed{4.10 \text{ m/s}}$$

(c) Does the block fall off at or before the top of the track? The block falls if $a_c < g$.

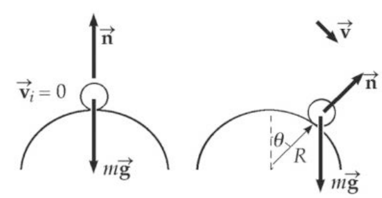
$$a_c = \frac{v_T^2}{R} = \frac{(4.10 \text{ m/s})^2}{1.00 \text{ m}} = 16.8 \text{ m/s}^2$$

Therefore $a_c > g$ and the $\boxed{\text{block stays on the track}}$.

P8.66 m = mass of pumpkin

R = radius of silo top

$$\sum F_r = ma_r \Rightarrow n - mg\cos\theta = -m\frac{v^2}{R}$$



ANS. FIG. P8.66

When the pumpkin first loses contact with the surface, $n = 0$.

Thus, at the point where it leaves the surface: $v^2 = Rg\cos\theta$.

Choose $U_g = 0$ in the $\theta = 90.0°$ plane. Then applying conservation of energy for the pumpkin-Earth system between the starting point and the point where the pumpkin leaves the surface gives

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv^2 + mgR\cos\theta = 0 + mgR$$

Using the result from the force analysis, this becomes

$$\frac{1}{2}\text{mRg}\cos\theta + \text{mgR}\cos\theta = \text{mgR}, \text{ which reduces to}$$

$$\cos\theta = \frac{2}{3}, \text{ and gives } \theta = \cos^{-1}(2/3) = \boxed{48.2°}$$

as the angle at which the pumpkin will lose contact with the surface.

P8.67 Convert the speed to metric units:

$$v = (100\ \text{km/h})\left(\frac{1\ 000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3\ 600\ \text{s}}\right) = 27.8\ \text{m/s}$$

Write Equation 8.2 for this situation, treating the car and surrounding air as an isolated system with a nonconservative force acting:

$$\Delta K + \Delta U_{\text{grav}} + \Delta U_{\text{fuel}} + \Delta E_{\text{int}} = 0$$

The power of the engine is a measure of how fast it can convert chemical potential energy in the fuel to other forms. The magnitude of the change in energy to other forms is equal to the negative of the change in potential energy in the fuel: $\Delta E_{\text{other forms}} = -\Delta U_{\text{fuel}}$. Therefore, if the car moves a distance *d* along the hill,

$$\begin{aligned} P &= -\frac{\Delta U_{\text{fuel}}}{\Delta t} = -\frac{\left(-\Delta K - \Delta U_{\text{grav}} - \Delta E_{\text{int}}\right)}{\Delta t} \\ &= \frac{0 + (mgd\sin 3.2° - 0) + \frac{1}{2}D\rho Av^2 d}{\Delta t} \\ &= mgv\sin 3.2° + \frac{1}{2}D\rho Av^3 \end{aligned}$$

where we have recognized $d/\Delta t$ as the speed *v* of the car. Substituting numerical values,

$$\begin{aligned} P = {} & (1\ 500\ \text{kg})(9.80\ \text{m/s}^2)(27.8\ \text{m/s})\sin 3.2° \\ & + \frac{1}{2}(0.330)(1.20\ \text{kg/m}^3)(2.50\ \text{m}^2)(27.8\ \text{m/s})^3 \end{aligned}$$

$$\boxed{P = 33.4\ \text{kW} = 44.8\ \text{hp}}$$

The actual power will be larger than this because additional energy coming from the engine is used to do work against internal friction in the moving parts of the car and rolling friction with the road. In addition, some energy from the engine is radiated away by sound. Finally, some of the energy from the fuel raises the internal energy of the engine, and energy leaves the warm engine by heat into the cooler air.

P8.68 (a) Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg, and the ball swings equally high on both sides.

(b) The ball will swing in a circle of radius $R = (L - d)$ about the peg. If the ball is to travel in the circle, the minimum centripetal acceleration at the top of the circle must be that of gravity:

$$\frac{mv^2}{R} = g \;\rightarrow\; v^2 = g(L-d)$$

When the ball is released from rest, $U_i = mgL$, and when it is at the top of the circle, $U_i = mg2(L - d)$, where height is measured from the bottom of the swing. By energy conservation,

$$mgL = mg2(L-d) + \frac{1}{2}mv^2$$

From this and the condition on v^2 we find $\boxed{d = \frac{3L}{5}}$.

P8.69 If the spring is just barely able to lift the lower block from the table, the spring lifts it through no noticeable distance, but exerts on the block a force equal to its weight Mg. The extension of the spring, from $|\vec{F}_s| = kx$, must be Mg/k. Between an initial point at release and a final point when the moving block first comes to rest, we have

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$$

$$0 + mg\left(-\frac{4mg}{k}\right) + \frac{1}{2}k\left(\frac{4mg}{k}\right)^2 = 0 + mg\left(\frac{Mg}{k}\right) + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$-\frac{4m^2g^2}{k} + \frac{8m^2g^2}{k} = \frac{mMg^2}{k} + \frac{M^2g^2}{2k}$$

$$4m^2 = mM + \frac{M^2}{2}$$

$$\frac{M^2}{2} + mM - 4m^2 = 0$$

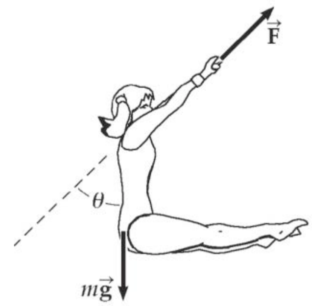
$$M = \frac{-m \pm \sqrt{m^2 - 4\left(\frac{1}{2}\right)\left(-4m^2\right)}}{2\left(\frac{1}{2}\right)} = -m \pm \sqrt{9m^2}$$

Only a positive mass is physical, so we take $M = m(3-1) = \boxed{2m}$.

P8.70 The force needed to hang on is equal to the force F the trapeze bar exerts on the performer. From the free-body diagram for the performer's body, as shown,

$$F - mg\cos\theta = m\frac{v^2}{\ell}$$

or $$F = mg\cos\theta + m\frac{v^2}{\ell}$$



ANS. FIG. P8.70

At the bottom of the swing, $\theta = 0°$, so

$$F = mg + m\frac{v^2}{\ell}$$

The performer cannot sustain a tension of more than $1.80mg$. What is the force F at the bottom of the swing? To find out, apply conservation of mechanical energy of the performer-Earth system as the performer moves between the starting point and the bottom:

$$mg\ell(1 - \cos 60.0°) = \frac{1}{2}mv^2 \rightarrow \frac{mv^2}{\ell} = 2mg(1 - \cos 60.0°) = mg$$

Hence, $F = mg + m\frac{v^2}{\ell} = mg + mg = 2mg$ at the bottom.

$\boxed{\text{The tension at the bottom is greater than the performer can withstand; therefore the situation is impossible.}}$

***P8.71** We first determine the energy output of the runner:

$$= (0.600\ \text{J/kg}\cdot\text{step})(60.0\ \text{kg})\left(\frac{1\ \text{step}}{1.50\ \text{m}}\right) = 24.0\ \text{J/m}$$

From this we calculate the force exerted by the runner per step:

$$F = (24.0\ \text{J/m})(1\ \text{N}\cdot\text{m/J}) = 24.0\ \text{N}$$

Then, from the definition of power, $P = Fv$, we obtain

$$v = \frac{P}{F} = \frac{70.0\ \text{W}}{24.0\ \text{N}} = \boxed{2.92\ \text{m/s}}$$

P8.72 (a) At the top of the loop the car and riders are in free fall:

$$\sum F_y = ma_y:$$

$$mg \text{ down} = \frac{mv^2}{R} \text{ down}$$

$$v = \sqrt{Rg}$$

Energy of the car-riders-Earth system is conserved between release and top of loop:

$$K_i + U_{gi} = K_f + U_{gf}:$$

$$0 + mgh = \frac{1}{2}mv^2 + mg(2R)$$

$$gh = \frac{1}{2}Rg + g(2R)$$

$$\boxed{h = \frac{5R}{2}}$$

(b) Let *h* now represent the height $\geq 2.5\text{ R}$ of the release point. At the bottom of the loop we have

$$mgh = \frac{1}{2}mv_b^2$$

or $\quad v_b^2 = 2gh$

then, from $\sum F_y = ma_y$:

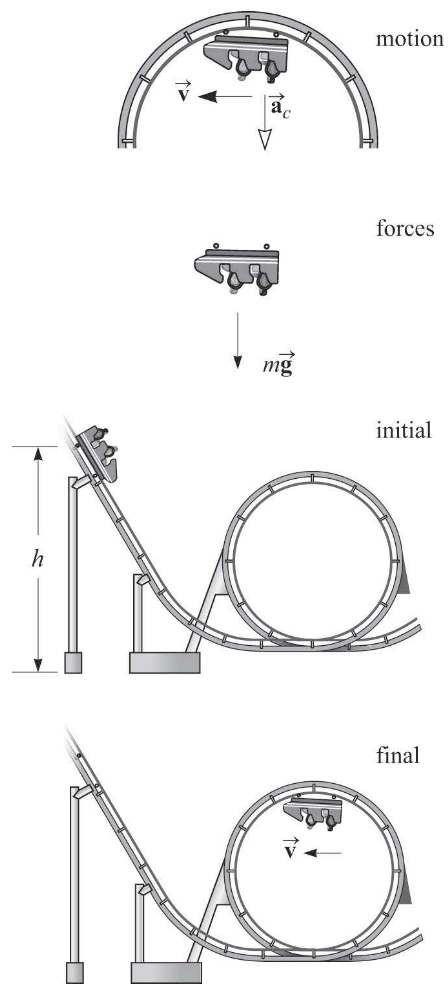
$$n_b - mg = \frac{mv_b^2}{R}(\text{up})$$

$$n_b = mg + \frac{m(2gh)}{R}$$

At the top of the loop,

$$mgh = \frac{1}{2}mv_t^2 + mg(2R)$$

$$v_t^2 = 2gh - 4gR$$



ANS. FIG. P8.72

from $\sum F_y = ma_y$:

$$-n_t - mg = -\frac{mv_t^2}{R}$$

$$n_t = -mg + \frac{m}{R}(2gh - 4gR)$$

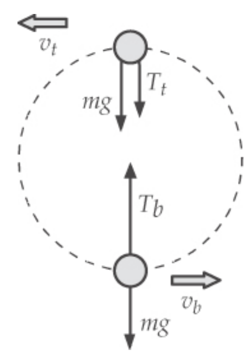
$$n_t = \frac{m(2gh)}{R} - 5mg$$

Then the normal force at the bottom is larger by

$$n_b - n_t = mg + \frac{m(2gh)}{R} - \frac{m(2gh)}{R} + 5mg = \boxed{6mg}$$

Note that this is the same result we will obtain for the difference in the tension in the string at the top and bottom of a vertical circle in Problem 73.

P8.73 Applying Newton's second law at the bottom (b) and top (t) of the circle gives



ANS. FIG. P8.73

$$T_b - mg = \frac{mv_b^2}{R} \quad \text{and} \quad -T_t - mg = -\frac{mv_t^2}{R}$$

Adding these gives

$$T_b = T_t + 2mg + \frac{m(v_b^2 - v_t^2)}{R}$$

Also, energy must be conserved and $\Delta U + \Delta K = 0$.

So, $\quad \frac{m(v_b^2 - v_t^2)}{2} + (0 - 2mgR) = 0 \quad$ and $\quad \frac{m(v_b^2 - v_t^2)}{R} = 4mg$

Substituting into the above equation gives $\boxed{T_b = T_t + 6mg}$.

P8.74 (a) $\boxed{\text{No.}}$ The system of the airplane and the surrounding air is nonisolated. There are two forces acting on the plane that move through displacements, the thrust due to the engine (acting across the boundary of the system) and a resistive force due to the air (acting within the system). Since the air resistance force is nonconservative, some of the energy in the system is transformed to internal energy in the air and the surface of the airplane. Therefore, the change in kinetic energy of the plane is less than the positive work done by the engine thrust. So,

$\boxed{\text{mechanical energy is not conserved in this case.}}$

(b) Since the plane is in level flight, $U_{gf} = U_{gi}$ and the conservation of energy for nonisolated systems reduces to

$$\sum W_{\text{other forces}} = W = \Delta K + \Delta U + \Delta E_{\text{int}}$$

or

$$W = W_{\text{thrust}} = K_f - K_i - fs$$

$$F(\cos 0°)s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 - f(\cos 180°)s$$

This gives

$$v_f = \sqrt{v_i^2 + \frac{2(F-f)s}{m}}$$

$$= \sqrt{(60.0\text{ m/s})^2 + \frac{2\left[(7.50-4.00)\times 10^4\text{ N}\right](500\text{ m})}{1.50\times 10^4\text{ kg}}}$$

$$v_f = \boxed{77.0\text{ m/s}}$$

P8.75 (a) As at the end of the process analyzed in Example 8.8, we begin with a 0.800-kg block at rest on the end of a spring with stiffness constant 50.0 N/m, compressed 0.092 4 m. The energy in the spring is (1/2)(50 N/m)(0.092 4 m)^2 = 0.214 J. To push the block back to the unstressed spring position would require work against friction of magnitude 3.92 N (0.092 4 m) = 0.362 J.

Because 0.214 J is less than 0.362 J, the spring cannot push the object back to $x = 0$.

(b) The block approaches the spring with energy

$$\frac{1}{2}mv^2 = \frac{1}{2}(0.800\text{ kg})(1.20\text{ m/s})^2 = 0.576\text{ J}$$

It travels against friction by equal distances in compressing the spring and in being pushed back out, so half of the initial kinetic energy is transformed to internal energy in its motion to the right and the rest in its motion to the left. The spring must possess one-half of this energy at its maximum compression:

$$\frac{0.576\text{ J}}{2} = \frac{1}{2}(50.0\text{ N/m})x^2$$

so $x = 0.107$ m

For the compression process we have the conservation of energy equation

$$0.576\text{ J} + \mu_k\, 7.84\text{ N}\left(0.107\text{ m}\right)\cos 180° = 0.288\text{ J}$$

$$\text{so } \mu_k = 0.288\text{ J}/0.841\text{ J} = \boxed{0.342}$$

As a check, the decompression process is described by

$$0.288\text{ J} + \mu_k\, 7.84\text{ N}\left(0.107\text{ m}\right)\cos 180° = 0$$

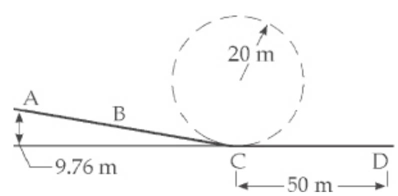
which gives the same answer for the coefficient of friction.

***P8.76** As it moves at constant speed, the bicycle is in equilibrium. The forward friction force is equal in magnitude to the air resistance, which we write as av^2, where a is a proportionality constant. The exercising woman exerts the friction force on the ground; by Newton's third law, it is this same magnitude again. The woman's power output is $P = Fv = av^3 = ch$, where c is another constant and h is her heart rate. We are given $a(22\text{ km/h})^3 = c(90\text{ beats/min})$. For her minimum heart rate we have $av_{min}^3 = c\left(136\text{ beats/min}\right)$. By division $\left(\dfrac{v_{min}}{22\text{ km/h}}\right)^3 = \dfrac{136}{90}$.

$$v_{min} = \left(\frac{136}{90}\right)^{1/3}\left(22\text{ km/h}\right) = \boxed{25.2\text{ km/h}}$$

Similarly, $v_{max} = \left(\dfrac{166}{90}\right)^{1/3}\left(22\text{ km/h}\right) = \boxed{27.0\text{ km/h}}$.

P8.77 (a) Conservation of energy for the sled-rider-Earth system, between A and C:



ANS. FIG. P8.77

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}m\left(2.50\text{ m/s}\right)^2 + m\left(9.80\text{ m/s}^2\right)\left(9.76\text{ m}\right) = \frac{1}{2}mv_C^2 + 0$$

$$v_C = \sqrt{\left(2.50\text{ m/s}\right)^2 + 2\left(9.80\text{ m/s}^2\right)\left(9.76\text{ m}\right)} = \boxed{14.1\text{ m/s}}$$

(b) Incorporating the loss of mechanical energy during the portion of the motion in the water, we have, for the entire motion between A and D (the rider's stopping point),

$$K_i + U_{gi} - f_k d = K_f + U_{gf}:$$

$$\frac{1}{2}(80.0\text{ kg})(2.50\text{ m/s})^2 + (80.0\text{ kg})(9.80\text{ m/s}^2)(9.76\text{ m}) - f_k d = 0 + 0$$

$$-f_k d = 7.90\times 10^3\text{ J}$$

The water exerts a friction force

$$f_k = \frac{7.90\times 10^3\text{ J}}{d} = \frac{7.90\times 10^3\text{ N}\cdot\text{m}}{50.0\text{ m}} = 158\text{ N}$$

and also a normal force of

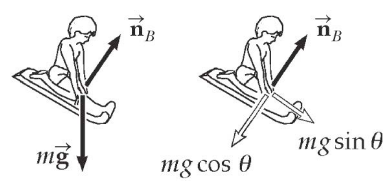
$$n = mg = (80.0\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$$

The magnitude of the water force is

$$\sqrt{(158\text{ N})^2 + (784\text{ N})^2} = \boxed{800\text{ N}}$$

(c) The angle of the slide is

$$\theta = \sin^{-1}\left(\frac{9.76\text{ m}}{54.3\text{ m}}\right) = 10.4°$$



ANS. FIG. P8.77(c)

For forces perpendicular to the track at B,

$$\sum F_y = ma_y: \qquad n_B - mg\cos\theta = 0$$

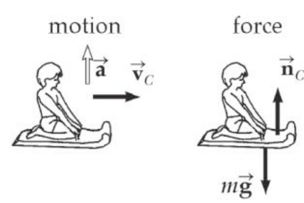
$$n_B = (80.0\text{ kg})(9.80\text{ m/s}^2)\cos 10.4° = \boxed{771\text{ N}}$$

(d) $\sum F_y = ma_y$:

$$+n_C - mg = \frac{mv_C^2}{r}$$

$$n_C = (80.0\text{ kg})(9.80\text{ m/s}^2) + \frac{(80.0\text{ kg})(14.1\text{ m/s})^2}{20.0\text{ m}}$$

$$n_C = \boxed{1.57\times 10^3\text{ N up}}$$



ANS. FIG. P8.77(d)

The rider pays for the thrills of a giddy height at A, and a high speed and tremendous splash at C. As a bonus, he gets the quick change in direction and magnitude among the forces we found in parts (d), (b), and (c).

P8.78 (a) Maximum speed occurs after the needle leaves the spring, before it enters the body. We assume the needle is fired horizontally.

ANS. FIG. P8.78(a)

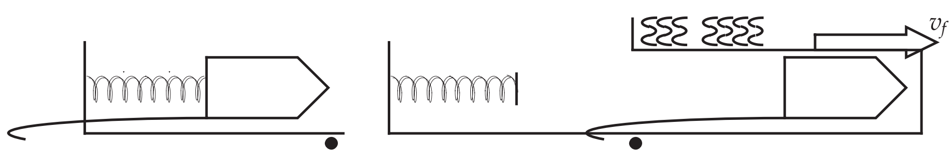
$$K_i + U_i - f_k d = K_f + U_f$$

$$0 + \frac{1}{2}kx^2 - 0 = \frac{1}{2}mv_{max}^2 + 0$$

$$\frac{1}{2}(375\ \text{N/m})(0.081\ \text{m})^2 = \frac{1}{2}(0.005\,6\ \text{kg})v_{max}^2$$

$$\left(\frac{2(1.23\ \text{J})}{0.005\,6\ \text{kg}}\right)^{1/2} = v_{max} = \boxed{21.0\ \text{m/s}}$$

(b) The same energy of 1.23 J as in part (a) now becomes partly internal energy in the soft tissue, partly internal energy in the organ, and partly kinetic energy of the needle just before it runs into the stop. We write a conservation of energy equation to describe this process:



ANS. FIG. P8.78(b)

$$K_i + U_i - f_{k1}d_1 - f_{k2}d_2 = K_f + U_f$$

$$0 + \frac{1}{2}kx^2 - f_{k1}d_1 - f_{k2}d_2 = \frac{1}{2}mv_f^2 + 0$$

$$1.23\ \text{J} - 7.60\ \text{N}(0.024\ \text{m}) - 9.20\ \text{N}(0.035\ \text{m}) = \frac{1}{2}(0.005\,6\ \text{kg})v_f^2$$

$$\left(\frac{2(1.23\ \text{J} - 0.182\ \text{J} - 0.322\ \text{J})}{0.005\,6\ \text{kg}}\right)^{1/2} = v_f = \boxed{16.1\ \text{m/s}}$$

Challenge Problems

P8.79 (a) Let *m* be the mass of the whole board. The portion on the rough surface has mass *mx*/*L*. The normal force supporting it is $\frac{mxg}{L}$ and the friction force is $\frac{\mu_k mgx}{L} = ma$. Then

$$\boxed{a = \frac{\mu_k gx}{L} \text{ opposite to the motion}}$$

(b) In an incremental bit of forward motion *dx*, the kinetic energy converted into internal energy is $f_k dx = \frac{\mu_k mgx}{L} dx$. The whole energy converted is

$$\frac{1}{2}mv^2 = \int_0^L \frac{\mu_k mgx}{L} dx = \frac{\mu_k mg}{L} \frac{x^2}{2}\bigg|_0^L = \frac{\mu_k mgL}{2}$$

$$\boxed{v = \sqrt{\mu_k gL}}$$

P8.80 (a) $U_g = mgy = (64.0 \text{ kg})(9.80 \text{ m/s}^2)y = \boxed{(627 \text{ N})y}$

(b) At the original height and at all heights above 65.0 m – 25.8 m = 39.2 m, the cord is unstretched and $\boxed{U_s = 0}$. Below 39.2 m, the cord extension *x* is given by *x* = 39.2 m – *y*, so the elastic energy is

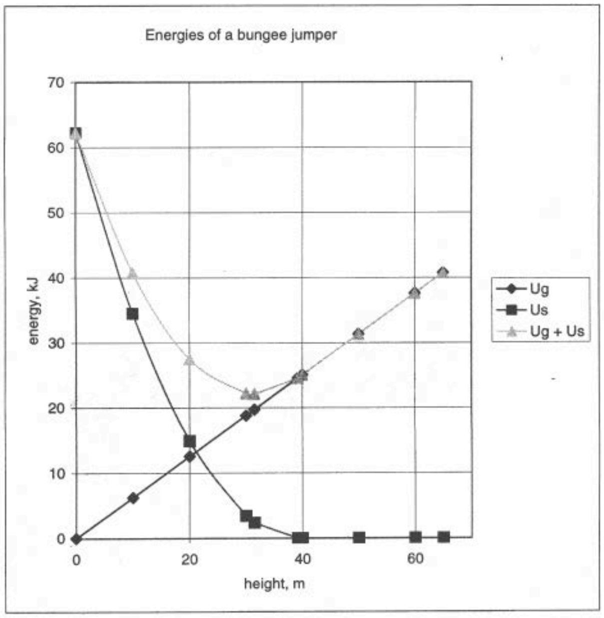
$$U_s = \frac{1}{2}kx^2 = \boxed{\frac{1}{2}(81.0 \text{ N/m})(39.2 \text{ m} - y)^2}.$$

(c) For *y* > 39.2 m, $U_g + U_s = \boxed{(627 \text{ N})y}$

For $y \le 39.2$ m,

$$U_g + U_s = (627 \text{ N})y + 40.5 \text{ N/m}\left(1\,537 \text{ m}^2 - (78.4 \text{ m})y + y^2\right)$$
$$= \boxed{(40.5 \text{ N/m})y^2 - (2\,550 \text{ N})y + 62\,200 \text{ J}}$$

(d) See the graph in ANS. FIG. P8.80(d) below.



ANS. FIG. P8.80(d)

(e) At minimum height, the jumper has zero kinetic energy and the system has the same total energy as it had when the jumper was at his starting point. $K_i + U_i = K_f + U_f$ becomes

$$(627\text{ N})(65.0\text{ m}) = (40.5\text{ N/m})y_f^2 - (2\ 550\text{ N})y_f + 62\ 200\text{ J}$$

Suppressing units,

$$0 = 40.5y_f^2 - 2\ 550y_f + 21\ 500$$

$$y_f = \boxed{10.0\text{ m}}\ \left[\text{the solution 52.9 m is unphysical}\right]$$

(f) The total potential energy has a minimum, representing a $\boxed{\text{stable equilibrium}}$ position. To find it, we require $\frac{dU}{dy} = 0.$

Suppressing units, we get

$$\frac{d}{dy}\left(40.5y^2 - 2\ 550y + 62\ 200\right) = 0 = 81y - 2\ 550$$

$$y = \boxed{31.5\text{ m}}$$

(g) Maximum kinetic energy occurs at minimum potential energy. Between the takeoff point and this location, we have

$$K_i + U_i = K_f + U_f$$

Suppressing units,

$$0 + 40\ 800$$
$$= \frac{1}{2}(64.0)v_{max}^2 + 40.5(31.5)^2 - 2\ 550(31.5) + 62\ 200$$
$$v_{max} = \left(\frac{2(40\ 800 - 22\ 200)}{64.0\ \text{kg}}\right)^{1/2} = \boxed{24.1\ \text{m/s}}$$

P8.81 The geometry reveals $D = L\sin\theta + L\sin\phi$,

$$50.0\ \text{m} = 40.0\ \text{m}(\sin 50° + \sin\phi),\ \phi = 28.9°$$

(a) From takeoff to landing for the Jane-Earth system:

$$\Delta K + \Delta U + \Delta E_{int} = 0$$
$$\left(0 - \frac{1}{2}mv_i^2\right) + \left[mg(-L\cos\phi) - mg(-L\cos\theta)\right] + FD = 0$$
$$\frac{1}{2}mv_i^2 + mg(-L\cos\theta) + FD(-1) = 0 + mg(-L\cos\phi)$$
$$\frac{1}{2}(50.0\ \text{kg})v_i^2 + (50.0\ \text{kg})(9.80\ \text{m/s}^2)(-40.0\ \text{m})\cos 50°$$
$$-(110\ \text{N})(50.0\ \text{m})$$
$$= (50.0\ \text{kg})(9.80\ \text{m/s}^2)(-40.0\ \text{m})\cos 28.9°$$
$$\frac{1}{2}(50.0\ \text{kg})v_i^2 - 1.26\times10^4\ \text{J} - 5.5\times10^3\ \text{J} = -1.72\times10^4\ \text{J}$$
$$v_i = \sqrt{\frac{2(947\ \text{J})}{50.0\ \text{kg}}} = \boxed{6.15\ \text{m/s}}$$

(b) For the swing back:

$$\Delta K + \Delta U = \Delta E_{mech}$$
$$\left(0 - \frac{1}{2}mv_i^2\right) + \left[mg(-L\cos\theta) - mg(-L\cos\phi)\right] = FD$$
$$\frac{1}{2}mv_i^2 + mg(-L\cos\phi) + FD(+1) = 0 + mg(-L\cos\theta)$$
$$\frac{1}{2}(130\ \text{kg})v_i^2 + (130\ \text{kg})(9.80\ \text{m/s}^2)(-40.0\ \text{m})\cos 28.9°$$
$$+(110\ \text{N})(50.0\ \text{m})$$
$$= (130\ \text{kg})(9.80\ \text{m/s}^2)(-40.0\ \text{m})\cos 50°$$

$$\frac{1}{2}(130\text{ kg})v_i^2 - 4.46\times10^4\text{ J} + 5\,500\text{ J} = -3.28\times10^4\text{ J}$$

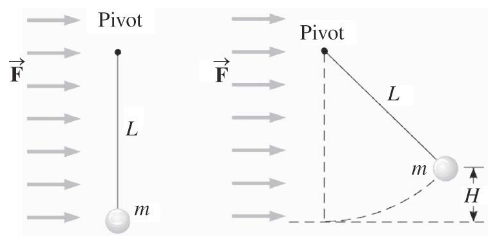
$$v_i = \sqrt{\frac{2(6\,340\text{ J})}{130\text{ kg}}} = \boxed{9.87\text{ m/s}}$$

P8.82 (a) Take the original point where the ball is released and the final point where its upward swing stops at height H and horizontal displacement

$$x = \sqrt{L^2 - (L-H)^2} = \sqrt{2LH - H^2}$$

Since the wind force is purely horizontal, it does work

$$W_{\text{wind}} = \int \vec{\mathbf{F}}\cdot d\vec{\mathbf{s}} = F\int dx = F\sqrt{2LH - H^2}$$



ANS FIG. P8.82

The work-energy theorem can be written:

$$K_i + U_{gi} + W_{\text{wind}} = K_f + U_{gf}\text{, or}$$

$$0 + 0 + F\sqrt{2LH - H^2} = 0 + mgH$$

giving

$$F^2 2LH - F^2H^2 = m^2g^2H^2$$

Here the solution $H = 0$ represents the lower turning point of the ball's oscillation, and the upper limit is at $F^2(2L) = (F^2 + m^2g^2)H$. Solving for H yields

$$H = \frac{2LF^2}{F^2 + m^2g^2} = \frac{2L}{1 + (mg/F)^2}$$

$$= \frac{2(0.800\text{ m})}{1 + (0.300\text{ kg})^2(9.8\text{ m/s}^2)^2/F^2} = \boxed{\frac{1.60\text{ m}}{1 + 8.64\text{ N}^2/F^2}}$$

(b) $H = 1.6\text{ m}[1 + 8.64/1]^{-1} = \boxed{0.166\text{ m}}$

(c) $H = 1.6\text{ m}[1 + 8.64/100]^{-1} = \boxed{1.47\text{ m}}$

(d) As $F \to 0$, $\boxed{H \to 0\text{ as is reasonable.}}$

(e) As $F \to \infty$, $\boxed{H \to 1.60 \text{ m}}$, which would be hard to approach experimentally.

(f) Call θ the equilibrium angle with the vertical and T the tension in the string.

$$\sum F_x = 0 \Rightarrow T\sin\theta = F, \text{ and}$$
$$\sum F_y = 0 \Rightarrow T\cos\theta = mg$$

Dividing: $\tan\theta = \dfrac{F}{mg}$

Then

$$\cos\theta = \frac{mg}{\sqrt{(mg)^2 + F^2}} = \frac{1}{\sqrt{1+(F/mg)^2}} = \frac{1}{\sqrt{1+F^2/8.64 \text{ N}^2}}$$

Therefore, $H_{eq} = L(1-\cos\theta) = \boxed{(0.800 \text{ m})\left(1 - \dfrac{1}{\sqrt{1+F^2/8.64 \text{ N}^2}}\right)}$

(g) For $F = 10$ N, $H_{eq} = 0.800 \text{ m}[1-(1+100/8.64)^{-1/2}] = \boxed{0.574 \text{ m}}$

(h) As $F \to \infty$, $\tan\theta \to \infty$, $\theta \to 90.0°$, $\cos\theta \to 0$, and $H_{eq} \to \boxed{0.800 \text{ m}}$.

A very strong wind pulls the string out horizontal, parallel to the ground.

P8.83 The coaster-Earth system is isolated as the coaster travels up the circle. Find how high the coaster travels from the bottom:

$$K_i + U_i = K_f + U_f$$
$$\frac{1}{2}mv^2 + 0 = 0 + mgh \to h = \frac{v^2}{2g} = \frac{(15.0 \text{ m/s})^2}{2g} = 11.5 \text{ m}$$

For this situation, the coaster stops at height 11.5 m, which is lower than the height of 24 m at the top of the circular section; in fact, it is close to halfway to the top. The passengers will be supported by the normal force from the backs of their seats. Because of the usual position of a seatback, there may be a slight downhill incline of the seatback that would tend to cause the passengers to slide out. Between the force the passengers can exert by hanging on to a part of the car and the friction between their backs and the back of their seat, the passengers should be able to avoid sliding out of the cars. Therefore, this situation is $\boxed{\text{less dangerous}}$ than that in the original higher-speed situation, where the coaster is upside down.

P8.84 (a) Let mass m_1 of the chain laying on the table and mass m_2 hanging off the edge. For the hanging part of the chain, apply the particle in equilibrium model in the vertical direction:

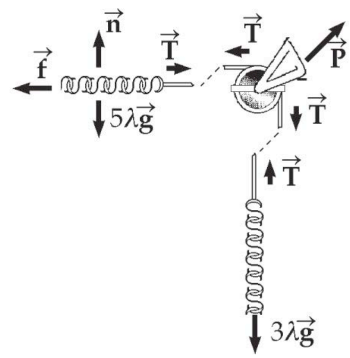
$$m_2 g - T = 0 \quad \textbf{[1]}$$

For the part of the chain on the table, apply the particle in equilibrium model in both directions:

$$n - m_1 g = 0 \quad \textbf{[2]}$$

$$T - f_s = 0 \quad \textbf{[3]}$$

Assume that the length of chain hanging over the edge is such that the chain is on the verge of slipping. Add equations [1] and [3], impose the assumption of impending motion, and substitute equation [2]:



ANS. FIG. P8.84

$$n - m_1 g = 0$$

$$f_s = m_2 g \quad \rightarrow \quad \mu_s n = m_2 g$$

$$\rightarrow \quad \mu_s m_1 g = m_2 g$$

$$\rightarrow \quad m_2 = \mu_s m_1 = 0.600 m_1$$

From the total length of the chain of 8.00 m, we see that

$$m_1 + m_2 = 8.00\lambda$$

where λ is the mass of a one meter length of chain. Substituting for m_2,

$$m_1 + 0.600 m_1 = 8.00\lambda \rightarrow 1.60 m_1 = 8.00\lambda \rightarrow m_1 = 5.00\lambda$$

From this result, we find that $\boxed{m_2 = 3.00\lambda}$ and we see that 3.00 m of chain hangs off the table in the case of impending motion.

(b) Let x represent the variable distance the chain has slipped since the start.

Then length $(5 - x)$ remains on the table, with now

$$\sum F_y = 0: \qquad +n - (5 - x)\lambda g = 0 \quad \rightarrow \quad n = (5 - x)\lambda g$$

$$f_k = \mu_k n = 0.4(5 - x)\lambda g = 2\lambda g - 0.4 x \lambda g$$

Consider energies of the chain-Earth system at the initial moment when the chain starts to slip, and a final moment when $x = 5$, when the last link goes over the brink. Measure heights above the final position of the leading end of the chain. At the moment the final link slips off, the center of the chain is at $y_f = 4$ meters.

Originally, 5 meters of chain is at height 8 m and the middle of the dangling segment is at height $8-\frac{3}{2}=6.5$ m.

$$K_i+U_i+\Delta E_{mech}=K_f+U_f:$$

$$0+\left(m_1gy_1+m_2gy_2\right)_i-\int_i^f f_k dx=\left(\frac{1}{2}mv^2+mgy\right)_f$$

$$(5\lambda g)8+(3\lambda g)6.5-\int_0^5(2\lambda g-0.4x\lambda g)dx=\frac{1}{2}(8\lambda)v^2+(8\lambda g)4$$

$$40.0g+19.5g-2.00g\int_0^5 dx+0.400g\int_0^5 x\,dx=4.00v^2+32.0g$$

$$27.5g-2.00gx\Big|_0^5+0.400g\frac{x^2}{2}\Bigg|_0^5=4.00v^2$$

$$27.5g-2.00g(5.00)+0.400g(12.5)=4.00v^2$$

$$22.5g=4.00v^2$$

$$v=\sqrt{\frac{(22.5\text{ m})(9.80\text{ m/s}^2)}{4.00}}=\boxed{7.42\text{ m/s}}$$

P8.85 (a) For a 5.00-m cord the spring constant is described by $F=kx$, $mg=k(1.50\text{ m})$. For a longer cord of length L the stretch distance is longer so the spring constant is smaller in inverse proportion:

$$k=\left(\frac{5.00\text{ m}}{L}\right)\left(\frac{mg}{1.50\text{ m}}\right)=3.33mg/L$$

From the isolated system model,

$$\left(K+U_g+U_s\right)_i=\left(K+U_g+U_s\right)_f$$

$$0+mgy_i+0=0+mgy_f+\frac{1}{2}kx_f^2$$

$$mg\left(y_i-y_f\right)=\frac{1}{2}kx_f^2=\frac{1}{2}(3.33)\left(\frac{mg}{L}\right)x_f^2$$

here $y_i-y_f=55\text{ m}=L+x_f$. Substituting,

$$(55.0\text{ m})L=\frac{1}{2}(3.33)(55.0\text{ m}-L)^2$$

$$(55.0\text{ m})L=5.04\times10^3\text{ m}^2-(183\text{ m})L+1.67L^2$$

Suppressing units, we have

$$0 = 1.67L^2 - 238L + 5.04\times 10^3 = 0$$

$$L = \frac{238 \pm \sqrt{238^2 - 4(1.67)(5.04\times 10^3)}}{2(1.67)} = \frac{238 \pm 152}{3.33} = \boxed{25.8\text{ m}}$$

Only the value of L less than 55 m is physical.

(b) From part (a), $k = 3.33\left(\frac{mg}{25.8\text{ m}}\right)$, with

$$x_{max} = x_f = 55.0\text{ m} - 25.8\text{ m} = 29.2\text{ m}$$

From Newton's second law,

$$\sum F = ma: \qquad +kx_{max} - mg = ma$$

$$3.33\frac{mg}{25.8\text{ m}}(29.2\text{ m}) - mg = ma$$

$$a = 2.77g = \boxed{27.1\text{ m/s}^2}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P8.2 (a) $\Delta K + \Delta U = 0$, $v = \sqrt{2gh}$; (b) $v = \sqrt{2gh}$

P8.4 (a) 1.85×10^4 m, 5.10×10^4 m; (b) 1.00×10^7 J

P8.6 (a) 5.94 m/s, 7.67 m/s; (b) 147 J

P8.8 (a) $\sqrt{\dfrac{2(m_1 - m_2)gh}{m_1 + m_2}}$; (b) $\dfrac{2m_1 h}{m_1 + m_2}$

P8.10 (a) 1.11×10^9 J; (b) 0.2

P8.12 2.04 m

P8.14 (a) −168 J; (b) 184 J; (c) 500 J; (d) 148 J; (e) 5.65 m/s

P8.16 (a) 650 J; (b) 588 J; (c) 0; (d) 0; (e) 62.0 J; (f) 1.76 m/s

P8.18 (a) 22.0 J, $E = K + U = 30.0$ J + 10.0 J = 40.0 J; (b) Yes; (c) The total mechanical energy has decreased, so a nonconservative force must have acted.

P8.20 (a) $v_B = 1.65$ m/s^2; (b) green bead, see P8.20 for full explanation

P8.22 3.74 m/s

P8.24 (a) 0.381 m; (b) 0.371 m; (c) 0.143 m

P8.26 (a) 24.5 m/s; (b) Yes. This is too fast for safety; (c) 206 m; (d) see P8.26(d) for full explanation

P8.28 (a) 1.24×10^3 W; (b) 0.209

P8.30 (a) 8.01 W; (b) see P8.30(b) for full explanation

P8.32 2.03×10^8 s, 5.64×10^4 h

P8.34 194 m

P8.36 The power of the sports car is four times that of the older-model car.

P8.38 (a) 5.91×10^3 W; (b) 1.11×10^4 W

P8.40 (a) 854; (b) 0.182 hp; (c) This method is impractical compared to limiting food intake.

P8.42 ~10^2 W

P8.44 (a) 0.225 J; (b) −0.363 J; (c) no; (d) It is possible to find an effective coefficient of friction but not the actual value of μ since n and f vary with position.

P8.46 (a) 2.49 m/s; (b) 5.45 m/s; (c) 1.23 m; (d) no; (e) Some of the kinetic energy of m_2 is transferred away as sound and to internal energy in m_1 and the floor.

P8.48 We find that her arms would need to be 1.36 m long to perform this task. This is significantly longer than the human arm.

P8.50 (a) 0.403 m or –0.357 m (b) From a perch at a height of 2.80 m above the top of a pile of mattresses, a 46.0-kg child jumps upward at 2.40 m/s. The mattresses behave as a linear spring with force constant 19.4 kN/m. Find the maximum amount by which they are compressed when the child lands on them; (c) 0.023 2 m; (d) This result is the distance by which the mattresses compress if the child just stands on them. It is the location of the equilibrium position of the oscillator.

P8.52 (a) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_f^2$; (b) $-mgh - \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right)$; (c) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_f^2 + mgh$

P8.54 $\frac{\rho A v^3}{2}$; $F = \frac{\rho A v^2}{2}$; see P8.54 for full explanation

P8.56 (a) 16.5 m; (b) See ANS. FIG. P8.56

P8.58 Unrestrained passengers will fall out of the cars

P8.60 (a) See P8.60(a) for full explanation; (b) see P8.60(b) for full explanation

P8.62 (a) 0.378 m; (b) 2.30 m/s; (c) 1.08 m

P8.64 1.24 m/s

P8.66 48.2°

P8.68 $\frac{3L}{5}$

P8.70 The tension at the bottom is greater than the performer can withstand.

P8.72 (a) 5R/2; (b) 6*mg*

P8.74 (a) No, mechanical energy is not conserved in this case; (b) 77.0 m/s

P8.76 25.2 km/h and 27.0 km/h

P8.78 (a) 21.0 m/s; (b) 16.1 m/s

P8.80 (a) (627 N)y; (b) $U_s = 0$, $\frac{1}{2}(81\text{ N/m})(39.2\text{m} - y)^2$; (c) (627 N)$y$, (40.5 N/m) y^2 – (2 550 N)y + 62 200 J; (d) See ANS. FIG. P7.78(d); (e) 10.0 m; (f) stable equilibrium, 31.5 m; (g) 24.1 m/s

P8.82 (a) $\frac{1.60\text{ m}}{1+8.64\text{ N}^2/\text{F}^2}$; (b) 0.166 m; (c) 1.47 m; (d) $H \to 0$ as is reasonable; (e) $H \to 1.60$ m; (f) $(0.800\text{ m})\left(1-\frac{1}{\sqrt{1+\text{F}^2/8.64\text{ N}^2}}\right)$; (g) 0.574 m; (h) 0.800 m

P8.84 (a) 3.00λ; (b) 7.42 m/s

9

Linear Momentum and Collisions

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ9.1 Think about how much the vector momentum of the Frisbee changes in a horizontal plane. This will be the same in magnitude as your momentum change. Since you start from rest, this quantity directly controls your final speed. Thus (b) is largest and (c) is smallest. In between them, (e) is larger than (a) and (a) is larger than (c). Also (a) is equal to (d), because the ice can exert a normal force to prevent you from recoiling straight down when you throw the Frisbee up. The assembled answer is b > e > a = d > c.

OQ9.2 (a) No: mechanical energy turns into internal energy in the coupling process.

(b) No: the Earth feeds momentum into the boxcar during the downhill rolling process.

(c) Yes: total energy is constant as it turns from gravitational into kinetic.

(d) Yes: If the boxcar starts moving north, the Earth, very slowly, starts moving south.

(e) No: internal energy appears.

(f) Yes: Only forces internal to the two-car system act.

OQ9.3 (i) Answer (c). During the short time the collision lasts, the total system momentum is constant. Whatever momentum one loses the other gains.

(ii) Answer (a). The problem implies that the tractor's momentum is negligible compared to the car's momentum before the collision. It also implies that the car carries most of the kinetic energy of the system. The collision slows down the car and speeds up the tractor, so that they have the same final speed. The faster-moving car loses more energy than the slower tractor gains because a lot of the car's original kinetic energy is converted into internal energy.

OQ9.4 Answer (a). We have $m_1 = 2$ kg, $v_{1i} = 4$ m/s; $m_2 = 1$ kg, and $v_{1i} = 0$. We find the velocity of the 1-kg mass using the equation derived in Section 9.4 for an elastic collision:

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i}$$

$$v_{2f} = \left(\frac{4\text{ kg}}{3\text{ kg}}\right)(4\text{ m/s}) + \left(\frac{1\text{ kg}}{3\text{ kg}}\right)(0) = 5.33\text{ m/s}$$

OQ9.5 Answer (c). We choose the original direction of motion of the cart as the positive direction. Then, $v_i = 6$ m/s and $v_f = -2$ m/s. The change in the momentum of the cart is

$$\begin{aligned}\Delta p = mv_f - mv_i = m(v_f - v_i) &= (5\text{ kg})(-2\text{ m/s} - 6\text{ m/s}) \\ &= -40\text{ kg}\cdot\text{m/s}.\end{aligned}$$

OQ9.6 Answer (c). The impulse given to the ball is $I = F_{avg}\Delta t = mv_f - mv_i$. Choosing the direction of the final velocity of the ball as the positive direction, this gives

$$\begin{aligned}F_{avg} = \frac{m(v_f - v_i)}{\Delta t} &= \frac{(57.0 \times 10^{-3}\text{ kg})[25.0\text{ m/s} - (-21.0\text{ m/s})]}{0.060\text{ s}} \\ &= 43.7\text{ kg}\cdot\text{m/s}^2 = 43.7\text{ N}\end{aligned}$$

OQ9.7 Answer (a). The magnitude of momentum is proportional to speed and the kinetic energy is proportional to speed squared. The speed of the rocket becomes 4 times larger, so the kinetic energy becomes 16 times larger.

OQ9.8 Answer (d). The magnitude of momentum is proportional to speed and the kinetic energy is proportional to speed squared. The speed of the rocket becomes 2 times larger, so the magnitude of the momentum becomes 2 times larger.

OQ9.9 Answer (c). The kinetic energy of a particle may be written as

$$\mathrm{KE} = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

The ratio of the kinetic energies of two particles is then

$$\frac{(\mathrm{KE})_2}{(\mathrm{KE})_1} = \frac{p_2^2/2m_2}{p_1^2/2m_1} = \left(\frac{p_2}{p_1}\right)^2\left(\frac{m_1}{m_2}\right)$$

We see that, if the magnitudes of the momenta are equal ($p_2 = p_1$), the kinetic energies will be equal only if the masses are also equal. The correct response is then (c).

OQ9.10 Answer (d). Expressing the kinetic energy as $KE = p^2/2m$, we see that the ratio of the magnitudes of the momenta of two particles is

$$\frac{p_2}{p_1} = \frac{\sqrt{2m_2(\mathrm{KE})_2}}{\sqrt{2m_1(\mathrm{KE})_1}} = \sqrt{\left(\frac{m_2}{m_1}\right)\frac{(\mathrm{KE})_2}{(\mathrm{KE})_1}}$$

Thus, we see that if the particles have equal kinetic energies [$(KE)_2 = (KE)_1$], the magnitudes of their momenta are equal only if the masses are also equal. However, momentum is a *vector quantity* and we can say the two particles have equal momenta only it both the magnitudes and directions are equal, making choice (d) the correct answer.

OQ9.11 Answer (b). Before collision, the bullet, mass $m_1 = 10.0$ g, has speed $v_{1i} = v_b$, and the block, mass $m_2 = 200$ g, has speed $v_{2i} = 0$. After collision, the objects have a common speed (velocity) $v_{1f} = v_{2f} = v$. The collision of the bullet with the block is completely inelastic:

$$m_1v_{1i} + m_2v_2 = m_1v_{1f} + m_2v_{2f}$$

$$m_1v_b = (m_1 + m_2)v\,, \quad \text{so} \qquad v_b = v\left(\frac{m_1 + m_2}{m_1}\right)$$

The kinetic friction, $f_k = \mu_k n$, slows down the block with acceleration of magnitude $\mu_k g$. The block slides to a stop through a distance d = 8.00 m. Using $v_f^2 = v_i^2 + 2a(x_f - x_i)$, we find the speed of the block just after the collision:

$$v = \sqrt{2(0.400)(9.80\ \text{m/s}^2)(8.00\ \text{m})} = 7.92\ \text{m/s}.$$

Using the results above, the speed of the bullet before collision is

$$v_b = (7.92\ \text{m/s})\left(\frac{10 + 200}{10.0}\right) = 166\ \text{m/s}.$$

OQ9.12 Answer (c). The masses move through the same distance under the same force. Equal net work inputs imply equal kinetic energies.

OQ9.13 Answer (a). The same force gives the larger mass a smaller acceleration, so the larger mass takes a longer time interval to move through the same distance; therefore, the impulse given to the larger mass is larger, which means the larger mass will have a greater final momentum.

OQ9.14 Answer (d). Momentum of the ball-Earth system is conserved. Mutual gravitation brings the ball and the Earth together into one system. As the ball moves downward, the Earth moves upward, although with an acceleration on the order of 10^{25} times smaller than that of the ball. The two objects meet, rebound, and separate.

OQ9.15 Answer (d). Momentum is the same before and after the collision. Before the collision the momentum is

$$m_1 v_1 + m_2 v_2 = (3\ \text{kg})(+2\ \text{m/s}) + (2\ \text{kg})(-4\ \text{m/s}) = -2\ \text{kg}\cdot\text{m/s}$$

OQ9.16 Answer (a). The ball gives more rightward momentum to the block when the ball reverses its momentum.

OQ9.17 Answer (c). Assuming that the collision was head-on so that, after impact, the wreckage moves in the original direction of the car's motion, conservation of momentum during the impact gives

$$(m_c + m_t)v_f = m_c v_{0c} + m_t v_{0t} = m_c v + m_t(0)$$

or

$$v_f = \left(\frac{m_c}{m_c + m_t}\right)v = \left(\frac{m}{m + 2m}\right)v = \frac{v}{3}$$

OQ9.18 Answer (c). Billiard balls all have the same mass and collisions between them may be considered to be elastic. The dual requirements of conservation of kinetic energy and conservation of momentum in a one-dimensional, elastic collision are summarized by the two relations:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \textbf{[1]}$$

and

$$v_{1i} - v_{2i} = \left(v_{1f} - v_{2f}\right) \quad \textbf{[2]}$$

In this case, $m_1 = m_2$ and the masses cancel out of the first equation. Call the blue ball #1 and the red ball #2 so that $v_{1i} = -3v$, $v_{2i} = +v$, $v_{1f} = v_{\text{blue}}$, and $v_{2f} = v_{\text{red}}$. Then, the two equations become

$$-3v + v = v_{\text{blue}} + v_{\text{red}} \quad \text{or} \quad v_{\text{blue}} + v_{\text{red}} = v \quad \textbf{[1]}$$

and

$$-3v - v = -\left(v_{\text{blue}} - v_{\text{red}}\right) \quad \text{or} \quad \left(v_{\text{blue}} - v_{\text{red}}\right) = 4v \quad \textbf{[2]}$$

Adding the final versions of these equations yields $2v_{\text{blue}} = 2v$, or $v_{\text{blue}} = v$. Substituting this result into either [1] or [2] above then yields $v_{\text{red}} = -3v$.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ9.1 The passenger must undergo a certain momentum change in the collision. This means that a certain impulse must be exerted on the passenger by the steering wheel, the window, an air bag, or something. By increasing the distance over which the momentum change occurs, the time interval during which this change occurs is also increased, resulting in the force on the passenger being decreased.

CQ9.2 If the golfer does not “follow through,” the club is slowed down by the golfer before it hits the ball, so the club has less momentum available to transfer to the ball during the collision.

CQ9.3 Its speed decreases as its mass increases. There are no external horizontal forces acting on the box, so its momentum cannot change as it moves along the horizontal surface. As the box slowly fills with water, its mass increases with time. Because the product mv must be constant, and because m is increasing, the speed of the box must decrease. Note that the vertically falling rain has no horizontal momentum of its own, so the box must “share” its momentum with the rain it catches.

CQ9.4 (a) It does not carry force, force requires another object on which to act.

(b) It cannot deliver more kinetic energy than it possesses. This would violate the law of energy conservation.

(c) It can deliver more momentum in a collision than it possesses in its flight, by bouncing from the object it strikes.

CQ9.5 Momentum conservation is not violated if we choose as our system the planet along with you. When you receive an impulse forward, the Earth receives the same size impulse backwards. The resulting acceleration of the Earth due to this impulse is much smaller than your acceleration forward, but the planet's backward momentum is equal in magnitude to your forward momentum. If we choose you as the system, momentum conservation is not violated because you are not an isolated system.

CQ9.6 The rifle has a much lower speed than the bullet and much less kinetic energy. Also, the butt distributes the recoil force over an area much larger than that of the bullet.

CQ9.7 The time interval over which the egg is stopped by the sheet (more for a faster missile) is much longer than the time interval over which the egg is stopped by a wall. For the same change in momentum, the longer the time interval, the smaller the force required to stop the egg. The sheet increases the time interval so that the stopping force is never too large.

CQ9.8 (a) Assuming that both hands are never in contact with a ball, and one hand is in contact with any one ball 20% of the time, the total contact time with the system of three balls is 3(20%) = 60% of the time. The center of mass of the balls is in free fall, moving up and then down with the acceleration due to gravity, during the 40% of the time when the juggler's hands are empty. During the 60% of the time when the juggler is engaged in catching and tossing, the center of mass must accelerate up with a somewhat smaller average acceleration. The center of mass moves around in a little closed loop with a parabolic top and likely a circular bottom, making three revolutions for every one revolution that one ball makes.

(b) On average, in one cycle of the system, the center of mass of the balls does not change position, so its average acceleration is zero (i.e., the average net force on the system is zero). Letting T represent the time for one cycle and F_g the weight of one ball, we have $F_J(0.60T) = 3F_gT$, and $F_J = 5F_g$. The average force exerted by the juggler is five times the weight of one ball.

CQ9.9 (a) In empty space, the center of mass of a rocket-plus-fuel system does not accelerate during a burn, because no outside force acts on this system. The rocket body itself does accelerate as it blows exhaust containing momentum out the back.

(b) According to the text's 'basic expression for rocket propulsion,' the change in speed of the rocket body will be larger than the speed of the exhaust relative to the rocket, if the final mass is less than 37% of the original mass.

CQ9.10 To generalize broadly, around 1740 the English favored position (a), the Germans position (b), and the French position (c). But in France Emilie de Chatelet translated Newton's *Principia* and argued for a more inclusive view. A Frenchman, Jean D'Alembert, is most responsible for showing that each theory is consistent with the others. All the theories are equally correct. Each is useful for giving a mathematically simple and conceptually clear solution for some problems. There is another comprehensive mechanical theory, the angular impulse–angular momentum theorem, which we will glimpse in Chapter 11. It identifies the product of the torque of a force and the time it acts as the cause of a change in motion, and change in angular momentum as the effect.

We have here an example of how scientific theories are different from what people call a theory in everyday life. People who think that different theories are mutually exclusive should bring their thinking up to date to around 1750.

CQ9.11 No. Impulse, $\vec{\mathbf{F}}\Delta t$, depends on the force and the time interval during which it is applied.

CQ9.12 No. Work depends on the force and on the displacement over which it acts.

CQ9.13 (a) Linear momentum is conserved since there are no external forces acting on the system. The fragments go off in different directions and their vector momenta add to zero.

(b) Kinetic energy is not conserved because the chemical potential energy initially in the explosive is converted into kinetic energy of the pieces of the bomb.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 9.1 Linear Momentum

P9.1 (a) The momentum is $p = mv$, so $v = p/m$ and the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$$

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{\frac{2K}{m}}$ so $p = mv = m\sqrt{\frac{2K}{m}} = \boxed{\sqrt{2mK}}$.

P9.2 $K = p^2/2m$, and hence, $p = \sqrt{2mK}$. Thus,

$$m = \frac{p^2}{2\cdot K} = \frac{(25.0\text{ kg}\cdot\text{m/s})^2}{2(275\text{ J})} = \boxed{1.14\text{ kg}}$$

and

$$v = \frac{p}{m} = \frac{\sqrt{2m(K)}}{m} = \sqrt{\frac{2(K)}{m}} = \sqrt{\frac{2(275\text{ J})}{1.14\text{ kg}}} = \boxed{22.0\text{ m/s}}$$

P9.3 We apply the impulse-momentum theorem to relate the change in the horizontal momentum of the sled to the horizontal force acting on it:

$$\Delta p_x = F_x\Delta t \rightarrow F_x = \frac{\Delta p_x}{\Delta t} = \frac{mv_{xf} - mv_{xi}}{\Delta t}$$

$$F_x = \frac{-(17.5\text{ kg})(3.50\text{ m/s})}{8.75\text{ s}}$$

$$F_x = \boxed{7.00\text{ N}}$$

***P9.4** We are given $m = 3.00$ kg and $\vec{\mathbf{v}} = (3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})$ m/s.

(a) The vector momentum is then

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} = (3.00\text{ kg})\left[(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})\text{ m/s}\right]$$
$$= (9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}})\text{ kg}\cdot\text{m/s}$$

Thus, $\boxed{p_x = 9.00\text{ kg}\cdot\text{m/s}}$ and $\boxed{p_y = -12.0\text{ kg}\cdot\text{m/s}}$.

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00\text{ kg}\cdot\text{m/s})^2 + (12.0\text{ kg}\cdot\text{m/s})^2}$

$= \boxed{15.0\text{ kg}\cdot\text{m/s}}$

at an angle of

$$\theta = \tan^{-1}\left(\frac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$$

P9.5 We apply the impulse-momentum theorem to find the average force the bat exerts on the baseball:

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\Delta t \rightarrow \vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = m\left(\frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{\Delta t}\right)$$

Choosing the direction toward home plate as the positive *x* direction, we have $\vec{\mathbf{v}}_i = (45.0 \text{ m/s})\hat{\mathbf{i}}$, $\vec{\mathbf{v}}_f = (55.0 \text{ m/s})\hat{\mathbf{j}}$, and $\Delta t = 2.00$ ms:

$$\vec{\mathbf{F}}_{\text{on ball}} = m\frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{\Delta t} = (0.145 \text{ kg})\frac{(55.0 \text{ m/s})\hat{\mathbf{j}} - (45.0 \text{ m/s})\hat{\mathbf{i}}}{2.00 \times 10^{-3} \text{ s}}$$

$$\vec{\mathbf{F}}_{\text{on ball}} = \left(-3.26\hat{\mathbf{i}} + 3.99\hat{\mathbf{j}}\right) \text{ N}$$

By Newton's third law,

$$\vec{\mathbf{F}}_{\text{on bat}} = -\vec{\mathbf{F}}_{\text{on ball}} \quad \text{so} \quad \boxed{\vec{\mathbf{F}}_{\text{on bat}} = \left(+3.26\hat{\mathbf{i}} - 3.99\hat{\mathbf{j}}\right) \text{ N}}$$

Section 9.2 Analysis Model: Isolated system (Momentum)

P9.6 (a) The girl-plank system is isolated, so horizontal momentum is conserved.

We measure momentum relative to the ice: $\vec{\mathbf{p}}_{gi} + \vec{\mathbf{p}}_{pi} = \vec{\mathbf{p}}_{gf} + \vec{\mathbf{p}}_{pf}$.

The motion is in one dimension, so we can write,

$$v_{gi}\hat{\mathbf{i}} = v_{gp}\hat{\mathbf{i}} + v_{pi}\hat{\mathbf{i}} \rightarrow v_{gi} = v_{gp} + v_{pi}$$

where v_{gi} denotes the velocity of the girl relative to the ice, v_{gp} the velocity of the girl relative to the plank, and v_{pi} the velocity of the plank relative to the ice. The momentum equation becomes

$$0 = m_g v_{gi}\hat{\mathbf{i}} + m_p v_{pi}\hat{\mathbf{i}} \rightarrow 0 = m_g v_{gi} + m_p v_{pi}$$

$$0 = m_g\left(v_{gp} + v_{pi}\right) + m_p v_{pi}$$

$$0 = m_g v_{gp} + \left(m_g + m_p\right)v_{pi} \rightarrow v_{pi} = -\left(\frac{m_g}{m_g + m_p}\right)v_{gp}$$

solving for the velocity of the plank gives

$$v_{pi} = -\left(\frac{m_g}{m_g + m_p}\right)v_{gp} = -\left(\frac{45.0\text{ kg}}{45.0\text{ kg} + 150\text{ kg}}\right)(1.50\text{ m/s})$$

$$\boxed{v_{pi} = -0.346\text{ m/s}}$$

(b) Using our result above, we find that

$$v_{gi} = v_{gp} + v_{pi} = (1.50\text{ m/s}) + (-0.346\text{ m/s})$$

$$\boxed{v_{gi} = 1.15\text{ m/s}}$$

P9.7 (a) The girl-plank system is isolated, so horizontal momentum is conserved.

We measure momentum relative to the ice: $\vec{\mathbf{p}}_{gi} + \vec{\mathbf{p}}_{pi} = \vec{\mathbf{p}}_{gf} + \vec{\mathbf{p}}_{pf}$.

The motion is in one dimension, so we can write

$$v_{gi}\hat{\mathbf{i}} = v_{gp}\hat{\mathbf{i}} + v_{pi}\hat{\mathbf{i}} \rightarrow v_{gi} = v_{gp} + v_{pi}$$

where v_{gi} denotes the velocity of the girl relative to the ice, v_{gp} the velocity of the girl relative to the plank, and v_{pi} the velocity of the plank relative to the ice. The momentum equation becomes

$$0 = m_g v_{gi}\hat{\mathbf{i}} + m_p v_{pi}\hat{\mathbf{i}} \rightarrow 0 = m_g v_{gi} + m_p v_{pi}$$

$$0 = m_g\left(v_{gp} + v_{pi}\right) + m_p v_{pi}$$

$$0 = m_g v_{gp} + \left(m_g + m_p\right)v_{pi}$$

solving for the velocity of the plank gives

$$\boxed{v_{pi} = -\left(\frac{m_g}{m_g + m_p}\right)v_{gp}}$$

(b) Using our result above, we find that

$$v_{gi} = v_{gp} + v_{pi} = v_{gp}\frac{\left(m_g + m_p\right)}{m_g + m_p} - \frac{m_g}{m_g + m_p}v_{gp}$$

$$v_{gi} = \frac{\left(m_g + m_p\right)v_{gp} - m_g v_{gp}}{m_g + m_p}$$

$$v_{gi} = \frac{m_g v_{gp} + m_p v_{gp} - m_g v_{gp}}{m_g + m_p}$$

$$\boxed{v_{gi} = \left(\frac{m_p}{m_g + m_p}\right) v_{gp}}$$

P9.8 (a) Brother and sister exert equal-magnitude oppositely-directed forces on each other for the same time interval; therefore, the impulses acting on them are equal and opposite. Taking east as the positive direction, we have

impulse on boy: $I = F\Delta t = \Delta p = (65.0 \text{ kg})(-2.90 \text{ m/s}) = -189 \text{ N}\cdot\text{s}$

impulse on girl: $I = -F\Delta t = -\Delta p = +189 \text{ N}\cdot\text{s} = mv_f$

Her speed is then

$$v_f = \frac{I}{m} = \frac{189 \text{ N}\cdot\text{s}}{40.0 \text{ kg}} = 4.71 \text{ m/s}$$

meaning $\boxed{\text{she moves at 4.71 m/s east}}$.

(b) original chemical potential energy in girl's body = total final kinetic energy

$$U_{\text{chemical}} = \frac{1}{2}m_{\text{boy}}v_{\text{boy}}^2 + \frac{1}{2}m_{\text{girl}}v_{\text{girl}}^2$$

$$= \frac{1}{2}(65.0 \text{ kg})(2.90 \text{ m/s})^2 + \frac{1}{2}(40.0 \text{ kg})(4.71 \text{ m/s})^2$$

$$= \boxed{717 \text{ J}}$$

(c) $\boxed{\text{Yes.}}$ System momentum is conserved with the value zero.

(d) The forces on the two siblings are internal forces, which cannot change the momentum of the system— $\boxed{\text{the system is isolated}}$.

(e) $\boxed{\text{Even though there is motion afterward, the final momenta are of equal magnitude in opposite directions so the final momentum of the system is still zero.}}$

***P9.9** We assume that the velocity of the blood is constant over the 0.160 s. Then the patient's body and pallet will have a constant velocity of $\frac{6\times10^{-5}\text{ m}}{0.160\text{ s}} = 3.75\times10^{-4}\text{ m/s}$ in the opposite direction. Momentum conservation gives

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}:$$

$$0 = m_{\text{blood}}(0.500\text{ m/s}) + (54.0\text{ kg})(-3.75\times10^{-4}\text{ m/s})$$

$$m_{\text{blood}} = 0.040\ 5\text{ kg} = \boxed{40.5\text{ g}}$$

P9.10 I have mass 72.0 kg and can jump to raise my center of gravity 25.0 cm. I leave the ground with speed given by

$$v_f^2 - v_i^2 = 2a(x_f - x_i): \qquad 0 - v_i^2 = 2(-9.80\text{ m/s}^2)(0.250\text{ m})$$

$$v_i = 2.20\text{ m/s}$$

Total momentum of the system of the Earth and me is conserved as I push the planet down and myself up:

$$0 = (5.98\times10^{24}\text{ kg})(-v_e) + (85.0\text{ kg})(2.20\text{ m/s})$$

$$v_e \sim \boxed{10^{-23}\text{ m/s}}$$

P9.11 (a) For the system of two blocks $\Delta p = 0$, or $p_i = p_f$. Therefore,

$$0 = mv_m + (3m)(2.00\text{ m/s})$$

Solving gives $v_m = \boxed{-6.00\text{ m/s}}$ (motion toward the left).

(b) $\frac{1}{2}kx^2 = \frac{1}{2}mv_M^2 + \frac{1}{2}(3m)v_{3M}^2$

$$= \frac{1}{2}(0.350\text{ kg})(-6.00\text{ m/s})^2 + \frac{3}{2}(0.350\text{ kg})(2.00\text{ m/s})^2$$

$$= \boxed{8.40\text{ J}}$$

(c) $\boxed{\text{The original energy is in the spring.}}$

(d) A force had to be exerted over a displacement to compress the spring, transferring energy into it by work.
$\boxed{\text{The cord exerts force, but over no displacement.}}$

(e) $\boxed{\text{System momentum is conserved with the value zero.}}$

(f) The forces on the two blocks are internal forces, which cannot change the momentum of the system— $\boxed{\text{the system is isolated.}}$

(g) $\boxed{\text{Even though there is motion afterward, the final momenta are of equal magnitude in opposite directions so the final momentum of the system is still zero.}}$

Section 9.3 Analysis Model: Nonisolated system (Momentum)

P9.12 (a) $I = F_{avg}\Delta t$, where *I* is the impulse the man must deliver to the child:

$$I = F_{avg}\Delta t = \Delta p_{child} = m_{child}\left|v_f - v_i\right| \to F_{avg} = \frac{m_{child}\left|v_f - v_i\right|}{\Delta t}$$

Solving for the average force gives

$$F_{avg} = \frac{m_{child}\left|v_f - v_i\right|}{\Delta t} = \frac{(12.0\text{ kg})\left|0 - 60\text{ mi/h}\right|}{0.10\text{ s}}\left(\frac{0.447\text{ m/s}}{1\text{ mi/h}}\right)$$
$$= \boxed{3.22 \times 10^3\text{ N}}$$

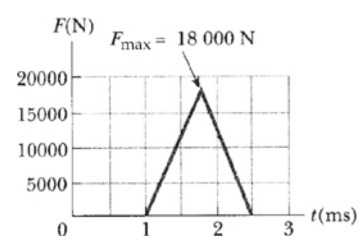
or

$$F_{avg} = \left(3.22 \times 10^3\text{ N}\right)\left(\frac{0.224\,8\text{ lb}}{1\text{ N}}\right) \approx \boxed{720\text{ lb}}$$

(b) $\boxed{\text{The man's claim is nonsense.}}$ He would not be able to exert a force of this magnitude on the child. In reality, the violent forces during the collision would tear the child from his arms.

(c) $\boxed{\text{These devices are essential for the safety of small children.}}$

P9.13 (a) The impulse delivered to the ball is equal to the area under the *F*-*t* graph. We have a triangle and so to get its area we multiply half its height times its width:



ANS. FIG. P9.13

$$I = \int F dt = \text{area under curve}$$

$$I = \frac{1}{2}\left(1.50 \times 10^{-3}\text{ s}\right)\left(18\,000\text{ N}\right) = \boxed{13.5\text{ N}\cdot\text{s}}$$

(b) $F = \dfrac{13.5\ \text{N}\cdot\text{s}}{1.50\times10^{-3}\ \text{s}} = \boxed{9.00\ \text{kN}}$

P9.14 (a) The impulse the floor exerts on the ball is equal to the change in momentum of the ball:

$$\begin{aligned}\Delta\vec{\mathbf{p}} &= m\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i\right) = m\left(v_f - v_i\right)\hat{\mathbf{j}}\\ &= (0.300\ \text{kg})\left[(5.42\ \text{m/s}) - (-5.86\ \text{m/s})\right]\hat{\mathbf{j}}\\ &= \boxed{3.38\ \text{kg}\cdot\text{m/s}\ \hat{\mathbf{j}}}\end{aligned}$$

(b) Estimating the contact time interval to be 0.05 s, from the impulse-momentum theorem, we find

$$\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{3.38\ \text{kg}\cdot\text{m/s}\ \hat{\mathbf{j}}}{0.05\ \text{s}} \rightarrow \boxed{\vec{\mathbf{F}} = 7\times10^{2}\ \text{N}\ \hat{\mathbf{j}}}$$

P9.15 (a) The mechanical energy of the isolated spring-mass system is conserved:

$$K_i + U_{si} = K_f + U_{sf}$$

$$0 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + 0$$

$$v = x\sqrt{\frac{k}{m}}$$

(b) $I = \left|\vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i\right| = mv_f - 0 = mx\sqrt{\dfrac{k}{m}} = x\sqrt{km}$

(c) For the glider, $W = K_f - K_i = \dfrac{1}{2}mv^2 - 0 = \dfrac{1}{2}kx^2$

The mass makes $\boxed{\text{no difference}}$ to the work.

***P9.16** We take the *x* axis directed toward the pitcher.

(a) In the *x* direction, $p_{xi} + I_x = p_{xf}$:

$$\begin{aligned}I_x &= p_{xf} - p_{xi}\\ &= (0.200\ \text{kg})(40.0\ \text{m/s})\cos 30.0^\circ\\ &\qquad - (0.200\ \text{kg})(15.0\ \text{m/s})(-\cos 45.0^\circ)\\ &= 9.05\ \text{N}\cdot\text{s}\end{aligned}$$

In the *y* direction, $p_{yi} + I_y = p_{yf}$:

$$\begin{aligned} I_y &= p_{yf} - p_{yi} \\ &= (0.200 \text{ kg})(40.0 \text{ m/s})\sin 30.0° \\ &\qquad - (0.200 \text{ kg})(15.0 \text{ m/s})(-\sin 45.0°) \\ &= 6.12 \text{ N}\cdot\text{s} \end{aligned}$$

Therefore, $\vec{\mathbf{I}} = \boxed{\left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right) \text{N}\cdot\text{s}}$

(b) $\vec{\mathbf{I}} = \frac{1}{2}\left(0 + \vec{\mathbf{F}}_m\right)(4.00 \text{ ms}) + \vec{\mathbf{F}}_m(20.0 \text{ ms}) + \frac{1}{2}\vec{\mathbf{F}}_m(4.00 \text{ ms})$

$\vec{\mathbf{F}}_m \times 24.0 \times 10^{-3} \text{ s} = \left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right) \text{N}\cdot\text{s}$

$\vec{\mathbf{F}}_m = \boxed{\left(377\hat{\mathbf{i}} + 255\hat{\mathbf{j}}\right) \text{N}}$

***P9.17** (a) From the kinematic equations,

$$\Delta t = \frac{\Delta x}{v_{avg}} = \frac{2\Delta x}{v_f + v_i} = \frac{2(1.20 \text{ m})}{0 + 25.0 \text{ m/s}} = \boxed{9.60 \times 10^{-2} \text{ s}}$$

(b) We find the average force from the momentum-impulse theorem:

$$F_{avg} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{(1\ 400 \text{ kg})(25.0 \text{ m/s} - 0)}{9.60 \times 10^{-2} \text{ s}} = \boxed{3.65 \times 10^5 \text{ N}}$$

(c) Using the particle under constant acceleration model,

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{25.0 \text{ m/s} - 0}{9.60 \times 10^{-2} \text{ s}} = \left(260 \text{ m/s}^2\right)\left(\frac{1 \text{ g}}{9.80 \text{ m/s}^2}\right) = \boxed{26.5g}$$

P9.18 We assume that the initial direction of the ball is in the –*x* direction.

(a) The impulse delivered to the ball is given by

$$\begin{aligned} \vec{\mathbf{I}} &= \Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i \\ &= (0.060\ 0 \text{ kg})(40.0 \text{ m/s})\hat{\mathbf{i}} - (0.060\ 0 \text{ m/s})(20.0 \text{ m/s})\left(-\hat{\mathbf{i}}\right) \\ &= \boxed{3.60\hat{\mathbf{i}} \text{ N}\cdot\text{s}} \end{aligned}$$

(b) We choose the tennis ball as a nonisolated system for energy. Let the time interval be from just before the ball is hit until just after. Equation 9.2 for conservation of energy becomes

$$\Delta K + \Delta E_{\text{int}} = T_{\text{MW}}$$

Solving for the energy sum $\Delta E_{\text{int}} - T_{\text{MW}}$ and substituting gives

$$\Delta E_{\text{int}} - T_{\text{MW}} = -\Delta K = -\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) = \frac{1}{2}m\left(v_i^2 - v_f^2\right)$$

Substituting numerical values gives,

$$\Delta E_{\text{int}} - T_{\text{MW}} = -\frac{1}{2}(0.060\,0\text{ kg})\left[(20.0\text{ m/s})^2 - (40.0\text{ m/s})^2\right]$$
$$= 36.0\text{ J}$$

There is no way of knowing how the energy splits between ΔE_{int} and T_{MW} without more information.

P9.19 (a) The impulse is in the x direction and equal to the area under the F-t graph:

$$I = \left(\frac{0+4\text{ N}}{2}\right)(2\text{ s}-0) + (4\text{ N})(3\text{ s}-2\text{ s}) + \left(\frac{4\text{ N}+0}{2}\right)(5\text{ s}-3\text{ s})$$
$$= 12.0\text{ N}\cdot\text{s}$$

$$\boxed{\vec{\mathbf{I}} = 12.0\text{ N}\cdot\text{s }\hat{\mathbf{i}}}$$

(b) From the momentum-impulse theorem,

$$m\vec{\mathbf{v}}_i + \vec{\mathbf{F}}\Delta t = m\vec{\mathbf{v}}_f$$

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}\Delta t}{m} = 0 + \frac{12.0\,\hat{\mathbf{i}}\text{ N}\cdot\text{s}}{2.50\text{ kg}} = \boxed{4.80\,\hat{\mathbf{i}}\text{ m/s}}$$

(c) From the same equation,

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}\Delta t}{m} = -2.00\,\hat{\mathbf{i}}\text{ m/s} + \frac{12.0\,\hat{\mathbf{i}}\text{ N}\cdot\text{s}}{2.50\text{ kg}} = \boxed{2.80\,\hat{\mathbf{i}}\text{ m/s}}$$

(d) $\vec{\mathbf{F}}_{\text{avg}}\Delta t = 12.0\hat{\mathbf{i}}\text{ N}\cdot\text{s} = \vec{\mathbf{F}}_{\text{avg}}(5.00\text{ s}) \quad \rightarrow \quad \vec{\mathbf{F}}_{\text{avg}} = \boxed{2.40\hat{\mathbf{i}}\text{ N}}$

P9.20 (a) A graph of the expression for force shows a parabola opening down, with the value zero at the beginning and end of the 0.800-s interval. We integrate the given force to find the impulse:

$$I = \int_0^{0.800\text{s}} F\,dt$$
$$= \int_0^{0.800\text{s}} (9\,200\text{ t N/s} - 11\,500\text{ t}^2\text{ N/s}^2)\,dt$$
$$= \left[\frac{1}{2}(9\,200\text{ N/s})t^2 - \frac{1}{3}(11\,500\text{ N/s}^2)t^3\right]_0^{0.800\text{s}}$$
$$= \frac{1}{2}(9\,200\text{ N/s})(0.800\text{ s})^2 - \frac{1}{3}(11\,500\text{ N/s}^2)(0.800\text{ s})^3$$
$$= 2\,944\text{ N}\cdot\text{s} - 1\,963\text{ N}\cdot\text{s} = 981\text{ N}\cdot\text{s}$$

The athlete imparts a downward impulse to the platform, so the platform imparts to her an impulse of $\boxed{981\text{ N}\cdot\text{s, up.}}$

(b) We could find her impact speed as a free-fall calculation, but we choose to write it as a conservation-of-energy calculation:

$$mgy_{top} = \frac{1}{2}mv_{impact}^2$$

$$v_{impact} = \sqrt{2gy_{top}} = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(0.600 \text{ m}\right)}$$

$$= \boxed{3.43 \text{ m/s, down}}$$

(c) Gravity, as well as the platform, imparts impulse to her during the interaction with the platform

$$I = \Delta p$$

$$I_{grav} + I_{platform} = mv_f - mv_i$$

$$-mg\Delta t + I_{platform} = mv_f - mv_i$$

solving for the final velocity gives

$$v_f = v_i - mg\Delta t + \frac{I_{platform}}{m}$$

$$= \left(-3.43 \text{ m/s}\right) - \left(9.80 \text{ m/s}^2\right)\left(0.800 \text{ s}\right) + \frac{981 \text{ N}\cdot\text{s}}{65.0 \text{ kg}}$$

$$= \boxed{3.83 \text{ m/s, up}}$$

Note that the athlete is putting a lot of effort into jumping and does not exert any force "on herself." The usefulness of the force platform is to measure her effort by showing the force she exerts on the floor.

(d) Again energy is conserved in upward flight:

$$mgy_{top} = \frac{1}{2}mv_{takeoff}^2$$

which gives

$$y_{top} = \frac{v_{takeoff}^2}{2g} = \frac{\left(3.83 \text{ m/s}\right)^2}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{0.748 \text{ m}}$$

P9.21 After 3.00 s of pouring, the bucket contains

(3.00 s)(0.250 L/s) = 0.750 liter

of water, with mass (0.750 L)(1 kg/1 L) = 0.750 kg, and feeling gravitational force (0.750 kg)(9.80 m/s^2) = 7.35 N. The scale through the bucket must exert 7.35 N upward on this stationary water to support its weight. The scale must exert another 7.35 N to support the 0.750-kg

bucket itself.

Water is entering the bucket with speed given by

$$mgy_{top} = \frac{1}{2}mv_{impact}^2$$

$$v_{impact} = \sqrt{2gy_{top}} = \sqrt{2(9.80\text{ m/s}^2)(2.60\text{ m})}$$
$$= 7.14\text{ m/s, downward}$$

The scale exerts an extra upward force to stop the downward motion of this additional water, as described by

$$mv_{impact} + F_{extra}t = mv_f$$

The rate of change of momentum is the force itself:

$$\left(\frac{dm}{dt}\right)v_{impact} + F_{extra} = 0$$

which gives

$$F_{extra} = -\left(\frac{dm}{dt}\right)v_{impact} = -(0.250\text{ kg/s})(-7.14\text{ m/s}) = 1.78\text{ N}$$

Altogether the scale must exert 7.35 N + 7.35 N + 1.78 N = $\boxed{16.5\text{ N}}$

Section 9.4 Collisions in One Dimension

P9.22 (a) Conservation of momentum gives

$$m_T v_{Tf} + m_C v_{Cf} = m_T v_{Ti} + m_C v_{Ci}$$

Solving for the final velocity of the truck gives

$$v_{Tf} = \frac{m_T v_{Ti} + m_C(v_{Ci} - v_{Cf})}{m_T}$$
$$= \frac{(9\ 000\text{ kg})(20.0\text{ m/s}) + (1\ 200\text{ kg})[(25.0 - 18.0)\text{ m/s}]}{9\ 000\text{ kg}}$$
$$v_{Tf} = \boxed{20.9\text{ m/s East}}$$

(b) We compute the change in mechanical energy of the car-truck system from

$$\Delta KE = KE_f - KE_i = \left[\frac{1}{2}m_C v_{Cf}^2 + \frac{1}{2}m_T v_{Tf}^2\right] - \left[\frac{1}{2}m_C v_{Ci}^2 + \frac{1}{2}m_T v_{Ti}^2\right]$$

$$= \frac{1}{2}\left[m_C\left(v_{Cf}^2 - v_{Ci}^2\right) + m_T\left(v_{Tf}^2 - v_{Ti}^2\right)\right]$$

$$= \frac{1}{2}\Big\{(1\ 200\ \text{kg})\left[(18.0\ \text{m/s})^2 - (25.0\ \text{m/s})^2\right]$$

$$+ (9\ 000\ \text{kg})\left[(20.9\ \text{m/s})^2 - (20.0\ \text{m/s})^2\right]\Big\}$$

$$\Delta KE = \boxed{-8.68 \times 10^3\ \text{J}}$$

Note: If 20.9 m/s were used to determine the energy lost instead of 20.9333 as the answer to part (a), the answer would be very different. We have kept extra digits in all intermediate answers until the problem is complete.

(c) The mechanical energy of the car-truck system has decreased. Most of the energy was transformed to internal energy with some being carried away by sound.

P9.23 Momentum is conserved for the bullet-block system:

$$mv + 0 = (m + M)v_f$$

$$v = \left(\frac{m + M}{m}\right)v_f = \left(\frac{10.0 \times 10^{-3}\ \text{kg} + 5.00\ \text{kg}}{10.0 \times 10^{-3}\ \text{kg}}\right)(0.600\ \text{m/s})$$

$$= \boxed{301\ \text{m/s}}$$

P9.24 The collision is completely inelastic.

(a) Momentum is conserved by the collision:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$mv_1 + (2m)v_2 = mv_f + 2mv_f = 3mv_f$$

$$v_f = \frac{mv_1 + 2mv_2}{3m} \rightarrow \boxed{v_f = \frac{1}{3}(v_1 + 2v_2)}$$

(b) We compute the change in mechanical energy of the car-truck system from

$$\Delta K = K_f - K_i = \frac{1}{2}(3m)v_f^2 - \left[\frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2\right]$$

$$\Delta K = \frac{3m}{2}\left[\frac{1}{3}(v_1 + 2v_2)\right]^2 - \left[\frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2\right]$$

$$\Delta K = \frac{3m}{2}\left(\frac{v_1^2}{9} + \frac{4v_1v_2}{9} + \frac{4v_2^2}{9}\right) - \frac{mv_1^2}{2} - mv_2^2$$

$$= m\left(\frac{v_1^2}{6} + \frac{2v_1v_2}{3} + \frac{2v_2^2}{3} - \frac{v_1^2}{2} - v_2^2\right)$$

$$\Delta K = m\left(\frac{v_1^2}{6} + \frac{4v_1v_2}{6} + \frac{4v_2^2}{6} - \frac{3v_1^2}{6} - \frac{6v_2^2}{6}\right)$$

$$= m\left(-\frac{2v_1^2}{6} + \frac{4v_1v_2}{6} - \frac{2v_2^2}{6}\right)$$

$$\Delta K = \boxed{-\frac{m}{3}(v_1^2 + v_2^2 - 2v_1v_2)}$$

***P9.25** (a) We write the law of conservation of momentum as

$$mv_{1i} + 3mv_{2i} = 4mv_f$$

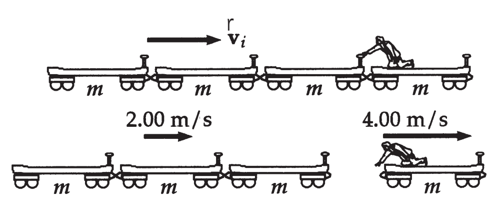
or $$v_f = \frac{4.00 \text{ m/s} + 3(2.00 \text{ m/s})}{4} = \boxed{2.50 \text{ m/s}}$$

(b) $$K_f - K_i = \frac{1}{2}(4m)v_f^2 - \left[\frac{1}{2}mv_{1i}^2 + \frac{1}{2}(3m)v_{2i}^2\right]$$

$$= \frac{1}{2}(2.50 \times 10^4 \text{ kg})[4(2.50 \text{ m/s})^2 - (4.00 \text{ m/s})^2 - 3(2.00 \text{ m/s})^2]$$

$$= \boxed{-3.75 \times 10^4 \text{ J}}$$

***P9.26** (a) The internal forces exerted by the actor do not change the total momentum of the system of the four cars and the movie actor. Conservation of momentum gives



ANS. FIG. P9.26

$$(4m)v_i = (3m)(2.00 \text{ m/s}) + m(4.00 \text{ m/s})$$

$$v_i = \frac{6.00 \text{ m/s} + 4.00 \text{ m/s}}{4} = \boxed{2.50 \text{ m/s}}$$

(b) $W_{actor} = K_f - K_i$

$$= \frac{1}{2}\left[(3m)(2.00\ \text{m/s})^2 + m(4.00\ \text{m/s})^2\right] - \frac{1}{2}(4m)(2.50\ \text{m/s})^2$$

$$W_{actor} = \frac{(2.50\times 10^4\ \text{kg})}{2}(12.0 + 16.0 - 25.0)(\text{m/s})^2 = \boxed{37.5\ \text{kJ}}$$

(c) The event considered here is the time reversal of the perfectly inelastic collision in the previous problem. The same momentum conservation equation describes both processes.

P9.27 (a) From the text's analysis of a one-dimensional elastic collision with an originally stationary target, the *x* component of the neutron's velocity changes from v_i to $v_{1f} = (1 - 12)v_i/13 = -11v_i/13$. The *x* component of the target nucleus velocity is $v_{2f} = 2v_i/13$.

The neutron started with kinetic energy $\frac{1}{2}m_1v_i^2$.

The target nucleus ends up with kinetic energy $\frac{1}{2}(12m_1)\left(\frac{2v_i}{13}\right)^2$.

Then the fraction transferred is

$$\frac{\frac{1}{2}(12m_1)(2v_i/13)^2}{\frac{1}{2}m_1v_i^2} = \frac{48}{169} = \boxed{0.284}$$

Because the collision is elastic, the other 71.6% of the original energy stays with the neutron. The carbon is functioning as a *moderator* in the reactor, slowing down neutrons to make them more likely to produce reactions in the fuel.

(b) The final kinetic energy of the neutron is

$$K_n = (0.716)(1.60\times 10^{-13}\ \text{J}) = \boxed{1.15\times 10^{-13}\ \text{J}}$$

and the final kinetic energy of the carbon nucleus is

$$K_C = (0.284)(1.60\times 10^{-13}\ \text{J}) = \boxed{4.54\times 10^{-14}\ \text{J}}$$

***P9.28** Let's first analyze the situation in which the wood block, of mass $m_w = 1.00$ kg, is held in a vise. The bullet of mass $m_b = 7.00$ g is initially moving with speed v_b and then comes to rest in the block due to the kinetic friction force f_k between the block and the bullet as the bullet

deforms the wood fibers and moves them out of the way. The result is an increase in internal energy in the wood and the bullet. Identify the wood and the bullet as an isolated system for energy during the collision:

$$\Delta K + \Delta E_{\text{int}} = 0$$

Substituting for the energies:

$$\left(0 - \frac{1}{2}m_b v_b^2\right) + f_k d = 0 \qquad \textbf{[1]}$$

where d = 8.00 cm is the depth of penetration of the bullet in the wood.

Now consider the second situation, where the block is sitting on a frictionless surface and the bullet is fired into it. Identify the wood and the bullet as an isolated system for energy during the collision:

$$\Delta K + \Delta E_{\text{int}} = 0$$

Substituting for the energies:

$$\left[\frac{1}{2}(m_b + m_w)v_f^2 - \frac{1}{2}m_b v_b^2\right] + f_k d' = 0 \qquad \textbf{[2]}$$

where v_f is the speed with which the block and imbedded bullet slide across the table after the collision and d' is the depth of penetration of the bullet in this situation. Identify the wood and the bullet as an isolated system for momentum during the collision:

$$\Delta p = 0 \quad \rightarrow \quad p_i = p_f \quad \rightarrow \quad m_b v_b = (m_b + m_w)v_f \qquad \textbf{[3]}$$

Solving equation [3] for v_b, we obtain

$$v_b = \frac{(m_b + m_w)v_f}{m_b} \qquad \textbf{[4]}$$

Solving equation [1] for $f_k d$ and substituting for v_b from equation [4] above:

$$f_k d = \frac{1}{2}m_b v_b^2 = \frac{1}{2}m_b\left[\frac{(m_b + m_w)v_f}{m_b}\right]^2 = \frac{1}{2}\frac{(m_b + m_w)^2}{m_b}v_f^2 \qquad \textbf{[5]}$$

Solving equation [2] for $f_k d'$ and substituting for v_b from equation [4]:

$$f_k d' = -\left[\frac{1}{2}(m_b + m_w)v_f^2 - \frac{1}{2}m_b v_b^2\right]$$

$$= -\left[\frac{1}{2}(m_b + m_w)v_f^2 - \frac{1}{2}m_b\left[\frac{(m_b + m_w)v_f}{m_b}\right]^2\right]$$

$$f_k d' = \frac{1}{2}\left[\frac{m_w}{m_b}(m_b + m_w)\right]v_f^2 \qquad \textbf{[6]}$$

Dividing equation [6] by [5] gives

$$\frac{f_k d'}{f_k d} = \frac{d'}{d} = \frac{\frac{1}{2}\left[\frac{m_w}{m_b}(m_b + m_w)\right]v_f^2}{\frac{1}{2}\left[\frac{(m_b + m_w)^2}{m_b}\right]v_f^2} = \frac{m_w}{m_b + m_w}$$

Solving for d' and substituting numerical values gives

$$d' = \left(\frac{m_w}{m_b + m_w}\right)d = \left[\frac{1.00\text{ kg}}{0.007\ 00\text{ kg} + 1.00\text{ kg}}\right](8.00\text{ cm}) = \boxed{7.94\text{ cm}}$$

***P9.29** (a) The speed v of both balls just before the basketball reaches the ground may be found from $v_{yf}^2 = v_{yi}^2 + 2a_y\Delta y$ as

$$v = \sqrt{v_{yi}^2 + 2a_y\Delta y} = \sqrt{0 + 2(-g)(-h)} = \sqrt{2gh}$$

$$= \sqrt{2(9.80\text{ m/s}^2)(1.20\text{ m})} = \boxed{4.85\text{ m/s}}$$

(b) Immediately after the basketball rebounds from the floor, it and the tennis ball meet in an elastic collision. The velocities of the two balls just before collision are

for the tennis ball (subscript *t*): $v_{ti} = -v$

and for the basketball (subscript *b*): $v_{bi} = +v$

We determine the velocity of the tennis ball immediately after this elastic collision as follows:

Momentum conservation gives

$$m_t v_{tf} + m_b v_{bf} = m_t v_{ti} + m_b v_{bi}$$

or $$m_t v_{tf} + m_b v_{bf} = (m_b - m_t)v \qquad \textbf{[1]}$$

From the criteria for a perfectly elastic collision:

$$v_{ti} - v_{bi} = -\left(v_{tf} - v_{bf}\right)$$

or $\quad v_{bf} = v_{tf} + v_{ti} - v_{bi} = v_{tf} - 2v \qquad$ **[2]**

Substituting equation [2] into [1] gives

$$m_t v_{tf} + m_b\left(v_{tf} - 2v\right) = \left(m_b - m_t\right)v$$

or the upward speed of the tennis ball immediately after the collision is

$$v_{tf} = \left(\frac{3m_b - m_t}{m_t + m_b}\right)v = \left(\frac{3m_b - m_t}{m_t + m_b}\right)\sqrt{2gh}$$

The vertical displacement of the tennis ball during its rebound following the collision is given by $v_{yf}^2 = v_{yi}^2 + 2a_y\Delta y$ as

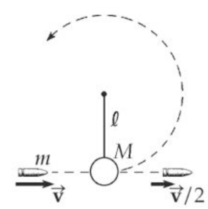
$$\Delta y = \frac{v_{yf}^2 - v_{yi}^2}{2a_y} = \frac{0 - v_{tf}^2}{2\left(-g\right)} = \left(\frac{1}{\cancel{2g}}\right)\left(\frac{3m_b - m_t}{m_t + m_b}\right)^2\left(\cancel{2g}h\right)$$

$$= \left(\frac{3m_b - m_t}{m_t + m_b}\right)^2 h$$

Substituting,

$$\Delta y = \left[\frac{3\left(590\text{ g}\right) - \left(57.0\text{ g}\right)}{57.0\text{ g} + 590\text{ g}}\right]^2\left(1.20\text{ m}\right) = \boxed{8.41\text{ m}}$$

P9.30 Energy is conserved for the bob-Earth system between bottom and top of the swing. At the top the stiff rod is in compression and the bob nearly at rest.



ANS. FIG. P9.30

$$K_i + U_i = K_f + U_f: \qquad \frac{1}{2}Mv_b^2 + 0 = 0 + Mg2\ell$$

$$v_b^2 = 4g\ell \qquad \text{so} \qquad v_b = 2\sqrt{g\ell}$$

Momentum of the bob-bullet system is conserved in the collision:

$$mv = m\frac{v}{2} + M\left(2\sqrt{g\ell}\right) \;\rightarrow\; \boxed{v = \frac{4M}{m}\sqrt{g\ell}}$$

P9.31 The collision between the clay and the wooden block is completely inelastic. Momentum is conserved by the collision. Find the relation between the speed of the clay (C) just before impact and the speed of the clay+block (CB) just after impact:

$$\vec{\mathbf{p}}_{Bi} + \vec{\mathbf{p}}_{Ci} = \vec{\mathbf{p}}_{Bf} + \vec{\mathbf{p}}_{Cf} \rightarrow m_B v_{Bi} + m_C v_{Ci} = m_B v_{Bf} + m_C v_{Cf}$$

$$M(0) + mv_C = mv_{CB} + Mv_{CB} = (m + M)v_{CB}$$

$$v_C = \frac{(m + M)}{m} v_{CB}$$

Now use conservation of energy in the presence of friction forces to find the relation between the speed v_{CB} just after impact and the distance the block slides before stopping:

$$\Delta K + \Delta E_{int} = 0: \quad 0 - \frac{1}{2}(m + M)v_{CB}^2 - fd = 0$$

$$\text{and} \quad -fd = -\mu nd = -\mu(m + M)gd$$

$$\rightarrow \frac{1}{2}(m + M)v_{CB}^2 = \mu(m + M)gd \rightarrow v_{CB} = \sqrt{2\mu gd}$$

Combining our results, we have

$$v_C = \frac{(m + M)}{m}\sqrt{2\mu gd}$$

$$= \frac{(12.0 \text{ g} + 100 \text{ g})}{12.0 \text{ g}}\sqrt{2(0.650)\left(9.80 \text{ m/s}^2\right)(7.50 \text{ m})}$$

$$\boxed{v_C = 91.2 \text{ m/s}}$$

P9.32 The collision between the clay and the wooden block is completely inelastic. Momentum is conserved by the collision. Find the relation between the speed of the clay (C) just before impact and the speed of the clay+block (CB) just after impact:

$$\vec{\mathbf{p}}_{Bi} + \vec{\mathbf{p}}_{Ci} = \vec{\mathbf{p}}_{Bf} + \vec{\mathbf{p}}_{Cf} \rightarrow m_B v_{Bi} + m_C v_{Ci} = m_B v_{Bf} + m_C v_{Cf}$$

$$M(0) + mv_C = mv_{CB} + Mv_{CB} = (m + M)v_{CB}$$

$$v_C = \frac{(m + M)}{m} v_{CB}$$

Now use conservation of energy in the presence of friction forces to find the relation between the speed v_{CB} just after impact and the distance the block slides before stopping:

$$\Delta K + \Delta E_{int} = 0: \quad 0 - \frac{1}{2}(m + M)v_{CB}^2 - fd = 0$$

$$\text{and} \qquad -fd = -\mu nd = -\mu(m + M)gd$$

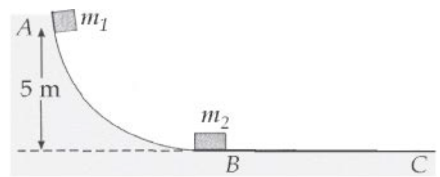
Then,

$$\frac{1}{2}(m+M)v_{CB}^2 = \mu(m+M)gd \rightarrow v_{CB} = \sqrt{2\mu gd}$$

Combining our results, we have

$$\boxed{v_C = \frac{(m+M)}{m}\sqrt{2\mu gd}}$$

P9.33 The mechanical energy of the isolated block-Earth system is conserved as the block of mass m_1 slides down the track. First we find v_1, the speed of m_1 at B before collision:



ANS. FIG. P9.33

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m_1v_1^2 + 0 = 0 + m_1gh$$

$$v_1 = \sqrt{2(9.80 \text{ m/s}^2)(5.00 \text{ m})} = 9.90 \text{ m/s}$$

Now we use the text's analysis of one-dimensional elastic collisions to find v_{1f}, the speed of m_1 at B just after collision.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_1 = -\frac{1}{3}(9.90) \text{ m/s} = -3.30 \text{ m/s}$$

Now the 5-kg block bounces back up to its highest point after collision according to

$$m_1gh_{max} = \frac{1}{2}m_1v_{1f}^2$$

which gives

$$h_{max} = \frac{v_{1f}^2}{2g} = \frac{(-3.30 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{0.556 \text{ m}}$$

P9.34 (a) Using conservation of momentum, $\left(\sum\vec{\mathbf{p}}\right)_{\text{before}} = \left(\sum\vec{\mathbf{p}}\right)_{\text{after}}$, gives

$$(4.00 \text{ kg})(5.00 \text{ m/s}) + (10.0 \text{ kg})(3.00 \text{ m/s}) + (3.00 \text{ kg})(-4.00 \text{ m/s}) = [(4.00 + 10.0 + 3.00) \text{ kg}]v$$

Therefore, $v = +2.24$ m/s, or $\boxed{\text{2.24 m/s toward the right}}$.

(b) $\boxed{\text{No.}}$ For example, if the 10.0-kg and 3.00-kg masses were to

stick together first, they would move with a speed given by solving

$$(13.0\text{ kg})v_1 = (10.0\text{ kg})(3.00\text{ m/s}) + (3.00\text{ kg})(-4.00\text{ m/s})$$

or $v_1 = +1.38\text{ m/s}$

Then when this 13.0-kg combined mass collides with the 4.00-kg mass, we have

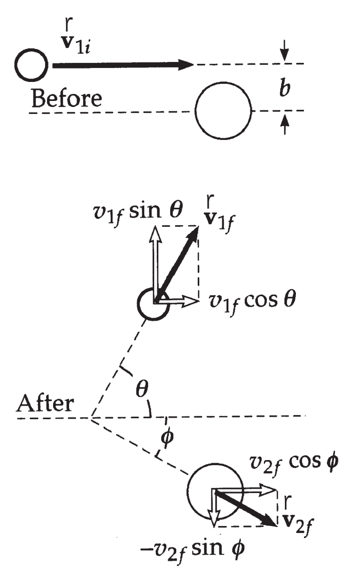
$$(17.0\text{ kg})v = (13.0\text{ kg})(1.38\text{ m/s}) + (4.00\text{ kg})(5.00\text{ m/s})$$

and $v = +2.24$ m/s, just as in part (a).

$\boxed{\text{Coupling order makes no difference to the final velocity.}}$

Section 9.5 Collisions in Two Dimensions

***P9.35** (a) We write equations expressing conservation of the x and y components of momentum, with reference to the figures on the right. Let the puck initially at rest be m_2. In the x direction,



ANS. FIG. P9.35

$$m_1 v_{1i} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\phi$$

which gives

$$v_{2f}\cos\phi = \frac{m_1 v_{1i} - m_1 v_{1f}\cos\theta}{m_2}$$

or

$$v_{2f}\cos\phi = \left(\frac{1}{0.300\text{ kg}}\right)\left[(0.200\text{ kg})(2.00\text{ m/s}) - (0.200\text{ kg})(1.00\text{ m/s})\cos 53.0°\right]$$

In the y direction,

$$0 = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\phi$$

which gives

$$v_{2f}\sin\phi = \frac{m_1 v_{1f}\sin\theta}{m_2}$$

or

$$0 = (0.200 \text{ kg})(1.00 \text{ m/s})\sin 53.0° - (0.300 \text{ kg})(v_{2f} \sin\phi)$$

From these equations, we find

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{0.532}{0.932} = 0.571 \quad \text{or} \quad \boxed{\phi = 29.7°}$$

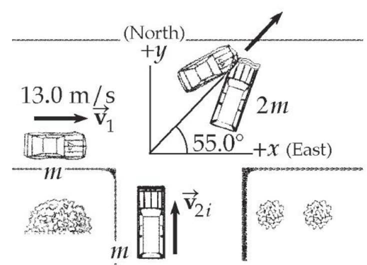
Then $v_{2f} = \dfrac{0.160 \text{ kg}\cdot\text{m/s}}{(0.300 \text{ kg})(\sin 29.7°)} = \boxed{1.07 \text{ m/s}}$

(b) $K_i = \dfrac{1}{2}(0.200 \text{ kg})(2.00 \text{ m/s})^2 = 0.400 \text{ J}$ and

$$K_f = \frac{1}{2}(0.200 \text{ kg})(1.00 \text{ m/s})^2 + \frac{1}{2}(0.300 \text{ kg})(1.07 \text{ m/s})^2 = 0.273 \text{ J}$$

$$f_{\text{lost}} = \frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = \frac{0.273 \text{ J} - 0.400 \text{ J}}{0.400 \text{ J}} = \boxed{-0.318}$$

P9.36 We use conservation of momentum for the system of two vehicles for both northward and eastward components, to find the original speed of car number 2.



ANS. FIG. P8.26

For the eastward direction:

$$m(13.0 \text{ m/s}) = 2mV_f \cos 55.0°$$

For the northward direction:

$$mv_{2i} = 2mV_f \sin 55.0°$$

Divide the northward equation by the eastward equation to find:

$$v_{2i} = (13.0 \text{ m/s}) \tan 55.0° = 18.6 \text{ m/s} = 41.5 \text{ mi/h}$$

Thus, the driver of the northbound car was untruthful. His original speed was more than 35 mi/h.

P9.37 We will use conservation of both the x component and the y component of momentum for the two-puck system, which we can write as a single vector equation.

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

Both objects have the same final velocity, which we call $\vec{\mathbf{v}}_f$. Doing the algebra and substituting to solve for the one unknown gives

$$\vec{\mathbf{v}}_f = \frac{m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i}}{m_1 + m_2}$$

$$= \frac{(3.00 \text{ kg})(5.00\hat{\mathbf{i}} \text{ m/s}) + (2.00 \text{ kg})(-3.00\hat{\mathbf{j}} \text{ m/s})}{3.00 \text{ kg} + 2.00 \text{ kg}}$$

and calculating gives $\vec{\mathbf{v}}_f = \dfrac{15.0\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}}{5.00}$ m/s = $\boxed{(3.00\hat{\mathbf{i}} - 1.20\hat{\mathbf{j}}) \text{ m/s}}$

P9.38 We write the conservation of momentum in the x direction, $p_{xf} = p_{xi}$, as

$$mv_O \cos 37.0° + mv_Y \cos 53.0° = m(5.00 \text{ m/s})$$

$$0.799v_O + 0.602v_Y = 5.00 \text{ m/s} \qquad \textbf{[1]}$$

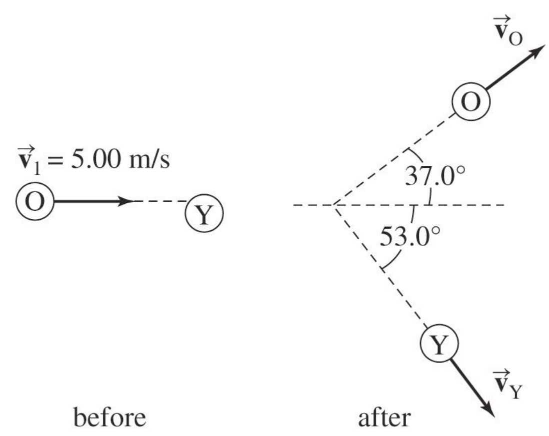
and the conservation of momentum in the y direction, $p_{yf} = p_{yi}$, as

$$mv_O \sin 37.0° - mv_Y \sin 53.0° = 0$$

$$0.602v_O = 0.799v_Y \qquad \textbf{[2]}$$

Solving equations [1] and [2] simultaneously gives,

$\boxed{v_O = 3.99 \text{ m/s}}$ and $\boxed{v_Y = 3.01 \text{ m/s}}$



ANS. FIG. P9.38

P9.39 ANS. FIG. P9.38 illustrates the collision. We write the conservation of momentum in the x direction, $p_{xf} = p_{xi}$, as

$$mv_O \cos\theta + mv_Y \cos(90.0° - \theta) = mv_i$$

$$v_O \cos\theta + v_Y \sin\theta = v_i \qquad \textbf{[1]}$$

and the conservation of momentum in the y direction, $p_{yf} = p_{yi}$, as

$$mv_O \sin\theta - mv_Y \cos(90.0° - \theta) = 0$$
$$v_O \sin\theta = v_Y \cos\theta \quad \textbf{[2]}$$

From equation [2],

$$v_O = v_Y\left(\frac{\cos\theta}{\sin\theta}\right) \quad \textbf{[3]}$$

Substituting into equation [1],

$$v_Y\left(\frac{\cos^2\theta}{\sin\theta}\right) + v_Y \sin\theta = v_i$$

so

$$v_Y\left(\cos^2\theta + \sin^2\theta\right) = v_i \sin\theta, \text{ and } \boxed{v_Y = v_i \sin\theta}$$

Then, from equation [3], $\boxed{v_O = v_i \cos\theta}$.

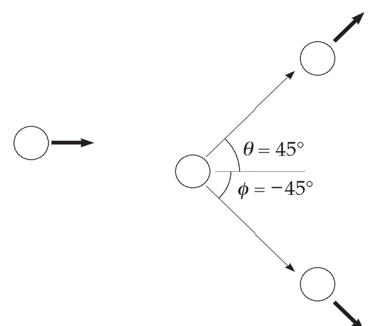
We did not need to write down an equation expressing conservation of mechanical energy. In this situation, the requirement on perpendicular final velocities is equivalent to the condition of elasticity.

***P9.40** (a) The vector expression for conservation of momentum, $\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$ gives $p_{xi} = p_{xf}$ and $p_{yi} = p_{yf}$.

$$mv_i = mv\cos\theta + mv\cos\phi \quad \textbf{[1]}$$

$$0 = mv\sin\theta + mv\sin\phi \quad \textbf{[2]}$$

From [2], $\sin\theta = -\sin\phi$ so $\theta = -\phi$.



ANS. FIG. P9.40

Furthermore, energy conservation for the system of two protons requires

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

so

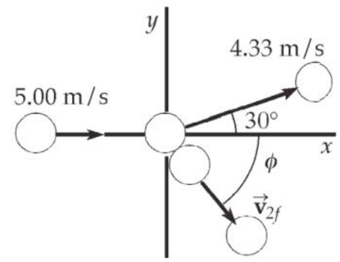
$$\boxed{v = \frac{v_i}{\sqrt{2}}}$$

(b) Hence, [1] gives

$$v_i = \frac{2v_i \cos\theta}{\sqrt{2}}$$

with $\theta = \boxed{45.0°}$ and $\phi = \boxed{-45.0°}$.

P9.41 By conservation of momentum for the system of the two billiard balls (with all masses equal), in the *x* and *y* directions separately,



ANS. FIG. P9.41

$$5.00\text{ m/s} + 0 = (4.33\text{ m/s})\cos 30.0° + v_{2fx}$$
$$v_{2fx} = 1.25\text{ m/s}$$
$$0 = (4.33\text{ m/s})\sin 30.0° + v_{2fy}$$
$$v_{2fy} = -2.16\text{ m/s}$$
$$\vec{\mathbf{v}}_{2f} = \boxed{2.50\text{ m/s at } -60.0°}$$

Note that we did not need to explicitly use the fact that the collision is perfectly elastic.

P9.42 (a) The opponent grabs the fullback and does not let go, so the two players move together at the end of their interaction—thus the collision is perfectly inelastic.

(b) First, we conserve momentum for the system of two football players in the *x* direction (the direction of travel of the fullback):

$$(90.0\text{ kg})(5.00\text{ m/s}) + 0 = (185\text{ kg})V\cos\theta$$

where θ is the angle between the direction of the final velocity V and the x axis. We find

$$V\cos\theta = 2.43\text{ m/s} \qquad \textbf{[1]}$$

Now consider conservation of momentum of the system in the *y* direction (the direction of travel of the opponent):

$$(95.0\text{ kg})(3.00\text{ m/s}) + 0 = (185\text{ kg})V\sin\theta$$

which gives

$$V\sin\theta = 1.54\text{ m/s} \qquad \textbf{[2]}$$

Divide equation [2] by [1]:

$$\tan\theta = \frac{1.54}{2.43} = 0.633$$

From which, $\boxed{\theta = 32.3°}$.

Then, either [1] or [2] gives $V = \boxed{2.88 \text{ m/s}}$.

(c) $K_i = \frac{1}{2}(90.0 \text{ kg})(5.00 \text{ m/s})^2 + \frac{1}{2}(95.0 \text{ kg})(3.00 \text{ m/s})^2 = 1.55 \times 10^3 \text{ J}$

$$K_f = \frac{1}{2}(185 \text{ kg})(2.88 \text{ m/s})^2 = 7.67 \times 10^2 \text{ J}$$

Thus, the kinetic energy lost is $\boxed{\text{786 J into internal energy}}$.

P9.43 (a) With three particles, the total final momentum of the system is $m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f} + m_3\vec{\mathbf{v}}_{3f}$ and it must be zero to equal the original momentum. The mass of the third particle is

$$m_3 = (17.0 - 5.00 - 8.40) \times 10^{-27} \text{ kg}$$

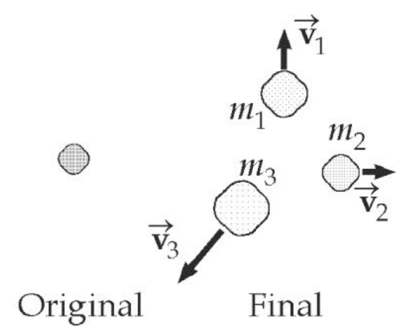
or $m_3 = 3.60 \times 10^{-27}$ kg

Solving $m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f} + m_3\vec{\mathbf{v}}_{3f} = 0$ for $\vec{\mathbf{v}}_{3f}$ gives

$$\vec{\mathbf{v}}_{3f} = -\frac{m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}}{m_3}$$

$$\vec{\mathbf{v}}_{3f} = -\frac{(3.36\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}) \times 10^{-20} \text{ kg} \cdot \text{m/s}}{3.60 \times 10^{-27} \text{ kg}}$$

$$= \boxed{(-9.33 \times 10^6 \hat{\mathbf{i}} - 8.33 \times 10^6 \hat{\mathbf{j}}) \text{ m/s}}$$



ANS. FIG. P9.43

(b) The original kinetic energy of the system is zero.

The final kinetic energy is $K = K_{1f} + K_{2f} + K_{3f}$.

The terms are

$$K_{1f} = \frac{1}{2}(5.00 \times 10^{-27} \text{ kg})(6.00 \times 10^6 \text{ m/s})^2 = 9.00 \times 10^{-14} \text{ J}$$

$$K_{2f} = \frac{1}{2}(8.40\times10^{-27}\text{ kg})(4.00\times10^{6}\text{ m/s})^2 = 6.72\times10^{-14}\text{ J}$$

$$K_{3f} = \frac{1}{2}(3.60\times10^{-27}\text{ kg}) \times\left[(-9.33\times10^{6}\text{ m/s})^2+(-8.33\times10^{6}\text{ m/s})^2\right] = 28.2\times10^{-14}\text{ J}$$

Then the system kinetic energy is

$$K = 9.00\times10^{-14}\text{ J} + 6.72\times10^{-14}\text{ J} + 28.2\times10^{-14}\text{ J} = \boxed{4.39\times10^{-13}\text{ J}}$$

P9.44 The initial momentum of the system is 0. Thus,

$$\left(1.20\text{m}\right)v_{Bi} = \text{m}\left(10.0\text{ m/s}\right)$$

and $v_{Bi} = 8.33\text{ m/s}$

From conservation of energy,

$$K_i = \frac{1}{2}\text{m}(10.0\text{ m/s})^2 + \frac{1}{2}(1.20\text{m})(8.33\text{ m/s})^2 = \frac{1}{2}\text{m}(183\text{ m}^2/\text{s}^2)$$

$$K_f = \frac{1}{2}\text{m}(v_G)^2 + \frac{1}{2}(1.20\text{m})(v_B)^2 = \frac{1}{2}\left(\frac{1}{2}\text{m}(183\text{ m}^2/\text{s}^2)\right)$$

or $v_G^2 + 1.20v_B^2 = 91.7\text{ m}^2/\text{s}^2$ **[1]**

From conservation of momentum,

$$\text{m}v_G = \left(1.20\text{m}\right)v_B$$

or $v_G = 1.20v_B$ **[2]**

Solving [1] and [2] simultaneously, we find

$$(1.20v_B)^2 + 1.20v_B^2 = 91.7\text{ m}^2/\text{s}^2$$

$$v_B = (91.7\text{ m}^2/\text{s}^2/2.64)^{1/2}$$

which gives

$\boxed{v_B = 5.89\text{ m/s}}$ (speed of blue puck after collision)

and $\boxed{v_G = 7.07\text{ m/s}}$ (speed of green puck after collision)

Section 9.6 The Center of Mass

P9.45 The *x* coordinate of the center of mass is

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0+0+0+0}{2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg}} = 0$$

and the *y* coordinate of the center of mass is

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i}$$

$$= \left(\frac{1}{2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg}}\right)$$

$$\times[(2.00 \text{ kg})(3.00 \text{ m}) + (3.00 \text{ kg})(2.50 \text{ m})$$

$$+ (2.50 \text{ kg})(0) + (4.00 \text{ kg})(-0.500 \text{ m})]$$

$$y_{CM} = 1.00 \text{ m}$$

Then $\boxed{\vec{\mathbf{r}}_{CM} = \left(0\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}\right) \text{ m}}$

P9.46 Let the *x* axis start at the Earth's center and point toward the Moon.

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{\left(5.97\times10^{24} \text{ kg}\right)(0) + \left(7.35\times10^{22} \text{ kg}\right)\left(3.84\times10^{8} \text{ m}\right)}{6.05\times10^{24} \text{ kg}}$$

$$= \boxed{4.66\times10^{6} \text{ m from the Earth's center}}$$

The center of mass is within the Earth, which has radius 6.37×10^6 m. It is 1.7 Mm below the point on the Earth's surface where the Moon is straight overhead.

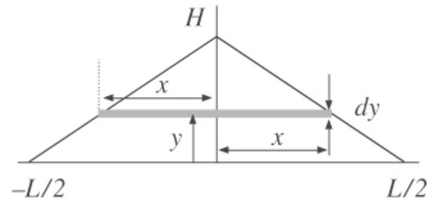
P9.47 The volume of the monument is that of a thick triangle of base $L = 64.8$ m, height $H = 15.7$ m, and width $W = 3.60$ m: $V = ½ LHW = 1.83 \times 10^3 \text{ m}^3$. The monument has mass $M = \rho V = (3\,800 \text{ kg/m}^3)V = 6.96 \times 10^6$ kg. The height of the center of mass (CM) is $y_{CM} = H/3$ (derived below). The amount of work done on the blocks is

$$U_g = Mgy_{CM}$$

$$= Mg\frac{H}{3} = \left(6.96\times10^{6} \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\left(\frac{15.7 \text{ m}}{3}\right)$$

$$= \boxed{3.57\times10^{8} \text{ J}}$$

We derive $y_{CM} = H/3$ here:



ANS. FIG. P9.47

We model the monument with the figure shown above. Consider the monument to be composed of slabs of infinitesimal thickness dy stacked on top of each other. A slab at height y has a infinitesimal volume element $dV = 2xWdy$, where W is the width of the monument and x is a function of height y.

The equation of the sloping side of the monument is

$$y = H - \frac{H}{L/2}x \rightarrow y = H - \frac{2H}{L}x \rightarrow y = H\left(1 - \frac{2}{L}x\right)$$

where x ranges from 0 to + $L/2$. Therefore,

$$x = \frac{L}{2}\left(1 - \frac{y}{H}\right)$$

where y ranges from 0 to H. The infinitesimal volume of a slab at height y is then

$$dV = 2xWdy = LW\left(1 - \frac{y}{H}\right)dy.$$

The mass contained in a volume element is $dm = \rho dV$.

Because of the symmetry of the monument, its CM lies above the origin of the coordinate axes at position y_{CM}:

$$y_{CM} = \frac{1}{M}\int_0^M y\,dm = \frac{1}{M}\int_0^V y\rho\,dV = \frac{1}{M}\int_0^H y\rho LW\left(1 - \frac{y}{H}\right)dy$$

$$y_{CM} = \frac{\rho LW}{M}\int_0^H \left(y - \frac{y^2}{H}\right)dy = \frac{\rho LW}{M}\left(\frac{y^2}{2} - \frac{y^3}{3H}\right)\Bigg|_0^H$$

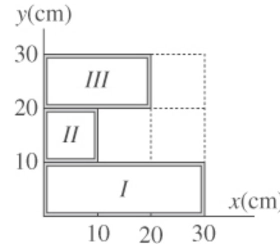
$$= \frac{\rho LW}{M}\left(\frac{H^2}{2} - \frac{H^3}{3H}\right)$$

$$y_{CM} = \frac{\rho LWH^2}{M}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{6}\frac{\rho LWH^2}{\left(\frac{1}{2}\rho LWH\right)} = \left(\frac{2}{1}\right)\frac{H}{6}$$

$$y_{CM} = \frac{H}{3}$$

where we have used $M = \rho\left(\frac{1}{2}LHW\right)$.

P9.48 We could analyze the object as nine squares, each represented by an equal-mass particle at its center. But we will have less writing to do if we think of the sheet as composed of three sections, and consider the mass of each section to be at the geometric center of that section. Define the mass per unit area to be σ, and number the rectangles as shown. We can then calculate the mass and identify the center of mass of each section.



ANS. FIG. P9.48

$$m_I = (30.0\text{ cm})(10.0\text{ cm})\sigma \quad \text{with} \quad CM_I = (15.0\text{ cm}, 5.00\text{ cm})$$

$$m_{II} = (10.0\text{ cm})(20.0\text{ cm})\sigma \quad \text{with} \quad CM_{II} = (5.00\text{ cm}, 20.0\text{ cm})$$

$$m_{III} = (10.0\text{ cm})(10.0\text{ cm})\sigma \quad \text{with} \quad CM_{III} = (15.0\text{ cm}, 25.0\text{ cm})$$

The overall center of mass is at a point defined by the vector equation:

$$\vec{\mathbf{r}}_{CM} \equiv \left(\sum m_i \vec{\mathbf{r}}_i\right) / \sum m_i$$

Substituting the appropriate values, $\vec{\mathbf{r}}_{CM}$ is calculated to be:

$$\vec{\mathbf{r}}_{CM} = \left(\frac{1}{\sigma\left(300\text{ cm}^2 + 200\text{ cm}^2 + 100\text{ cm}^2\right)}\right) \times \left\{\sigma[(300)(15.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}) + (200)(5.00\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}}) + (100)(15.0\hat{\mathbf{i}} + 25.0\hat{\mathbf{j}})]\text{ cm}^3\right\}$$

Calculating,

$$\vec{\mathbf{r}}_{CM} = \frac{4\,500\hat{\mathbf{i}} + 1\,500\hat{\mathbf{j}} + 1\,000\hat{\mathbf{i}} + 4\,000\hat{\mathbf{j}} + 1\,500\hat{\mathbf{i}} + 2\,500\hat{\mathbf{j}}}{600}\text{ cm}$$

and evaluating, $\vec{\mathbf{r}}_{CM} = \boxed{(11.7\hat{\mathbf{i}} + 13.3\hat{\mathbf{j}})\text{ cm}}$

P9.49 This object can be made by wrapping tape around a light, stiff, uniform rod.

(a) $$M = \int_0^{0.300\text{ m}} \lambda\, dx = \int_0^{0.300\text{ m}} \left[50.0 + 20.0x\right] dx$$

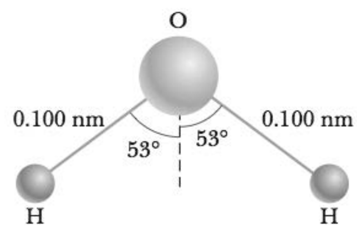
$$M = \left[50.0x + 10.0x^2\right]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$$

(b) $$x_{CM} = \frac{\int_{\text{all mass}} x\,dm}{M} = \frac{1}{M}\int_0^{0.300\text{ m}} \lambda x\,dx = \frac{1}{M}\int_0^{0.300\text{ m}} \left[50.0x + 20.0x^2\right]dx$$

$$x_{CM} = \frac{1}{15.9\text{ g}}\left[25.0x^2 + \frac{20x^3}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$$

***P9.50** We use a coordinate system centered in the oxygen (O) atom, with the *x* axis to the right and the *y* axis upward. Then, from symmetry,

$$\boxed{x_{CM} = 0}$$



ANS. FIG. P9.50

and

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i}$$
$$= \left(\frac{1}{15.999\text{ u} + 1.008\text{ u} + 1.008\text{ u}}\right)$$
$$\times[0 - (1.008\text{ u})(0.100\text{ nm})\cos 53.0°$$
$$-(1.008\text{ u})(0.100\text{ nm})\cos 53.0°]$$

$$\boxed{\text{The center of mass of the molecule lies on the dotted line shown in ANS. FIG. P9.50, 0.006 73 nm below the center of the O atom.}}$$

Section 9.7 Systems of Many Particles

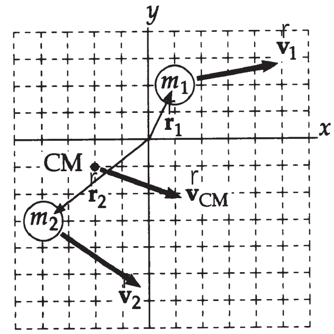
P9.51 (a) $$\vec{\mathbf{v}}_{CM} = \frac{\sum m_i \vec{\mathbf{v}}_i}{M} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{M}$$

$$= \left(\frac{1}{5.00\text{ kg}}\right)[(2.00\text{ kg})(2.00\hat{\mathbf{i}}\text{ m/s} - 3.00\hat{\mathbf{j}}\text{ m/s})$$
$$+ (3.00\text{ kg})(1.00\hat{\mathbf{i}}\text{ m/s} + 6.00\hat{\mathbf{j}}\text{ m/s})]$$

$$\vec{\mathbf{v}}_{CM} = \boxed{\left(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) $$\vec{\mathbf{p}} = M\vec{\mathbf{v}}_{CM} = (5.00\text{ kg})(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}})\text{ m/s} = \boxed{(7.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}})\text{ kg}\cdot\text{m/s}}$$

***P9.52** (a) ANS. FIG. P9.52 shows the position vectors and velocities of the particles.



ANS. FIG. P9.52

(b) Using the definition of the position vector at the center of mass,

$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$\vec{r}_{CM} = \left(\frac{1}{2.00\text{ kg} + 3.00\text{ kg}}\right)[(2.00\text{ kg})(1.00\text{ m},\ 2.00\text{ m}) + (3.00\text{ kg})(-4.00\text{ m},\ -3.00\text{ m})]$$

$$\vec{r}_{CM} = \boxed{(-2.00\hat{i} - 1.00\hat{j})\text{ m}}$$

(c) The velocity of the center of mass is

$$\vec{\mathbf{v}}_{CM} = \frac{\vec{\mathbf{P}}}{M} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}$$

$$= \left(\frac{1}{2.00\text{ kg} + 3.00\text{ kg}}\right)[(2.00\text{ kg})(3.00\text{ m/s},\ 0.50\text{ m/s}) + (3.00\text{ kg})(3.00\text{ m/s},\ -2.00\text{ m/s})]$$

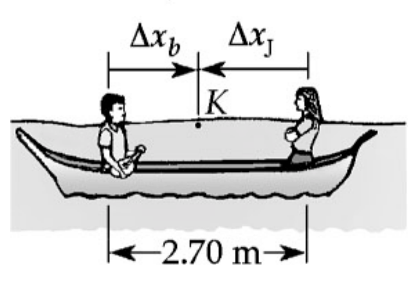
$$\vec{\mathbf{v}}_{CM} = \boxed{(3.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}})\text{ m/s}}$$

(d) The total linear momentum of the system can be calculated as $\vec{\mathbf{P}} = M\vec{\mathbf{v}}_{CM}$ or as $\vec{\mathbf{P}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$. Either gives

$$\vec{\mathbf{P}} = \boxed{(15.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}})\text{ kg}\cdot\text{m/s}}$$

P9.53 No outside forces act on the boat-plus-lovers system, so its momentum is conserved at zero and the center of mass of the boat-passengers system stays fixed:

$$x_{CM,i} = x_{CM,f}$$



ANS. FIG. P9.53

Define K to be the point where they kiss, and Δx_J and Δx_b as shown in the figure. Since Romeo moves with the boat (and thus $\Delta x_{Romeo} = \Delta x_b$), let m_b be the combined mass of Romeo and the boat. The front of the boat and the shore are to the right in this picture,

and we take the positive x direction to the right. Then,

$$m_J \Delta x_J + m_b \Delta x_b = 0$$

Choosing the x axis to point toward the shore,

$$(55.0 \text{ kg})\Delta x_J + (77.0 \text{ kg} + 80.0 \text{ kg})\Delta x_b = 0$$

and $\Delta x_J = -2.85 \Delta x_b$

As Juliet moves away from shore, the boat and Romeo glide toward the shore until the original 2.70-m gap between them is closed. We describe the relative motion with the equation

$$|\Delta x_J| + \Delta x_b = 2.70 \text{ m}$$

Here the first term needs absolute value signs because Juliet's change in position is toward the left. An equivalent equation is then

$$-\Delta x_J + \Delta x_b = 2.70 \text{ m}$$

Substituting, we find

$$+2.85\Delta x_b + \Delta x_b = 2.70 \text{ m}$$

so $\Delta x_b = \dfrac{2.70 \text{ m}}{3.85} = \boxed{0.700 \text{ m}}$ towards the shore

P9.54 The vector position of the center of mass is (suppressing units)

$$\vec{r}_{CM} = \frac{m_1 \vec{\mathbf{r}}_1 + m_2 \vec{\mathbf{r}}_2}{m_1 + m_2} = \frac{3.5\left[\left(3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)t + 2\hat{\mathbf{j}}t^2\right] + 5.5\left[3\hat{\mathbf{i}} - 2\hat{\mathbf{i}}t^2 + 6\hat{\mathbf{j}}t\right]}{3.5 + 5.5}$$

$$= \left(1.83 + 1.17t - 1.22t^2\right)\hat{\mathbf{i}} + \left(-2.5t + 0.778t^2\right)\hat{\mathbf{j}}$$

(a) At $t = 2.50$ s,

$$\vec{\mathbf{r}}_{CM} = (1.83 + 1.17 \cdot 2.5 - 1.22 \cdot 6.25)\hat{\mathbf{i}} + (-2.5 \cdot 2.5 + 0.778 \cdot 6.25)\hat{\mathbf{j}}$$

$$= \boxed{\left(-2.89\hat{\mathbf{i}} - 1.39\hat{\mathbf{j}}\right) \text{ cm}}$$

(b) The velocity of the center of mass is obtained by differentiating the expression for the vector position of the center of mass with respect to time:

$$\vec{\mathbf{v}}_{CM} = \frac{d\vec{\mathbf{r}}_{CM}}{dt} = (1.17 - 2.44t)\hat{\mathbf{i}} + (-2.5 + 1.56t)\hat{\mathbf{j}}$$

At $t = 2.50$ s,

$$\vec{\mathbf{v}}_{CM} = (1.17 - 2.44 \cdot 2.5)\hat{\mathbf{i}} + (-2.5 + 1.56 \cdot 2.5)\hat{\mathbf{j}}$$

$$= (-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}}) \text{ cm/s}$$

Now, the total linear momentum is the total mass times the velocity of the center of mass.

$$\vec{\mathbf{p}} = (9.00\ \text{g})(-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}})\ \text{cm/s}$$
$$= \boxed{(-44.5\hat{\mathbf{i}} + 12.5\hat{\mathbf{j}})\ \text{g}\cdot\text{cm/s}}$$

(c) As was shown in part (b), $\boxed{\left(-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}}\right)\ \text{cm/s}}$

(d) Differentiating again, $\vec{\mathbf{a}}_{CM} = \dfrac{d\vec{\mathbf{v}}_{CM}}{dt} = (-2.44)\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}$

The center of mass acceleration is $\boxed{\left(-2.44\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}\right)\ \text{cm/s}^2}$ at t = 2.50 s and at all times.

(e) The net force on the system is equal to the total mass times the acceleration of the center of mass:

$$\vec{\mathbf{F}}_{net} = (9.00\ \text{g})\left(-2.44\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}\right)\ \text{cm/s}^2 = \boxed{\left(-220\hat{\mathbf{i}} + 140\hat{\mathbf{j}}\right)\ \mu\text{N}}$$

P9.55 (a) Conservation of momentum for the two-ball system gives us:

$$(0.200\ \text{kg})(1.50\ \text{m/s}) + (0.300\ \text{kg})(-0.400\ \text{m/s})$$
$$= (0.200\ \text{kg})v_{1f} + (0.300\ \text{kg})v_{2f}$$

Relative velocity equation:

$$v_{2f} - v_{1f} = 1.90\ \text{m/s}$$

Then, suppressing units, we have

$$0.300 - 0.120 = 0.200v_{1f} + 0.300(1.90 + v_{1f})$$

$$v_{1f} = -0.780\ \text{m/s} \qquad v_{2f} = 1.12\ \text{m/s}$$

$$\boxed{\vec{\mathbf{v}}_{1f} = -0.780\hat{\mathbf{i}}\ \text{m/s}} \qquad \boxed{\vec{\mathbf{v}}_{2f} = 1.12\hat{\mathbf{i}}\ \text{m/s}}$$

(b) Before, $\vec{\mathbf{v}}_{CM} = \dfrac{(0.200\ \text{kg})(1.50\ \text{m/s})\hat{\mathbf{i}} + (0.300\ \text{kg})(-0.400\ \text{m/s})\hat{\mathbf{i}}}{0.500\ \text{kg}}$

$$\boxed{\vec{\mathbf{v}}_{CM} = (0.360\ \text{m/s})\hat{\mathbf{i}}}$$

Afterwards, the center of mass must move at the same velocity, because the momentum of the system is conserved.

Section 9.8 Deformable Systems

P9.56 (a) $\boxed{\text{Yes}}$ The only horizontal force on the vehicle is the frictional force exerted by the floor, so it gives the vehicle all of its final momentum, $(6.00\text{ kg})(3.00\hat{\mathbf{i}}\text{ m/s}) = \boxed{18.0\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}}$.

(b) $\boxed{\text{No. The friction force exerted by the floor on each stationary bit of caterpillar tread acts over no distance, so it does zero work.}}$

(c) $\boxed{\text{Yes, we could say that the final momentum of the cart came from the floor or from the Earth through the floor.}}$

(d) $\boxed{\text{No. The kinetic energy came from the original gravitational potential energy of the Earth-elevated load system, in the amount } KE = \left(\frac{1}{2}\right)(6.00\text{ kg})(3.00\text{ m/s})^2 = 27.0\text{ J.}}$

(e) $\boxed{\text{Yes. The acceleration is caused by the static friction force exerted by the floor that prevents the wheels from slipping backward.}}$

P9.57 (a) When the cart hits the bumper it immediately stops, and the hanging particle keeps moving with its original speed v_i. The particle swings up as a pendulum on a fixed pivot, keeping constant energy. Measure elevations from the pivot:

$$\frac{1}{2}mv_i^2 + mg(-L) = 0 + mg(-L\cos\theta)$$

Then $v_i = \boxed{\sqrt{2gL(1-\cos\theta)}}$

(b) The bumper continues to exert a force to the left until the particle has swung down to $\boxed{\text{its lowest point}}$. This leftward force is necessary to reverse the rightward motion of the particle and accelerate it to the left.

P9.58 (a) $\boxed{\text{Yes}}$ The floor exerts a force, larger than the person's weight over time as he is taking off.

(b) $\boxed{\text{No}}$ The work by the floor on the person is zero because the force exerted by the floor acts over zero distance.

(c) He leaves the floor with a speed given by $\frac{1}{2}mv^2 = mgy_f$, or

$$v = \sqrt{2gy_f} = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left(0.150\ \text{m}\right)} = 1.71\ \text{m/s}$$

so his momentum immediately after he leaves the floor is

$$p = mv = \left(60.0\ \text{kg}\right)\left(1.71\ \text{m/s up}\right) = \boxed{103\ \text{kg}\cdot\text{m/s up}}$$

(d) Yes. You could say that it came from the planet, that gained momentum 103 kg · m/s down, but it came through the force exerted by the floor over a time interval on the person, so it came through the floor or from the floor through direct contact.

(e) His kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(60.0\ \text{kg}\right)\left(1.71\ \text{m/s}\right)^2 = \boxed{88.2\ \text{J}}$$

(f) No. The energy came from chemical energy in the person's leg muscles. The floor did no work on the person.

P9.59 Consider the motion of the center of mass (CM) of the system of the two pucks. Because the pucks have equal mass *m*, the CM lies at the midpoint of the line connecting the pucks.

(a) The force *F* accelerates the CM to the right at the rate

$$a_{CM} = \frac{F}{2m}$$

According to Figure P9.59, when the force has moved through distance *d*, the CM has moves through distance $D_{CM} = d - \frac{1}{2}\ell$. We can find the speed of the CM, which is the same as the speed *v* of the pucks when they meet and stick together:

$$v_f^2 = v_i^2 + 2a_{CM}\left(x_f - x_i\right)$$

$$v_{CM}^2 = 0 + 2\left(\frac{F}{2m}\right)\left(d - \frac{1}{2}\ell\right) \rightarrow \qquad v = v_{CM} = \boxed{\sqrt{\frac{F\left(2d-\ell\right)}{2m}}}$$

(b) The force F does work on the system through distance d, the work done is $W = Fd$. Relate this work to the change in kinetic energy and internal energy:

$$\Delta K + \Delta E_{\text{int}} = W$$

where $\Delta K = \frac{1}{2}(2m)v_{\text{CM}}^2 = m\left[\frac{F(2d-\ell)}{2m}\right] = \frac{F(2d-\ell)}{2}$

$$\left[\frac{F(2d-\ell)}{2}\right] + \Delta E_{\text{int}} = Fd \rightarrow \Delta E_{\text{int}} = Fd - \left[\frac{F(2d-\ell)}{2}\right]$$

$$\Delta E_{\text{int}} = Fd - Fd + \frac{F\ell}{2}$$

$$\Delta E_{\text{int}} = \boxed{\frac{F\ell}{2}}$$

Section 9.9 Rocket Propulsion

P9.60 (a) The fuel burns at a rate given by

$$\frac{dM}{dt} = \frac{12.7\text{ g}}{1.90\text{ s}} = 6.68 \times 10^{-3}\text{ kg/s}$$

From the rocket thrust equation,

$$\text{Thrust} = v_e \frac{dM}{dt}:\;\; 5.26\text{ N} = v_e\left(6.68 \times 10^{-3}\text{ kg/s}\right)$$

$$v_e = \boxed{787\text{ m/s}}$$

(b) $v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$:

$$v_f - 0 = (787\text{ m/s})\ln\left(\frac{53.5\text{ g} + 25.5\text{ g}}{53.5\text{ g} + 25.5\text{ g} - 12.7\text{ g}}\right)$$

$$v_f = \boxed{138\text{ m/s}}$$

***P9.61** The force exerted on the water by the hose is

$$F = \frac{\Delta p_{\text{water}}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600\ \text{kg})(25.0\ \text{m/s}) - 0}{1.00\ \text{s}}$$
$$= \boxed{15.0\ \text{N}}$$

According to Newton's third law, the water exerts a force of equal magnitude back on the hose. Thus, the gardener must apply a 15.0-N force (in the direction of the velocity of the exiting water stream) to hold the hose stationary.

P9.62 (a) The thrust, *F*, is equal to the time rate of change of momentum as fuel is exhausted from the rocket.

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv_e)$$

Since the exhaust velocity v_e is a constant,

$$F = v_e(dm/dt), \text{ where } dm/dt = 1.50 \times 10^4\ \text{kg/s}$$

and $v_e = 2.60 \times 10^3\ \text{m/s}$.

Then $F = (2.60 \times 10^3\,\text{m/s})(1.50 \times 10^4\,\text{kg/s}) = \boxed{3.90 \times 10^7\ \text{N}}$

(b) Applying $\sum F = ma$ gives

$$\sum F_y = \text{Thrust} - Mg = Ma:$$
$$3.90 \times 10^7\ \text{N} - (3.00 \times 10^6\ \text{kg})(9.80\ \text{m/s}^2) = (3.00 \times 10^6\ \text{kg})a$$

$$a = \boxed{3.20\ \text{m/s}^2}$$

P9.63 In $v = v_e \ln \dfrac{M_i}{M_f}$ we solve for M_i.

(a) $M_i = e^{v/v_e} M_f \quad \rightarrow \quad M_i = e^5 (3.00 \times 10^3\ \text{kg}) = 4.45 \times 10^5\ \text{kg}$

The mass of fuel and oxidizer is

$$\Delta M = M_i - M_f = (445 - 3.00) \times 10^3\,\text{kg}$$
$$= \boxed{442\ \text{metric tons}}$$

(b) $\Delta M = e^2 (3.00\ \text{metric tons}) - 3.00\ \text{metric tons} = \boxed{19.2\ \text{metric tons}}$

(c) This is much less than the suggested value of 442/2.5. Mathematically, the logarithm in the rocket propulsion equation is not a linear function. Physically, a higher exhaust speed has an extra-large cumulative effect on the rocket body's final speed, by counting again and again in the speed the body attains second after second during its burn.

P9.64 (a) From the equation for rocket propulsion in the text,

$$v - 0 = v_e \ln\left(\frac{M_i}{M_f}\right) = -v_e \ln\left(\frac{M_f}{M_i}\right)$$

Now, $M_f = M_i - kt$, so $v = -v_e \ln\left(\frac{M_i - kt}{M_i}\right) = -v_e \ln\left(1 - \frac{k}{M_i}t\right)$

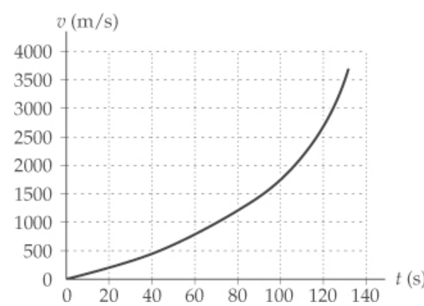
With the definition, $T_p \equiv \frac{M_i}{k}$, this becomes

$$v(t) = \boxed{-v_e \ln\left(1 - \frac{t}{T_p}\right)}$$

(b) With, $v_e = 1\,500$ m/s, and $T_p = 144$ s,

$$v = -(1\,500 \text{ m/s})\ln\left(1 - \frac{t}{144 \text{ s}}\right)$$

t (s)	*v* (m/s)
0	0
20	224
40	488
60	808
80	1 220
100	1 780
120	2 690
132	3 730



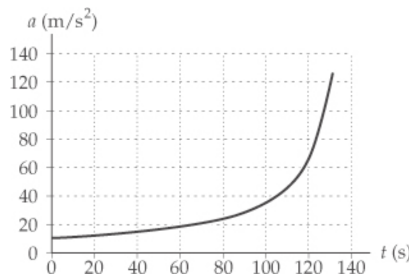
ANS. FIG. P9.64(b)

(c) $$a(t)=\frac{dv}{dt}=\frac{d\left[-v_e\ln\left(1-\frac{t}{T_p}\right)\right]}{dt}=-v_e\left(\frac{1}{1-\frac{t}{T_p}}\right)\left(-\frac{1}{T_p}\right)=\left(\frac{v_e}{T_p}\right)\left(\frac{1}{1-\frac{t}{T_p}}\right),$$

or $$a(t)=\boxed{\frac{v_e}{T_p-t}}$$

(d) With, $v_e = 1\,500$ m/s, and $T_p = 144$ s, $a=\dfrac{1\,500\text{ m/s}}{144\text{ s}-t}$.

t (s)	a (m/s^2)
0	10.4
20	12.1
40	14.4
60	17.9
80	23.4
100	34.1
120	62.5
132	125



ANS. FIG. P9.64(d)

(e) $$x(t)=0+\int_0^t v\,dt=\int_0^t\left[-v_e\ln\left(1-\frac{t}{T_p}\right)\right]dt=v_eT_p\int_0^t\ln\left[1-\frac{t}{T_p}\right]\left(-\frac{dt}{T_p}\right)$$

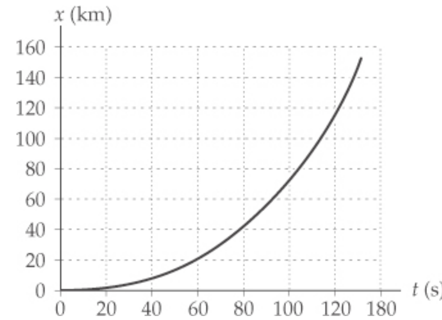
$$x(t)=v_eT_p\left[\left(1-\frac{t}{T_p}\right)\ln\left(1-\frac{t}{T_p}\right)-\left(1-\frac{t}{T_p}\right)\right]_0^t$$

$$x(t)=\boxed{v_e\left(T_p-t\right)\ln\left(1-\frac{t}{T_p}\right)+v_et}$$

(f) With, $v_e = 1.500$ m/s = 1.50 km/s, and $T_p = 144$ s,

$$x=1.50(144-t)\ln\left(1-\frac{t}{144}\right)+1.50t$$

t (s)	*x* (m)
0	0
20	2.19
40	9.23
60	22.1
80	42.2
100	71.7
120	115
132	153



ANS. FIG. P9.64(f)

Additional Problems

P9.65 (a) At the highest point, the velocity of the ball is zero, so momentum is also $\boxed{\text{zero}}$.

(b) Use $v_{yf}^2 = v_{yi}^2 + 2a(y_f - y_i)$ to find the maximum height H_{max}:

$$0 = v_i + 2(-g)H_{max}$$

or $$H_{max} = \frac{v_i^2}{2g}$$

Now, find the speed of the ball for $(y_f - y_i) = \frac{1}{2}H_{max}$:

$$v_f^2 = v_i^2 = 2(-g)\left(\frac{1}{2}H_{max}\right)$$

$$= v_i^2 - 2g\left(\frac{1}{2}\right)\left(\frac{v_i^2}{2g}\right) = v_i^2 - \frac{1}{2}v_i^2 = \frac{1}{2}v_i^2$$

which gives $v_f = \dfrac{v_i}{\sqrt{2}}$

Then, $p_f = mv_f = \boxed{\dfrac{mv_i}{\sqrt{2}}, \text{ upward}}$

P9.66 (a) The system is isolated because the skater is on frictionless ice — if it were otherwise, she would be able to move. Initially, the horizontal momentum of the system is zero, and this quantity is conserved; so when she throws the gloves in one direction, she will move in the opposite direction because the total momentum will remain zero. The system has total mass M. After the skater throws the gloves, the mass of the gloves, m, is moving with velocity $\vec{\mathbf{v}}_{\text{gloves}}$ and the mass of the skater less the gloves, $M - m$, is moving with velocity $\vec{\mathbf{v}}_{\text{girl}}$:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}$$

$$0 = (M - m)\vec{\mathbf{v}}_{\text{girl}} + m\vec{\mathbf{v}}_{\text{gloves}} \rightarrow \vec{\mathbf{v}}_{\text{girl}} = \boxed{-\left(\frac{m}{M - m}\right)\vec{\mathbf{v}}_{\text{gloves}}}$$

The term $M - m$ is the total mass less the mass of the gloves.

(b) $\boxed{\text{As she throws the gloves and exerts a force on them, the gloves exert an equal and opposite force on her (Newton's third law) that causes her to accelerate from rest to reach the velocity } \vec{\mathbf{v}}_{\text{girl}}.}$

P9.67 In $\vec{\mathbf{F}}\Delta t = \Delta(m\vec{\mathbf{v}})$, one component gives

$$\Delta p_y = m(v_{yf} - v_{yi}) = m(v\cos 60.0° - v\cos 60.0°) = 0$$

So the wall does not exert a force on the ball in the y direction. The other component gives

$$\begin{aligned}\Delta p_x &= m(v_{xf} - v_{xi}) = m(-v\sin 60.0° - v\sin 60.0°) \\ &= -2mv\sin 60.0° = -2(3.00\text{ kg})(10.0\text{ m/s})\sin 60.0° \\ &= -52.0\text{ kg}\cdot\text{m/s}\end{aligned}$$

So $$\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{\Delta p_x\hat{\mathbf{i}}}{\Delta t} = \frac{-52.0\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}}{0.200\text{ s}} = -260\hat{\mathbf{i}}\text{ N}$$

P9.68 (a) In the same symbols as in the text's Example, the original kinetic energy is

$$K_A = \frac{1}{2}m_1 v_{1A}^2$$

The example shows that the kinetic energy immediately after latching together is

$$K_B = \frac{1}{2}\left(\frac{m_1 v_{1A}^2}{m_1 + m_2}\right)$$

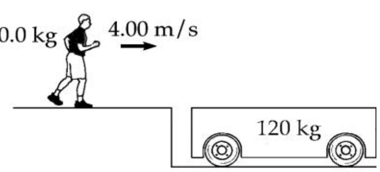
so the fraction of kinetic energy remaining as kinetic energy is

$$K_B/K_A = \boxed{m_1/(m_1+m_2)}$$

(b) Momentum is conserved in the collision so momentum after divided by momentum before is $\boxed{1.00}$.

(c) Energy is an entirely different thing from momentum. A comparison: When a photographer's single-use flashbulb flashes, a magnesium filament oxidizes. Chemical energy disappears. (Internal energy appears and light carries some energy away.) The measured mass of the flashbulb is the same before and after. It can be the same in spite of the 100% energy conversion, because energy and mass are totally different things in classical physics. In the ballistic pendulum, conversion of energy from mechanical into internal does not upset conservation of mass or conservation of momentum.

***P9.69** (a) Conservation of momentum for this totally inelastic collision gives



ANS. FIG. P9.69

$$m_p v_i = (m_p + m_c)v_f$$

$$(60.0\text{ kg})(4.00\text{ m/s}) = (120\text{ kg} + 60.0\text{ kg})v_f$$

$$\vec{\mathbf{v}}_f = \boxed{1.33\hat{\mathbf{i}}\text{ m/s}}$$

(b) To obtain the force of friction, we first consider Newton's second law in the *y* direction, $\sum F_y = 0$, which gives

$$n - (60.0\text{ kg})(9.80\text{ m/s}) = 0$$

or n = 588 N. The force of friction is then

$$f_k = \mu_k n = (0.400)(588\text{ N}) = 235\text{ N}$$

$$\vec{\mathbf{f}}_k = \boxed{-235\hat{\mathbf{i}}\text{ N}}$$

(c) The change in the person's momentum equals the impulse, or

$$p_i + I = p_f$$

$$mv_i + Ft = mv_f$$

$$(60.0\text{ kg})(4.00\text{ m/s}) - (235\text{ N})t = (60.0\text{ kg})(1.33\text{ m/s})$$

$$t = \boxed{0.680\text{ s}}$$

(d) The change in momentum of the person is

$$m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i = (60.0\text{ kg})(1.33 - 4.00)\hat{\mathbf{i}}\text{ m/s} = \boxed{-160\hat{\mathbf{i}}\text{ N}\cdot\text{s}}$$

The change in momentum of the cart is

$$(120\text{ kg})(1.33\text{ m/s}) - 0 = \boxed{+160\hat{\mathbf{i}}\text{ N}\cdot\text{s}}$$

(e) $$x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}[(4.00 + 1.33)\text{ m/s}](0.680\text{ s}) = \boxed{1.81\text{ m}}$$

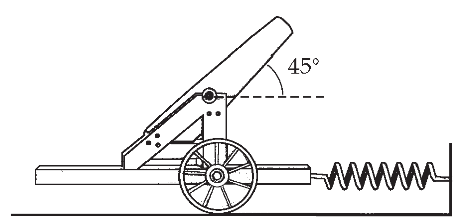
(f) $$x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(0 + 1.33\text{ m/s})(0.680\text{ s}) = \boxed{0.454\text{ m}}$$

(g) $$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(60.0\text{ kg})(1.33\text{ m/s})^2 - \frac{1}{2}(60.0\text{ kg})(4.00\text{ m/s})^2 = \boxed{-427\text{ J}}$$

(h) $$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(120\text{ kg})(1.33\text{ m/s})^2 - 0 = \boxed{107\text{ J}}$$

(i) The force exerted by the person on the cart must be equal in magnitude and opposite in direction to the force exerted by the cart on the person. The changes in momentum of the two objects must be equal in magnitude and must add to zero. Their changes in kinetic energy are different in magnitude and do not add to zero. The following represent two ways of thinking about why. The distance moved by the cart is different from the distance moved by the point of application of the friction force to the cart. The total change in mechanical energy for both objects together, –320 J, becomes +320 J of additional internal energy in this perfectly inelastic collision.

***P9.70** (a) Use conservation of the horizontal component of momentum for the system of the shell, the cannon, and the carriage, from just before to just after the cannon firing:



ANS. FIG. P9.70

$$p_{xf} = p_{xi}$$

$$m_{shell}v_{shell}\cos 45.0° + m_{cannon}v_{recoil} = 0$$

$$(200\text{ kg})(125\text{ m/s})\cos 45.0° + (5\,000\text{ kg})v_{recoil} = 0$$

or $\quad v_{recoil} = \boxed{-3.54\text{ m/s}}$

(b) Use conservation of energy for the system of the cannon, the carriage, and the spring from right after the cannon is fired to the instant when the cannon comes to rest.

$$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$$

$$0 + 0 + \frac{1}{2}kx_{max}^2 = \frac{1}{2}mv_{recoil}^2 + 0 + 0$$

$$x_{max} = \sqrt{\frac{mv_{recoil}^2}{k}} = \sqrt{\frac{(5\,000\text{ kg})(-3.54\text{ m/s})^2}{2.00\times 10^4\text{ N/m}}} = \boxed{1.77\text{ m}}$$

(c) $|F_{s,\,max}| = kx_{max}$

$$|F_{s,\,max}| = (2.00\times 10^4\text{ N/m})(1.77\text{ m}) = \boxed{3.54\times 10^4\text{ N}}$$

(d) $\boxed{\text{No.}}$ The spring exerts a force on the system during the firing. The force represents an impulse, so the momentum of the system is not conserved in the horizontal direction. Consider the vertical direction. There are two vertical forces on the system: the normal force from the ground and the gravitational force. During the firing, the normal force is larger than the gravitational force. Therefore, there is a net impulse on the system in the upward direction. The impulse accounts for the initial vertical momentum component of the projectile.

P9.71 (a) Momentum of the bullet-block system is conserved in the collision, so you can relate the speed of the block and bullet right after the collision to the initial speed of the bullet. Then, you can use conservation of mechanical energy for the bullet-block-Earth system to relate the speed after the collision to the maximum height.

(b) Momentum is conserved by the collision. Find the relation between the speed of the bullet v_i just before impact and the speed of the bullet + block v just after impact:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \rightarrow m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2i}$$

$$mv_i + M(0) = mv + Mv = (m+M)v$$

$$\rightarrow \qquad v_i = \frac{(m+M)}{m}v$$

For the bullet-block-Earth system, total energy is conserved. Find the relation between the speed of the bullet-block *v* and the height *h* the block climbs to:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}(m + M)v^2 + 0 = (m + M)gh \rightarrow v = \sqrt{2gh}$$

Combining our results, we find

$$v_i = \frac{m + M}{m}\sqrt{2gh} = \left(\frac{1.255\ \text{kg}}{0.005\ 00\ \text{kg}}\right)\sqrt{2(9.80\ \text{m/s}^2)(0.220\ \text{m})}$$

$$v_i = \boxed{521\ \text{m/s}}$$

P9.72 (a) Momentum of the bullet-block system is conserved in the collision, so you can relate the speed of the block and bullet right after the collision to the initial speed of the bullet. Then, you can use conservation of mechanical energy for the bullet-block-Earth system to relate the speed after the collision to the maximum height.

(b) Momentum is conserved by the collision. Find the relation between the speed of the bullet v_i just before impact and the speed of the bullet + block *v* just after impact:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2i}$$

$$mv_i + M(0) = mv + Mv = (m + M)v$$

$$\rightarrow \qquad v_i = \frac{(m + M)}{m}v$$

For the bullet-block-Earth system, total energy is conserved. Find the relation between the speed of the bullet-block *v* and the height *h* the block climbs to:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}(m + M)v^2 + 0 = (m + M)gh \rightarrow v = \sqrt{2gh}$$

Combining our results, we find $\boxed{v_i = \frac{m + M}{m}\sqrt{2gh}}$.

***P9.73** Momentum conservation for the system of the two objects can be written as

$$3mv_i - mv_i = mv_{1f} + 3mv_{2f}$$

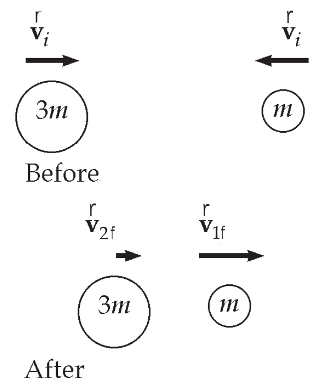
The relative velocity equation then gives

$$v_{1i} - v_{2i} = -v_{1f} + v_{2f}$$

or

$$-v_i - v_i = -v_{1f} + v_{2f}$$

$$2v_i = v_{1f} + 3v_{2f}$$



ANS. FIG. P9.73

Which gives

$$0 = 4v_{2f}$$

or $v_{1f} = \boxed{2v_i}$ and $v_{2f} = \boxed{0}$.

P9.74 (a) $\boxed{\text{The mass of the sleigh plus you is 270 kg. Your velocity is 7.50 m/s in the x direction. You unbolt a 15.0-kg seat and throw it back at the ravening wolves, giving it a speed of 8.00 m/s relative to you. Find the velocity of the sleigh afterward, and the velocity of the seat relative to the ground.}}$

(b) We substitute $v_{1f} = 8.00\text{ m/s} - v_{2f}$:

$$(270\text{ kg})(7.50\text{ m/s}) = (15.0\text{ kg})(-8.00\text{ m/s} + v_{2f}) + (255\text{ kg})v_{2f}$$

$$2\ 025\text{ kg}\cdot\text{m/s} = -120\text{ kg}\cdot\text{m/s} + (270\text{ kg})v_{2f}$$

$$v_{2f} = \frac{2\ 145\text{ m/s}}{270} = 7.94\text{ m/s}$$

$$v_{1f} = 8.00\text{ m/s} - 7.94\text{ m/s} = 0.055\ 6\text{ m/s}$$

$\boxed{\text{The final velocity of the seat is } -0.055\,6\hat{\mathbf{i}}\text{ m/s. That of the sleigh is } 7.94\hat{\mathbf{i}}\text{ m/s.}}$

(c) You transform potential energy stored in your body into kinetic energy of the system:

$$\Delta K + \Delta U_{\text{body}} = 0$$

$$\Delta U_{\text{body}} = -\Delta K = K_i - K_f$$

$$\Delta U_{\text{body}} = \frac{1}{2}(270 \text{ kg})(7.50 \text{ m/s})^2$$

$$-\left[\frac{1}{2}(15.0 \text{ kg})(0.0556 \text{ m/s})^2\right.$$

$$\left.+\frac{1}{2}(255 \text{ kg})(7.94 \text{ m/s})^2\right]$$

$$\Delta U_{\text{body}} = 7\ 594 \text{ J} - [0.023\ 1 \text{ J} + 8\ 047 \text{ J}]$$

$$\Delta U_{\text{body}} = \boxed{-453 \text{ J}}$$

P9.75 (a) When the spring is fully compressed, each cart moves with same velocity *v*. Apply conservation of momentum for the system of two gliders

$$p_i = p_f: \qquad m_1v_1 + m_2v_2 = (m_1 + m_2)v \;\rightarrow\; \boxed{v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}}$$

(b) Only conservative forces act; therefore, $\Delta E = 0$.

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}kx_m^2$$

Substitute for v from (a) and solve for x_m.

$$x_m^2 = \left(\frac{1}{k(m_1 + m_2)}\right)[(m_1 + m_2)m_1v_1^2 + (m_1 + m_2)m_2v_2^2$$

$$- (m_1v_1)^2 - (m_2v_2)^2 - 2m_1m_2v_1v_2]$$

$$x_m = \sqrt{\frac{m_1m_2(v_1^2 + v_2^2 - 2v_1v_2)}{k(m_1 + m_2)}} = \boxed{(v_1 - v_2)\sqrt{\frac{m_1m_2}{k(m_1 + m_2)}}}$$

(c) $m_1v_1 + m_2v_2 = m_1v_{1f} + m_2v_{2f}$

Conservation of momentum: $m_1(v_1 - v_{1f}) = m_2(v_{2f} - v_2)$ **[1]**

Conservation of energy: $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$

which simplifies to: $m_1(v_1^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_2^2)$

Factoring gives

$$m_1(v_1 - v_{1f})(v_1 + v_{1f}) = m_2(v_{2f} - v_2)\cdot(v_{2f} + v_2)$$

and with the use of the momentum equation (equation [1]),

this reduces to $v_1 + v_{1f} = v_{2f} + v_2$

or $v_{1f} = v_{2f} + v_2 - v_1$ **[2]**

Substituting equation [2] into equation [1] and simplifying yields

$$\boxed{v_{2f} = \frac{2m_1 v_1 + (m_2 - m_1)v_2}{m_1 + m_2}}$$

Upon substitution of this expression for into equation [2], one finds

$$\boxed{v_{1f} = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2}}$$

Observe that these results are the same as two equations given in the chapter text for the situation of a perfectly elastic collision in one dimension. Whatever the details of how the spring behaves, this collision ends up being just such a perfectly elastic collision in one dimension.

P9.76 We hope the momentum of the equipment provides enough recoil so that the astronaut can reach the ship before he loses life support! But can he do it?

Relative to the spacecraft, the astronaut has a momentum p = (150 kg)(20 m/s) = 3 000 kg · m/s away from the spacecraft. He must throw enough equipment away so that his momentum is reduced to at least zero relative to the spacecraft, so the equipment must have momentum of at least 3 000 kg · m/s relative to the spacecraft. If he throws the equipment at 5.00 m/s relative to himself in a direction away from the spacecraft, the velocity of the equipment will be 25.0 m/s away from the spacecraft. How much mass travelling at 25.0 m/s is necessary to equate to a momentum of 3 000 kg · m/s?

$$p = 3\ 000\ \text{kg} \cdot \text{m/s} = m(25.0\ \text{m/s})$$

which gives

$$m = \frac{3\ 000\ \text{kg} \cdot \text{m/s}}{25.0\ \text{m/s}} = 120\ \text{kg}$$

In order for his motion to reverse under these condition, the final mass of the astronaut and space suit is 30 kg, much less than is reasonable.

P9.77 Use conservation of mechanical energy for a block-Earth system in which the block slides down a frictionless surface from a height *h*:

$$\left(K+U_g\right)_i=\left(K+U_g\right)_f \rightarrow \frac{1}{2}mv^2+0=0+mgh \rightarrow v=\sqrt{2gh}$$

Note this also applies in reverse, a mass travelling at speed *v* will climb to a height *h* on a frictionless surface: $h=\dfrac{v^2}{2g}$.

From above, we see that because each block starts from the same height *h*, each block has the same speed *v* when it meets the other block:

$$v_1=v_2=v=\sqrt{2\left(9.80\text{ m/s}^2\right)\left(5.00\text{ m}\right)}=9.90\text{ m/s}$$

Apply conservation of momentum to the two-block system:

$$m_1v_{1f}+m_2v_{2f}=m_1v+m_2(-v)$$

$$m_1v_{1f}+m_2v_{2f}=\left(m_1-m_2\right)v \qquad \textbf{[1]}$$

For an elastic, head-on collision:

$$v_{1i}-v_{2i}=v_{1f}-v_{2f}$$

$$v-(-v)=v_{2f}-v_{1f}$$

$$v_{2f}=v_{1f}+2v \qquad \textbf{[2]}$$

Substituting equation [2] into [1] gives

$$m_1v_{1f}+m_2\left(v_{1f}+2v\right)=\left(m_1-m_2\right)v$$

$$\left(m_1+m_2\right)v_{1f}=\left(m_1-m_2\right)v-2m_2v$$

$$v_{1f}=\left(\frac{m_1-3m_2}{m_1+m_2}\right)v=\left[\frac{2.00\text{ kg}-3(4.00\text{ kg})}{2.00\text{ kg}+4.00\text{ kg}}\right](9.90\text{ m/s})$$

$$=-16.5\text{ m/s}$$

Using this result and equation [2], we have

$$v_{2f}=v_{1f}+2v=\left(\frac{m_1-3m_2}{m_1+m_2}\right)v+2v$$

$$v_{2f}=\left(\frac{3m_1-m_2}{m_1+m_2}\right)v=\left[\frac{3(2.00\text{ kg})-4.00\text{ kg}}{2.00\text{ kg}+4.00\text{ kg}}\right](9.90\text{ m/s})$$

$$=3.30\text{ m/s}$$

Using our result above, we find the height that each block rises to:

$$h_1 = \frac{v_{1f}^2}{2g} = \frac{(-16.5 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{13.9 \text{ m}}$$

and $$h_2 = \frac{v_{2f}^2}{2g} = \frac{(3.30 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{0.556 \text{ m}}$$

P9.78 (a) Proceeding step by step, we find the stone's speed just before collision, using energy conservation for the stone-Earth system:

$$m_a g y_i = \frac{1}{2} m_a v_i^2$$

which gives

$$v_i = \sqrt{2gh} = [2(9.80 \text{ m/s}^2)(1.80 \text{ m})]^{1/2} = 5.94 \text{ m/s}$$

Now for the elastic collision with the stationary cannonball, we use the specialized Equation 9.22 from the chapter text, with $m_1 = 80.0$ kg and $m_2 = m$:

$$v_{\text{cannonball}} = v_{2f} = \frac{2m_1 v_{1i}}{m_1 + m_2} = \frac{2(80.0 \text{ kg})(5.94 \text{ m/s})}{80.0 \text{ kg} + m}$$
$$= \frac{950 \text{ kg} \cdot \text{m/s}}{80.0 \text{ kg} + m}$$

The time for the cannonball's fall into the ocean is given by

$$\Delta y = v_{yi} t + \frac{1}{2} a_y t^2 \rightarrow -36.0 = \frac{1}{2}(-9.80) t^2 \rightarrow t = 2.71 \text{ s}$$

so its horizontal range is

$$R = v_{2f} t = (2.71 \text{ s})\left(\frac{950 \text{ kg} \cdot \text{m/s}}{80.0 \text{ kg} + m}\right)$$
$$= \boxed{\frac{2.58 \times 10^3 \text{ kg} \cdot \text{m}}{80.0 \text{ kg} + m}}$$

(b) The maximum value for R occurs for $m \rightarrow 0$, and is

$$R = \frac{2.58 \times 10^3 \text{ kg} \cdot \text{m}}{80.0 \text{ kg} + m} = \frac{2.58 \times 10^3 \text{ kg} \cdot \text{m}}{80.0 \text{ kg} + 0} = \boxed{32.2 \text{ m}}$$

(c) As indicated in part (b), the maximum range corresponds to $\boxed{m \rightarrow 0}$

(d) Yes, until the cannonball splashes down. No; the kinetic energy of the system is split between the stone and the cannonball after the collision and we don't know how it is split without using the conservation of momentum principle.

(e) The range is equal to the product of $v_{\text{cannonball}}$, the speed of the cannonball after the collision, and t, the time at which the cannonball reaches the ocean. But $v_{\text{cannonball}}$ is proportional to v_i, the speed of the stone just before striking the cannonball, which is, in turn, proportional to the square root of g. The time t at which the cannonball strikes the ocean is inversely proportional to the square root of g. Therefore, the product $R = (v_{\text{cannonball}})t$ is *independent* of g. At a location with weaker gravity, the stone would be moving more slowly before the collision, but the cannonball would follow the same trajectory, moving more slowly over a longer time interval.

P9.79 We will use the subscript 1 for the blue bead and the subscript 2 for the green bead. Conservation of mechanical energy for the blue bead-Earth system, $K_i + U_i = K_f + U_f$, can be written as

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh$$

where v_1 is the speed of the blue bead at point B just before it collides with the green bead. Solving for v_1 gives

$$v_1 = \sqrt{2gh} = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(1.50\text{ m}\right)} = 5.42\text{ m/s}$$

Now recall Equations 9.21 and 9.22 for an elastic collision:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$$

$$v_{2f} = \left(\frac{2m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i}$$

For this collision, the green bead is at rest, so $v_{2i} = 0$, and Equation 9.22 simplifies to

$$v_{2f} = \left(\frac{2m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i} = \left(\frac{2m_2}{m_1 + m_2}\right)v_{1i}$$

Plugging in gives

$$v_{2f} = \left(\frac{2(0.400 \text{ kg})}{0.400 \text{ kg} + 0.600 \text{ kg}}\right)(5.42 \text{ m/s}) = 4.34 \text{ m/s}$$

Now, we use conservation of the mechanical energy of the green bead after collision to find the maximum height the ball will reach. This gives

$$0 + m_2 g\, y_{\text{max}} = \frac{1}{2} m_2 v_{2f}^2 + 0$$

Solving for y_{max} gives

$$y_{\text{max}} = \frac{v_{2f}^2}{2g} = \frac{(4.34 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{0.960 \text{ m}}$$

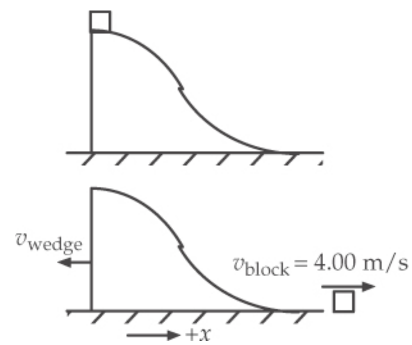
P9.80 (a) The initial momentum of the system is zero, which remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have

$$m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$$

or

$$(3.00 \text{ kg}) v_{\text{wedge}} + (0.500 \text{ kg})(+4.00 \text{ m/s}) = 0$$

so $v_{\text{wedge}} = \boxed{-0.667 \text{ m/s}}$



ANS. FIG. P9.80

(b) Using conservation of energy for the block-wedge-Earth system as the block slides down the smooth (frictionless) wedge, we have

$$\left[K_{\text{block}} + U_{\text{system}}\right]_i + \left[K_{\text{wedge}}\right]_i = \left[K_{\text{block}} + U_{\text{system}}\right]_f + \left[K_{\text{wedge}}\right]_f$$

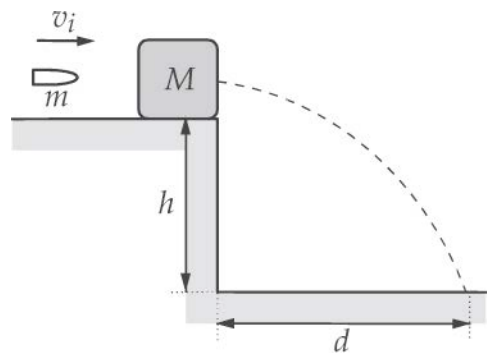
or $$\left[0 + m_1 g h\right] + 0 = \left[\frac{1}{2} m_1 (4.00 \text{ m/s})^2 + 0\right] + \frac{1}{2} m_2 (-0.667 \text{ m/s})^2$$

which gives $\boxed{h = 0.952 \text{ m}}$

***P9.81** Using conservation of momentum from just before to just after the impact of the bullet with the block:

$$mv_i = (M + m)v_f$$

or $$v_i = \left(\frac{M+m}{m}\right)v_f \qquad \textbf{[1]}$$



ANS. FIG. P9.81

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \quad \text{and} \quad h = \frac{1}{2}gt^2$$

Thus, $$t = \sqrt{\frac{2h}{g}} \quad \text{and} \quad v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting into [1] from above gives

$$v_i = \left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}} = \left(\frac{250\text{ g} + 8.00\text{ g}}{8.00\text{ g}}\right)\sqrt{\frac{(9.80\text{ m/s}^2)(2.00\text{ m})^2}{2(1.00\text{ m})}}$$

$$= \boxed{143\text{ m/s}}$$

P9.82 Refer to ANS. FIG. P9.81. Using conservation of momentum from just before to just after the impact of the bullet with the block:

$$mv_i = (M + m)v_f$$

or $$v_i = \left(\frac{M+m}{m}\right)v_f \qquad \textbf{[1]}$$

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \quad \text{and} \quad h = \frac{1}{2}gt^2$$

Thus, $$t = \sqrt{\frac{2h}{g}} \quad \text{and} \quad v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting into [1] from above gives $v_i = \boxed{\left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}}}$.

P9.83 (a) From conservation of momentum,

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2i}$$

$$(0.500\text{ kg})\left(2.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}} + 1.00\hat{\mathbf{k}}\right)\text{ m/s}$$
$$+ (1.50\text{ kg})\left(-1.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right)\text{ m/s}$$
$$= (0.500\text{ kg})\left(-1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 8.00\hat{\mathbf{k}}\right)\text{ m/s}$$
$$+ (1.50\text{ kg})\vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{2f} = \left(\frac{1}{1.50\text{ kg}}\right)\left[\left(-0.500\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}\right.$$
$$\left. + \left(0.500\hat{\mathbf{i}} - 1.50\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}\right]$$
$$= \boxed{0}$$

The original kinetic energy is

$$\frac{1}{2}(0.500\text{ kg})\left(2^2 + 3^2 + 1^2\right)\text{ m}^2/\text{s}^2$$
$$+ \frac{1}{2}(1.50\text{ kg})\left(1^2 + 2^2 + 3^2\right)\text{m}^2/\text{s}^2 = 14.0\text{ J}$$

The final kinetic energy is

$$\frac{1}{2}(0.500\text{ kg})\left(1^2 + 3^2 + 8^2\right)\text{ m}^2/\text{s}^2 + 0 = 18.5\text{ J}$$

different from the original energy so the collision is $\boxed{\text{inelastic}}$.

(b) We follow the same steps as in part (a):

$$\left(-0.500\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}$$
$$= (0.500\text{ kg})\left(-0.250\hat{\mathbf{i}} + 0.750\hat{\mathbf{j}} - 2.00\hat{\mathbf{k}}\right)\text{ m/s}$$
$$+ (1.50\text{ kg})\vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{2f} = \left(\frac{1}{1.50\text{ kg}}\right)\left(-0.5\hat{\mathbf{i}} + 1.5\hat{\mathbf{j}} - 4\hat{\mathbf{k}}\right)\text{kg}\cdot\text{m/s}$$
$$+ \left(0.125\hat{\mathbf{i}} - 0.375\hat{\mathbf{j}} + 1\hat{\mathbf{k}}\right)\text{kg}\cdot\text{m/s}$$
$$= \boxed{\left(-0.250\hat{\mathbf{i}} + 0.750\hat{\mathbf{j}} - 2.00\hat{\mathbf{k}}\right)\text{m/s}}$$

We see $\vec{\mathbf{v}}_{2f} = \vec{\mathbf{v}}_{1f}$ so the collision is $\boxed{\text{perfectly inelastic}}$.

(c) Again, from conservation of momentum,

$$\left(-0.500\hat{\mathbf{i}}+1.50\hat{\mathbf{j}}-4.00\hat{\mathbf{k}}\right)\ \text{kg}\cdot\text{m/s}$$
$$=\left(0.500\ \text{kg}\right)\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}+a\hat{\mathbf{k}}\right)\text{m/s}+\left(1.50\ \text{kg}\right)\vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{2f}=\left(\frac{1}{1.50\ \text{kg}}\right)\left(-0.500\hat{\mathbf{i}}+1.50\hat{\mathbf{j}}-4.00\hat{\mathbf{k}}\right)\ \text{kg}\cdot\text{m/s}$$
$$+\left(0.500\hat{\mathbf{i}}-1.50\hat{\mathbf{j}}-0.500a\hat{\mathbf{k}}\right)\ \text{kg}\cdot\text{m/s}$$
$$=\boxed{\left(-2.67-0.333a\right)\hat{\mathbf{k}}\ \text{m/s}}$$

Then, from conservation of energy:

$$14.0\ \text{J}=\frac{1}{2}\left(0.500\ \text{kg}\right)\left(1^2+3^2+a^2\right)\ \text{m}^2/\text{s}^2$$
$$+\frac{1}{2}\left(1.50\ \text{kg}\right)(2.67+0.333a)^2\ \text{m}^2/\text{s}^2$$
$$=2.50\ \text{J}+0.250a^2+5.33\ \text{J}+1.33a+0.0833a^2$$

This gives, suppressing units, a quadratic equation in a,

$$0=0.333a^2+1.33a-6.167=0$$

which solves to give

$$a=\frac{-1.33\pm\sqrt{1.33^2-4(0.333)(-6.167)}}{0.667}$$

With $\boxed{a=2.74}$,

$$\vec{\mathbf{v}}_{2f}=\left(-2.67-0.333(2.74)\right)\hat{\mathbf{k}}\ \text{m/s}=\boxed{-3.58\hat{\mathbf{k}}\ \text{m/s}}$$

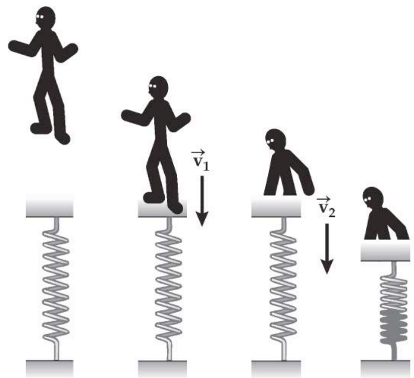
With $\boxed{a=-6.74}$,

$$\vec{\mathbf{v}}_{2f}=\left(-2.67-0.333(-6.74)\right)\hat{\mathbf{k}}\ \text{m/s}=\boxed{-0.419\hat{\mathbf{k}}\ \text{m/s}}$$

P9.84 Consider the motion of the firefighter during the three intervals: (1) before, (2) during, and (3) after collision with the platform.

(a) While falling a height of 4.00 m, her speed changes from $v_i = 0$ to v_1 as found from

$$\Delta E=\left(K_f+U_f\right)-\left(K_i-U_i\right)$$
$$K_f=\Delta E-U_f+K_i+U_i$$



ANS FIG. P9.84

When the initial position of the platform is taken as the zero level of gravitational potential, we have

$$\frac{1}{2}mv_1^2 = fh\cos(180°) - 0 + 0 + mgh$$

Solving for v_1 gives

$$\begin{aligned} v_1 &= \sqrt{\frac{2(-fh + mgh)}{m}} \\ &= \sqrt{\frac{2\left[-(300\text{ N})(4.00\text{ m}) + (75.0\text{ kg})(9.80\text{ m/s}^2)(4.00\text{ m})\right]}{75.0\text{ kg}}} \\ &= \boxed{6.81\text{ m/s}} \end{aligned}$$

(b) During the inelastic collision, momentum of the firefighter-platform system is conserved; and if v_2 is the speed of the firefighter and platform just after collision, we have $mv_1 = (m + M)v_2$, or

$$v_2 = \frac{m_1 v_1}{m + M} = \frac{(75.0\text{ kg})(6.81\text{ m/s})}{75.0\text{ kg} + 20.0\text{ kg}} = 5.38\text{ m/s}$$

Following the collision and again solving for the work done by nonconservative forces, using the distances as labeled in the figure, we have (with the zero level of gravitational potential at the initial position of the platform)

$$\Delta E = K_f + U_{fg} + U_{fs} - K_i - U_{ig} - U_{is}$$

or $$-fs = 0 + (m + M)g(-s) + \frac{1}{2}ks^2 - \frac{1}{2}(m + M)v^2 - 0 - 0$$

This results in a quadratic equation in s:

$2\,000s^2 - (931)s + 300s - 1\,375 = 0$

with solution $\boxed{s = 1.00 \text{ m}}$

P9.85 Each primate swings down according to

$$mgR = \frac{1}{2}mv_1^2 \quad \text{and} \quad MgR = \frac{1}{2}Mv_1^2 \quad \rightarrow \quad v_1 = \sqrt{2gR}$$

For the collision,

$$-mv_1 + Mv_1 = +(m + M)v_2$$

$$v_2 = \frac{M - m}{M + m}v_1$$

While the primates are swinging up,

$$\frac{1}{2}(M + m)v_2^2 = (M + m)gR(1 - \cos 35°)$$

$$v_2 = \sqrt{2gR(1 - \cos 35.0°)}$$

$$\sqrt{2gR(1 - \cos 35.0°)}(M + m) = (M - m)\sqrt{2gR}$$

$$0.425M + 0.425m = M - m$$

$$1.425m = 0.575M$$

which gives

$$\boxed{\frac{m}{M} = 0.403}$$

P9.86 (a) We can obtain the initial speed of the projectile by utilizing conservation of momentum:

$$m_1 v_{1A} + 0 = (m_1 + m_2)v_B$$

Solving for v_{1A} gives

$$v_{1A} = \frac{m_1 + m_2}{m_1}\sqrt{2gh}$$

$$v_{1A} \cong \boxed{6.29 \text{ m/s}}$$

(b) We begin with the kinematic equations in the x and y direction:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

And simplify by plugging in $x_0 = y_0 = 0$, $v_{y0} = 0$, $v_{x0} = v_{1A}$, $a_x = 0$, and $a_y = g$:

$$\frac{1}{2}gt^2 = y \text{ and } x = v_{1At}$$

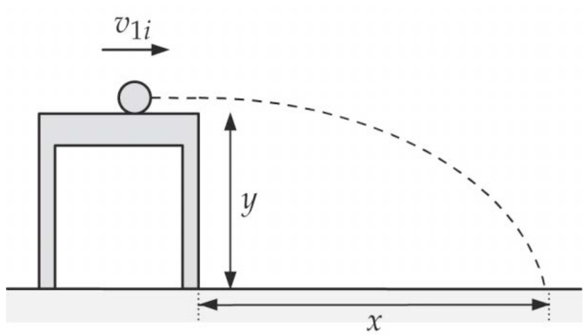
Combining them gives

$$v_{1A} = \frac{x}{\sqrt{2y/g}} = x\sqrt{\frac{g}{2y}}$$

Substituting the numerical values from the problem statement gives

$$v_{1A} = x\sqrt{\frac{g}{2y}} = (2.57\text{ m})\sqrt{\frac{9.80\text{ m/s}^2}{2(0.853\text{ m})}} = \boxed{6.16\text{ m/s}}$$

(c) Most of the 2% difference between the values for speed could be accounted for by air resistance.



ANS. FIG. P9.86

P9.87 The force exerted by the spring on each block is in magnitude.

$$|F_s| = kx = (3.85\text{ N/m})(0.08\text{ m}) = 0.308\text{ N}$$

(a) With no friction, the elastic energy in the spring becomes kinetic energy of the blocks, which have momenta of equal magnitude in opposite directions. The blocks move with constant speed after they leave the spring. From conservation of energy,

$$(K+U)_i = (K+U)_f$$

$$\frac{1}{2}kx^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$\frac{1}{2}(3.85\text{ N/m})(0.080\ 0\text{ m})^2$$
$$= \frac{1}{2}(0.250\text{ kg})v_{1f}^2 + \frac{1}{2}(0.500\text{ kg})v_{2f}^2 \qquad [1]$$

And from conservation of linear momentum,

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$0 = (0.250\text{ kg})v_{1f}(-\hat{\mathbf{i}}) + (0.500\text{ kg})v_{2f}\hat{\mathbf{i}}$$

$$v_{1f} = 2v_{2f}$$

Substituting this into [1] gives

$$0.012\ 3\text{ J} = \frac{1}{2}(0.250\text{ kg})\left(2v_{2f}\right)^2 + \frac{1}{2}(0.500\text{ kg})v_{2f}^2$$
$$= \frac{1}{2}(1.50\text{ kg})v_{2f}^2$$

Solving,

$$v_{2f} = \left(\frac{0.012\ 3\text{ J}}{0.750\text{ kg}}\right)^{1/2} = 0.128\text{ m/s} \qquad \boxed{\vec{\mathbf{v}}_{2f} = 0.128\hat{\mathbf{i}}\text{ m/s}}$$

$$v_{1f} = 2(0.128\text{ m/s}) = 0.256\text{ m/s} \qquad \boxed{\vec{\mathbf{v}}_{1f} = -0.256\hat{\mathbf{i}}\text{ m/s}}$$

(b) For the lighter block,

$$\sum F_y = ma_y, \quad n - 0.250\text{ kg}\left(9.80\text{ m/s}^2\right) = 0, \quad n = 2.45\text{ N},$$

$$f_k = \mu_k n = 0.1\left(2.45\text{ N}\right) = 0.245\text{ N}.$$

We assume that the maximum force of static friction is a similar size. Since 0.308 N is larger than 0.245 N, this block moves. For the heavier block, the normal force and the frictional force are twice as large: $f_k = 0.490$ N. Since 0.308 N is less than this, the heavier block stands still. In this case, the frictional forces exerted by the floor change the momentum of the two-block system. The lighter block will gain speed as long as the spring force is larger than the friction force: that is until the spring compression becomes x_f given by

$$\left|F_s\right| = kx,\ 0.245\text{ N} = (3.85\text{ N}\cdot\text{m})x_f,\ 0.063\ 6\text{ m} = x_f$$

Now for the energy of the lighter block as it moves to this maximum-speed point, we have

$$K_i + U_i - f_k d = K_f + U_f$$

$$0 + 0.012\,3\text{ J} - (0.245\text{ N})(0.08 - 0.063\,6\text{ m})$$
$$= \frac{1}{2}(0.250\text{ kg})v_f^2 + \frac{1}{2}(3.85\text{ N/m})(0.063\,6\text{ m})^2$$

$$0.012\,3\text{ J} - 0.004\,01\text{ J} = \frac{1}{2}(0.250\text{ kg})v_f^2 + 0.007\,80\text{ J}$$

$$\left(\frac{2(0.000\,515\text{ J})}{0.250\text{ kg}}\right)^{1/2} = v_f = 0.064\,2\text{ m/s}$$

Thus for the heavier block the maximum velocity is $\boxed{0}$ and for the lighter block, $\boxed{-0.064\,2\hat{\mathbf{i}}\text{ m/s}}$.

(c) For the lighter block, $f_k = 0.462(2.45\text{ N}) = 1.13\text{ N}$. The force of static friction must be at least as large. The 0.308-N spring force is too small to produce motion of either block. Each has $\boxed{0}$ maximum speed.

P9.88 The orbital speed of the Earth is

$$v_E = \frac{2\pi r}{T} = \frac{2\pi(1.496\times10^{11}\text{ m})}{3.156\times10^{7}\text{ s}} = 2.98\times10^{4}\text{ m/s}$$

In six months the Earth reverses its direction, to undergo momentum change

$$m_E|\Delta\vec{\mathbf{v}}_E| = 2m_Ev_E = 2(5.98\times10^{24}\text{ kg})(2.98\times10^{4}\text{ m/s})$$
$$= 3.56\times10^{29}\text{ kg}\cdot\text{m/s}$$

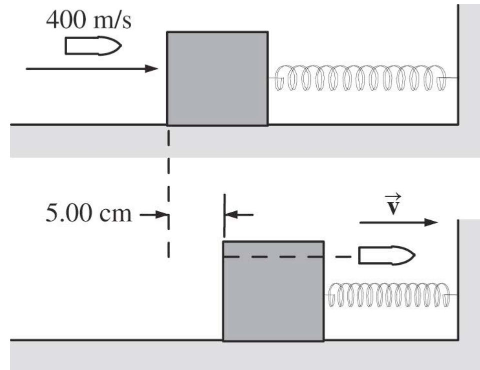
Relative to the center of mass, the Sun always has momentum of the same magnitude in the opposite direction. Its 6-month momentum change is the same size, $m_S|\Delta\vec{\mathbf{v}}_S| = 3.56\times10^{29}\text{ kg}\cdot\text{m/s}$

Then $$|\Delta\vec{\mathbf{v}}_S| = \frac{3.56\times10^{29}\text{ kg}\cdot\text{m/s}}{1.991\times10^{30}\text{ kg}} = \boxed{0.179\text{ m/s}}$$

ANS. FIG. P9.88

P9.89 (a) We find the speed when the bullet emerges from the block by using momentum conservation:

$$mv_i = MV_i + mv$$



ANS. FIG. P9.89

The block moves a distance of 5.00 cm. Assume for an approximation that the block quickly reaches its maximum velocity, V_i, and the bullet kept going with a constant velocity, v. The block then compresses the spring and stops. After the collision, the mechanical energy is conserved in the block-spring system:

$$\frac{1}{2}MV_i^2 = \frac{1}{2}kx^2$$

$$V_i = \sqrt{\frac{(900\text{ N/m})(5.00\times10^{-2}\text{ m})^2}{1.00\text{ kg}}} = 1.50\text{ m/s}$$

$$v = \frac{mv_i - MV_i}{m}$$

$$= \frac{(5.00\times10^{-3}\text{ kg})(400\text{ m/s}) - (1.00\text{ kg})(1.50\text{ m/s})}{5.00\times10^{-3}\text{ kg}}$$

$$v = \boxed{100\text{ m/s}}$$

(b) Identifying the system as the block and the bullet and the time interval from just before the collision to just after the collision,

$$\Delta K + \Delta E_{int} = 0 \quad \text{gives}$$

$$\Delta E_{int} = -\Delta K = -\left(\frac{1}{2}mv^2 + \frac{1}{2}MV_i^2 - \frac{1}{2}mv_i^2\right)$$

Then

$$\Delta E_{int} = -\left[\frac{1}{2}(0.005\ 00\text{ kg})(100\text{ m/s})^2 + \frac{1}{2}(1.00\text{ kg})(1.50\text{ m/s})^2 - \frac{1}{2}(0.005\ 00\text{ kg})(400\text{ m/s})^2\right]$$

$$= \boxed{374\text{ J}}$$

P9.90 (a) We have, from the impulse-momentum theorem, $\vec{\mathbf{p}}_i + \vec{\mathbf{F}}t = \vec{\mathbf{p}}_f$:

$$(3.00\text{ kg})(7.00\text{ m/s})\hat{\mathbf{j}} + \left(12.0\hat{\mathbf{i}}\text{ N}\right)(5.00\text{ s}) = (3.00\text{ kg})\vec{\mathbf{v}}_f$$

$$\vec{\mathbf{v}}_f = \boxed{\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) The particle's acceleration is

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t} = \frac{\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}} - 7.00\hat{\mathbf{j}}\right)\text{ m/s}}{5.00\text{ s}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$$

(c) From Newton's second law,

$$\vec{\mathbf{a}} = \frac{\sum\vec{\mathbf{F}}}{m} = \frac{12.0\hat{\mathbf{i}}\text{ N}}{3.00\text{ kg}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$$

(d) The vector displacement of the particle is

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$= \left(7.00\text{ m/s}\,\hat{\mathbf{j}}\right)(5.00\text{ s}) + \frac{1}{2}\left(4.00\text{ m/s}^2\hat{\mathbf{i}}\right)(5.00\text{ s})^2$$

$$\Delta\vec{\mathbf{r}} = \boxed{\left(50.0\hat{\mathbf{i}} + 35.0\hat{\mathbf{j}}\right)\text{ m}}$$

(e) Now, from the work-kinetic energy theorem, the work done on the particle is

$$W = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}} = \left(12.0\hat{\mathbf{i}}\text{ N}\right)\left(50.0\hat{\mathbf{i}}\text{ m} + 35.0\hat{\mathbf{j}}\text{ m}\right) = \boxed{600\text{ J}}$$

(f) The final kinetic energy of the particle is

$$\frac{1}{2}mv_f^2 = \frac{1}{2}(3.00\text{ kg})\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\cdot\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\text{ m}^2/\text{s}^2$$

$$\frac{1}{2}mv_f^2 = (1.50\text{ kg})\left(449\text{ m}^2/\text{s}^2\right) = \boxed{674\text{ J}}$$

(g) The final kinetic energy of the particle is

$$\frac{1}{2}mv_i^2 + W = \frac{1}{2}(3.00\text{ kg})(7.00\text{ m/s})^2 + 600\text{ J} = \boxed{674\text{ J}}$$

(h) The accelerations computed in different ways agree. The kinetic energies computed in different ways agree. The three theories are consistent.

P9.91 We note that the initial velocity of the target particle is zero (that is, $v_{2i} = 0$). Then, from conservation of momentum,

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + 0 \qquad \textbf{[1]}$$

For head-on elastic collisions, $v_{1i} - v_{2i} = (v_{1f} - v_{2f})$, and with $v_{2i} = 0$, this gives

$$v_{2f} = v_{1i} + v_{1f} \qquad \textbf{[2]}$$

Substituting equation [2] into [1] yields

$$m_1 v_{1f} + m_2 (v_{1i} + v_{1f}) = m_1 v_{1i}$$

or

$$(m_1 + m_2) v_{1f} = (m_1 - m_2) v_{1i}$$

which gives

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} \qquad \textbf{[3]}$$

Now, we substitute equation [3] into [2] to obtain

$$v_{2f} = v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} \qquad \textbf{[4]}$$

Equations [3] and [4] can now be used to answer both parts (a) and (b).

(a) If $m_1 = 2.00\text{ g}, m_2 = 1.00\text{ g}$, and $v_{1i} = 8.00\text{ m/s}$, then

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} = \left(\frac{2.00\text{ g} - 1.00\text{ g}}{2.00\text{ g} + 1.00\text{ g}} \right) (8.00\text{ m/s}) = \boxed{2.67\text{ m/s}}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} = \left[\frac{2(2.00\text{ g})}{2.00\text{ g} + 1.00\text{ g}} \right] (8.00\text{ m/s}) = \boxed{10.7\text{ m/s}}$$

(b) If $m_1 = 2.00\text{ g}, m_2 = 10.0\text{ g}$, and $v_{1i} = 8.00\text{ m/s}$, we find

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} = \left(\frac{2.00\text{ g} - 10.0\text{ g}}{2.00\text{ g} + 10.0\text{ g}} \right) (8.00\text{ m/s}) = \boxed{5.33\text{ m/s}}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} = \left[\frac{2(2.00\text{ g})}{2.00\text{ g} + 10.0\text{ g}} \right] (8.00\text{ m/s}) = \boxed{2.67\text{ m/s}}$$

(c) The final kinetic energy of the 2.00-g particle in each case is:

Case (a):

$$KE_{1f} = \frac{1}{2}m_1 v_{1f}^2 = \frac{1}{2}\left(2.00\times10^{-3}\text{ kg}\right)\left(2.67\text{ m/s}\right)^2 = \boxed{7.11\times10^{-3}\text{ J}}$$

Case (b):

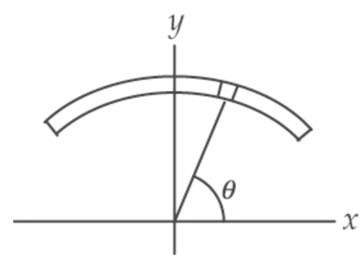
$$KE_{1f} = \frac{1}{2}m_1 v_{1f}^2 = \frac{1}{2}\left(2.00\times10^{-3}\text{ kg}\right)\left(5.33\text{ m/s}\right)^2 = \boxed{2.84\times10^{-2}\text{ J}}$$

Since the incident kinetic energy is the same in cases (a) and (b), we observe that

the incident particle loses more kinetic energy in case (a), in which the target mass is 1.00 g.

Challenge Problems

P9.92 Take the origin at the center of curvature. We have $L = \frac{1}{4}2\pi r$, $r = \frac{2L}{\pi}$. An incremental bit of the rod at angle θ from the x axis has mass given by $\frac{dm}{r d\theta} = \frac{m}{L}$, $dm = \frac{mr}{L}d\theta$, where we have used the definition of radian measure. Now



ANS. FIG. P9.92

$$y_{CM} = \frac{1}{M}\int_{\text{all mass}} y\,dm = \frac{1}{M}\int_{\theta=45°}^{135°} r\sin\theta\frac{Mr}{L}d\theta = \frac{r^2}{L}\int_{45°}^{135°}\sin\theta\,d\theta$$

$$= \left(\frac{2L}{\pi}\right)^2\frac{1}{L}(-\cos\theta)\Big|_{45°}^{135°} = \frac{4L}{\pi^2}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right) = \frac{4\sqrt{2}L}{\pi^2}$$

The top of the bar is above the origin by $r = \frac{2L}{\pi}$, so the center of mass is below the middle of the bar by

$$\frac{2L}{\pi} - \frac{4\sqrt{2}L}{\pi^2} = \frac{2}{\pi}\left(1-\frac{2\sqrt{2}}{\pi}\right)L = \boxed{0.063\ 5\ L}$$

P9.93 The x component of momentum for the system of the two objects is

$$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$$

$$-mv_i + 3mv_i = 0 + 3mv_{2x}$$

The y component of momentum of the system is

$$0 + 0 = -mv_{1y} + 3mv_{2y}$$

By conservation of energy of the system,

$$+\frac{1}{2}mv_i^2 + \frac{1}{2}3mv_i^2 = \frac{1}{2}mv_{1y}^2 + \frac{1}{2}3m\left(v_{2x}^2 + v_{2y}^2\right)$$

we have $v_{2x} = \dfrac{2v_i}{3}$

also $v_{1y} = 3v_{2y}$

So the energy equation becomes

$$4v_i^2 = 9v_{2y}^2 + \frac{4v_i^2}{3} + 3v_{2y}^2$$

$$\frac{8v_i^2}{3} = 12v_{2y}^2$$

or $v_{2y} = \dfrac{\sqrt{2}v_i}{3}$

(a) The object of mass m has final speed

$$v_{1y} = 3v_{2y} = \boxed{\sqrt{2}v_i}$$

and the object of mass $3m$ moves at

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\frac{4v_i^2}{9} + \frac{2v_i^2}{9}}$$

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \boxed{\sqrt{\frac{2}{3}}v_i}$$

(b) $\theta = \tan^{-1}\left(\dfrac{v_{2y}}{v_{2x}}\right)$

$$\theta = \tan^{-1}\left(\frac{\sqrt{2}v_i}{3}\frac{3}{2v_i}\right) = \boxed{35.3°}$$

P9.94 A picture one second later differs by showing five extra kilograms of sand moving on the belt.

(a) $\frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} = (0.750 \text{ m/s})(5.00 \text{ kg/s}) = \boxed{3.75 \text{ N}}$

(b) The only horizontal force on the sand is belt friction, which causes the momentum of the sand to change: $F = \frac{dp}{dt} = \boxed{3.75 \text{ N}}$ as above.

(c) The belt is in equilibrium:

$$\sum F_x = ma_x: \quad +F_{ext} - f = 0 \quad \text{and} \quad F_{ext} = \boxed{3.75 \text{ N}}$$

(d) $W = F\Delta r\cos\theta = (3.75 \text{ N})(0.750 \text{ m})\cos 0° = \boxed{2.81 \text{ J}}$

(e) $\frac{dK}{dt} = \frac{d\left(\frac{1}{2}mv^2\right)}{dt} = \frac{1}{2}v^2\frac{dm}{dt} = \frac{1}{2}(0.750 \text{ m/s})^2(5.00 \text{ kg/s}) = \boxed{1.41 \text{ J/s}}$

(f) One-half of the work input becomes kinetic energy of the moving sand and the other half becomes additional internal energy. The internal energy appears when the sand does not elastically bounce under the hopper, but has friction eliminate its horizontal motion relative to the belt. By contrast, all of the impulse input becomes momentum of the moving sand.

P9.95 Depending on the length of the cord and the time interval Δt for which the force is applied, the sphere may have moved very little when the force is removed, or we may have x_1 and x_2 nearly equal, or the sphere may have swung back, or it may have swung back and forth several times. Our solution applies equally to all of these cases.

(a) The applied force is constant, so the center of mass of the glider-sphere system moves with constant acceleration. It starts, we define, from $x = 0$ and moves to $(x_1 + x_2)/2$. Let v_1 and v_2 represent the horizontal components of velocity of glider and sphere at the moment the force stops. Then the velocity of the center of mass is $v_{CM} = (v_1 + v_2)/2$, and because the acceleration is constant we have

$$\frac{x_1 + x_2}{2} = \left(\frac{v_1 + v_2}{2}\right)\left(\frac{\Delta t}{2}\right)$$

which gives

$$\Delta t = 2\left(\frac{x_1 + x_2}{v_1 + v_2}\right)$$

The impulse-momentum theorem for the glider-sphere system is

$$F\Delta t = mv_1 + mv_2$$

or

$$2F\left(\frac{x_1 + x_2}{v_1 + v_2}\right) = m\left(v_1 + v_2\right)$$

$$2F\left(x_1 + x_2\right) = m\left(v_1 + v_2\right)^2$$

Dividing both sides by 4m and rearranging gives

$$\frac{2F\left(x_1 + x_2\right)}{4m} = \frac{m\left(v_1 + v_2\right)^2}{4m}$$

$$\frac{F\left(x_1 + x_2\right)}{2m} = \frac{\left(v_1 + v_2\right)^2}{4} = v_{CM}^2$$

or

$$v_{CM} = \boxed{\sqrt{\frac{F\left(x_1 + x_2\right)}{2m}}}$$

(b) The applied force does work that becomes, after the force is removed, kinetic energy of the constant-velocity center-of-mass motion plus kinetic energy of the vibration of the glider and sphere relative to their center of mass. The applied force acts only on the glider, so the work-energy theorem for the pushing process is

$$Fx_1 = \frac{1}{2}(2m)v_{CM}^2 + E_{vib}$$

Substitution gives

$$Fx_1 = \frac{1}{2}(2m)\left[\frac{F\left(x_1 + x_2\right)}{2m}\right] + E_{vib} = \frac{1}{2}Fx_1 + \frac{1}{2}Fx_2 + E_{vib}$$

Then,

$$E_{vib} = \frac{1}{2}Fx_1 - \frac{1}{2}Fx_2$$

When the cord makes its largest angle with the vertical, the vibrational motion is turning around. No kinetic energy is associated with the vibration at this moment, but only gravitational energy:

$$mgL\left(1 - \cos\theta\right) = F\left(x_1 - x_2\right)/2$$

Solving gives

$$\boxed{\theta = \cos^{-1}[1 - F(x_1 - x_2)/2mgL]}$$

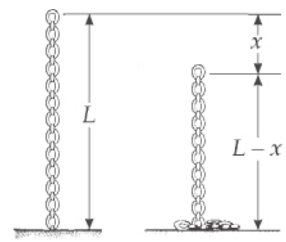
P9.96 The force exerted by the table is equal to the change in momentum of each of the links in the chain. By the calculus chain rule of derivatives,

$$F_1 = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}$$

We choose to account for the change in momentum of each link by having it pass from our area of interest just before it hits the table, so that

$$v\frac{dm}{dt} \neq 0 \text{ and } m\frac{dv}{dt} = 0$$

Since the mass per unit length is uniform, we can express each link of length dx as having a mass dm:



ANS. FIG. P9.96

$$dm = \frac{M}{L}dx$$

The magnitude of the force on the falling chain is the force that will be necessary to stop each of the elements dm.

$$F_1 = v\frac{dm}{dt} = v\left(\frac{M}{L}\right)\frac{dx}{dt} = \left(\frac{M}{L}\right)v^2$$

After falling a distance x, the square of the velocity of each link $v^2 = 2gx$ (from kinematics), hence

$$F_1 = \frac{2Mgx}{L}$$

The links already on the table have a total length x, and their weight is supported by a force F_2:

$$F_2 = \frac{Mgx}{L}$$

Hence, the *total* force on the chain is

$$F_{\text{total}} = F_1 + F_2 = \boxed{\frac{3Mgx}{L}}$$

That is, *the total force is three times the weight of the chain on the table at that instant.*

ANSWERS TO EVEN-NUMBERED PROBLEMS

P9.2 1.14 kg; 22.0 m/s

P9.4 (a) $p_x = 9.00\ \text{kg}\cdot\text{m/s}$, $p_y = -12.0\ \text{kg}\cdot\text{m/s}$; (b) 15.0 kg · m/s

P9.6 (a) $v_{pi} = -0.346\ \text{m/s}$; (b) $v_{gi} = 1.15\ \text{m/s}$

P9.8 (a) 4.71 m/s East; (b) 717 J

P9.10 10^{-23} m/s

P9.12 (a) 3.22×10^3 N, 720 lb; (b) not valid; (c) These devices are essential for the safety of small children.

P9.14 (a) $\Delta\vec{\mathbf{p}} = 3.38\ \text{kg}\cdot\text{m/s}\ \hat{\mathbf{j}}$; (b) $\vec{\mathbf{F}} = 7 \times 10^2\ \text{N}\hat{\mathbf{j}}$

P9.16 (a) $\left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right)\ \text{N}\cdot\text{s}$; (b) $\left(377\hat{\mathbf{i}} + 255\hat{\mathbf{j}}\right)\ \text{N}$

P9.18 (a) $3.60\hat{\mathbf{i}}\ \text{N}\cdot\text{s}$ away from the racket; (b) −36.0 **J**

P9.20 (a) $981\ \text{N}\cdot\text{s}$, up; (b) 3.43 m/s, down; (c) 3.83 m/s, up; (d) 0.748 m

P9.22 (a) 20.9 m/s East; (b) −8.68 × 103 J; (c) Most of the energy was transformed to internal energy with some being carried away by sound.

P9.24 (a) $v_f = \frac{1}{3}\left(v_1 + 2v_2\right)$; (b) $\Delta K = -\frac{m}{3}\left(v_1^2 + v_2^2 - 2v_1v_2\right)$

P9.26 (a) 2.50 m/s; (b) 37.5 kJ; (c) The event considered in this problem is the time reversal of the perfectly inelastic collision in Problem 9.25. The same momentum conservation equation describes both processes.

P9.28 7.94 cm

P9.30 $v = \frac{4M}{m}\sqrt{g\ell}$

P9.32 $v_c = \frac{\left(m + M\right)}{m}\sqrt{2\mu g d}$

P9.34 (a) 2.24 m/s toward the right; (b) No. Coupling order makes no difference to the final velocity.

P9.36 The driver of the northbound car was untruthful. His original speed was more than 35 mi/h.

P9.38 $v_O = 3.99\ \text{m/s}$ and $v_Y = 3.01\ \text{m/s}$

P9.40 $v = \dfrac{v_i}{\sqrt{2}}$, 45.0°, –45.0°

P9.42 The opponent grabs the fullback and does not let go, so the two players move together at the end of their interaction; (b) $\theta = 32.3°$, 2.88 m/s; (c) 786 J into internal energy

P9.44 $v_B = 5.89$ m/s; $v_G = 7.07$ m/s

P9.46 4.67×10^6 m from the Earth's center

P9.48 11.7 cm; 13.3 cm

P9.50 The center of mass of the molecule lies on the dotted line shown in ANS. FIG. P9.50, 0.006 73 nm below the center of the O atom.

P9.52 (a) See ANS. FIG. P8.42; (b) $\left(-2.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)$ m; (c) $\left(3.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)$ m/s; (d) $\left(15.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)$ kg · m/s

P9.54 (a) $\left(-2.89\hat{\mathbf{i}} - 1.39\hat{\mathbf{j}}\right)$cm; (b) $\left(-44.5\hat{\mathbf{i}} + 12.5\hat{\mathbf{j}}\right)$g · cm/s; (c) $\left(-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}}\right)$cm/s; (d) $\left(-2.44\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}\right)$cm/s^2; (e) $\left(-220\hat{\mathbf{i}} + 140\hat{\mathbf{j}}\right)\mu$N

P9.56 (a) Yes. $18.0\hat{\mathbf{i}}$ kg · m/s; (b) No. The friction force exerted by the floor on each stationary bit of caterpillar tread acts over no distance, so it does zero work; (c) Yes, we could say that the final momentum of the card came from the floor or from the Earth through the floor; (d) No. The kinetic energy came from the original gravitational potential energy of the Earth-elevated load system, in the amount 27.0 J; (e) Yes. The acceleration is caused by the static friction force exerted by the floor that prevents the wheels from slipping backward.

P9.58 (a) yes; (b) no; (c) 103 kg·m/s, up; (d) yes; (e) 88.2 J; (f) no, the energy came from chemical energy in the person's leg muscles

P9.60 (a) 787 m/s; (b) 138 m/s

P9.62 (a) 3.90×10^7 N; (b) 3.20 m/s^2

P9.64 (a) $-v_e \ln\left(1 - \dfrac{t}{T_p}\right)$; (b) See ANS. FIG. P9.64(b); (c) $\dfrac{v_e}{T_p - t}$; (d) See ANS. FIG. P9.64(d); (e) $v_e\left(T_p - t\right)\ln\left(1 - \dfrac{t}{T_p}\right) + v_e t$; (f) See ANS. FIG. P9.64(f)

P9.66 (a) $-\left(\frac{m}{M-m}\right)\vec{\mathbf{v}}_{\text{gloves}}$; (b) As she throws the gloves and exerts a force on them, the gloves exert an equal and opposite force on her that causes her to accelerate from rest to reach the velocity $\vec{\mathbf{v}}_{\text{girl}}$.

P9.68 (a) $K_E/K_A = m_1/(m_1+m_2)$; (b) 1.00; (c) See P9.68(c) for argument.

P9.70 (a) –3.54 m/s; (b) 1.77 m; (c) 3.54×10^4 N; (d) No

P9.72 (a) See P9.72(a) for description; (b) $v_i = \frac{m+M}{m}\sqrt{2gh}$

P9.74 (a) See P9.74 for complete statement; (b) The final velocity of the seat is $-0.055\,6\hat{\mathbf{i}}$ m/s. That of the sleigh is $7.94\hat{\mathbf{i}}$ m/s; (c) −453 J

P9.76 In order for his motion to reverse under these conditions, the final mass of the astronaut and space suit is 30 kg, much less than is reasonable.

P9.78 (a) $2.58\times10^3\ \text{kg}\cdot\text{m}/(80\ \text{kg}+m)$; (b) 32.2 m; (c) $m \to 0$; (d) See P9.78(d) for complete answer; (e) See P9.78(e) for complete answer.

P9.80 (a) –0.667 m/s; (b) h = 0.952 m

P9.82 $\left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}}$

P9.84 (a) 6.81 m/s; (b) s = 1.00 m

P9.86 (a) 6.29 m/s; (b) 6.16 m/s; (c) Most of the 2% difference between the values for speed could be accounted for by air resistance.

P9.88 0.179 m/s

P9.90 (a) $(20.0\hat{\mathbf{i}}+7.00\hat{\mathbf{j}})$ m/s; (b) $4.00\hat{\mathbf{i}}$ m/s^2; (c) $4.00\hat{\mathbf{i}}$ m/s^2;

(d) $(50.0\hat{\mathbf{i}}+35.0\hat{\mathbf{j}})$ m; (e) 600 J; (f) 674 J; (g) 674 J; (h) The accelerations computed in different ways agree. The kinetic energies computed in different ways agree. The three theories are consistent.

P9.92 0.063 5L

P9.94 (a) 3.75 N; (b) 3.75 N; (c) 3.75 N; (d) 2.81 J; (e) 1.41 J/s; (f) One-half of the work input becomes kinetic energy of the moving sand and the other half becomes additional internal energy. The internal energy appears when the sand does not elastically bounce under the hopper, but has friction eliminate its horizontal motion relative to the belt. By contrast, all of the impulse input becomes momentum of the moving sand.

P9.96 $\frac{3Mgx}{L}$

10

Rotation of a Rigid Object About a Fixed Axis

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ10.1 Answer (c). The wheel has a radius of 0.500 m and made 320 revolutions. The distance traveled is

$$s = r\theta = (0.500\text{ m})(320\text{ rev})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 1.00\times10^{3}\text{ m} = 1.00\text{ km}$$

OQ10.2 Answer (b). Any object moving in a circular path undergoes a constant change in the direction of its velocity. This change in the direction of velocity is an acceleration, always directed toward the center of the path, called the centripetal acceleration, $a_c = v^2/r = r\omega^2$. The tangential speed of the object is $v_t = r\omega$, where ω is the angular velocity. If ω is not constant, the object will have both an angular

acceleration, $\alpha_{\text{avg}} = \Delta\omega / \Delta t$, and a tangential acceleration, $a_t = r\alpha$. The only untrue statement among the listed choices is (b). Even when ω is constant, the object still has centripetal acceleration.

OQ10.3 Answer: b = e > a = d > c = 0. The tangential acceleration has magnitude $(3/\text{s}^2)r$, where r is the radius. It is constant in time. The radial acceleration has magnitude $\omega^2 r$, so it is $(4/\text{s}^2)r$ at the first and last moments mentioned and it is zero at the moment the wheel reverses.

OQ10.4 Answer (d). The angular displacement will be

$$\Delta\theta = \omega_{\text{avg}}\Delta t = \left(\frac{\omega_f + \omega_i}{2}\right)\Delta t$$
$$= \left(\frac{12.00 \text{ rad/s} + 4.00 \text{ rad/s}}{2}\right)(4.00 \text{ s}) = 32.0 \text{ rad}$$

OQ10.5 **(i)** Answer (d). The speedometer measures the number of revolutions per second of the tires. A larger tire will rotate fewer times to cover the same distance. The speedometer reading is assumed proportional to the rotation rate of the tires, $\omega = v/R$, for a standard tire radius R, but the actual reading is $\omega = v/(1.3)R$, or 1.3 times smaller. Example: When the car travels at 13 km/h, the speedometer reads 10 km/h.

(ii) Answer (d). If the driver uses the odometer reading to calculate fuel economy, this reading is a factor of 1.3 too small because the odometer assumes 1 rev = $2\pi R$ for a standard tire radius R, whereas the actual distance traveled is $1.3(2\pi R)$, so the fuel economy in miles per gallon will appear to be lower by a factor of 1.3. Example: If the car travels 13 km, the odometer will read 10 km. If the car actually makes 13 km/gal, the calculation will give 10 km/gal.

OQ10.6 **(i)** Answer (a). Smallest I is about the x axis, along which the larger-mass balls lie.

(ii) Answer (c). The balls all lie at a distance from the z axis, which is perpendicular to both the x and y axes and passes through the origin.

OQ10.7 Answer (a). The accelerations are not equal, but greater in case (a). The string tension above the 50-N object is less than its weight while the object is accelerating downward because it does not fall with the acceleration of gravity.

OQ10.8 Answers (a), (b), (e). The object must rotate with a nonzero and constant angular acceleration. Its moment of inertia would not

change unless there were a rearrangement of mass within the object.

OQ10.9 (**i**) Answer (a). The basketball has rotational as well as translational kinetic energy.

(**ii**) Answer (c). The motions of their centers of mass are identical.

(**iii**) Answer (a). The basketball-Earth system has more kinetic energy than the ice-Earth system due to the rotational kinetic energy of the basketball. Therefore, when the kinetic energy of both systems has transformed to gravitational potential energy when the objects momentarily come to rest at their highest point on the ramp, the basketball will be at a higher location, corresponding to the larger gravitational potential energy.

OQ10.10 **(i)** Answer (c). The airplane momentarily has zero torque acting on it. It was speeding up in its angular rotation before this instant of time and begins slowing down just after this instant.

(**ii**) Answer (b). Although the angular speed is zero at this instant, there is still an angular acceleration because the wound-up string applies a torque to the airplane. This is similar to a ball thrown upward, which we studied earlier: at the top of its flight, it momentarily comes to rest, but is still accelerating because the gravitational force is acting on it.

OQ10.11 Answer (e). The sphere of twice the radius has eight times the volume and eight times the mass, and the r^2 term in $I = \frac{2}{5}mr^2$ also becomes four times larger.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ10.1 Yes. For any object on which a net force acts but no net torque, the translational kinetic energy will change but the rotational kinetic energy will not. For example, if you drop an object, it will gain translational kinetic energy due to work done on the object by the gravitational force. Any rotational kinetic energy the object has is unaffected by dropping it.

CQ10.2 No, just as an object need not be moving to have mass.

CQ10.3 If the object is free to rotate about any axis, the object will start to rotate if the two forces act along <u>different</u> lines of action. Then the torques of the forces will not be equal in magnitude and opposite in direction.

CQ10.4 Attach an object, of known mass m, to the cord. You could measure

the time that it takes the object to fall a measured distance after being released from rest. Using this information, the linear acceleration of the mass can be calculated, and then the torque on the rotating object and its angular acceleration. It is assumed the mass of the cord has negligible effect on the motion as the cord unwinds.

CQ10.5 We have from Example 10.6 the means to calculate a and α. You could use $\omega = \alpha t$ and $v = at$.

CQ10.6 The moment of inertia depends on the distribution of mass with respect to a given axis. If the axis is changed, then each bit of mass that makes up the object is at a different distance from the axis than before. Compare the moments of inertia of a uniform rigid rod about axes perpendicular to the rod, first passing through its center of mass, then passing through an end. For example, if you wiggle repeatedly a meterstick back and forth about an axis passing through its center of mass, you will find it does not take much effort to reverse the direction of rotation. However, if you move the axis to an end, you will find it more difficult to wiggle the stick back and forth. The moment of inertia about the end is much larger, because much of the mass of the stick is farther from the axis.

CQ10.11 No, only if its angular velocity changes.

CQ10.12 Adding a small sphere of mass m to the end will increase the moment of inertia of the system from $(1/3)ML^2$ to $(1/3)ML^2 + mL^2$, and the initial potential energy would be $(1/2)MgL + mgL$. Following Example 10.11, the final angular speed ω would be

$$\omega = \sqrt{\frac{3g}{L}}\sqrt{\frac{M+2m}{M+3m}}$$

If $m = M$, $\omega = \sqrt{\frac{3g}{L}}\sqrt{\frac{M+2m}{M+3m}} = \sqrt{\frac{3g}{L}}\sqrt{\frac{3M}{4M}} = \sqrt{\frac{9g}{4L}}$

Therefore, ω would increase.

CQ10.13 (a) The sphere would reach the bottom first. (b) The hollow cylinder would reach the bottom last. First imagine that each object has the same mass and the same radius. Then they all have the same torque due to gravity acting on them. The one with the smallest moment of inertia will thus have the largest angular acceleration and reach the bottom of the plane first. Equation 10.52 describes the speed of an object rolling down an inclined plane. In the denominator, I_{CM} will be a numerical factor (e.g., 2/5 for the sphere) multiplied by MR^2. Therefore, the mass and radius will cancel in the equation and the center-of-mass speed will be independent of mass and radius.

CQ10.14 (a) Sewer pipe: $I_{CM} = MR^2$. (b) Embroidery hoop: $I_{CM} = MR^2$. (c) Door: $I = \frac{1}{3}MR^2$. (d) Coin: $I_{CM} = \frac{1}{2}MR^2$. The distribution of mass along lines parallel to the axis makes no difference to the moment of inertia.

CQ10.15 (a) The tricycle rolls forward. (b) The tricycle rolls forward. (c) The tricycle rolls backward. (d) The tricycle does not roll, but may skid forward. (e) The tricycle rolls backward. (f) To answer these questions, think about the torque of the string tension about an axis at the bottom of the wheel, where the rubber meets the road. This is the instantaneous axis of rotation in rolling. Cords A and B produce clockwise torques about this axis. Cords C and E produce counterclockwise torques. Cord D has zero lever arm.

CQ10.16 As one finger slides towards the center, the normal force exerted by the sliding finger on the ruler increases. At some point, this normal force will increase enough so that static friction between the sliding finger and the ruler will stop their relative motion. At this moment the other finger starts sliding along the ruler towards the center. This process repeats until the fingers meet at the center of the ruler.

Next step: Try a rod with a nonuniform mass distribution.

Next step: Wear a piece of sandpaper as a ring on one finger to change its coefficient of friction.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 10.1 Angular Position, Velocity, and Acceleration

P10.1 (a) The Earth rotates 2π radians (360°) on its axis in 1 day. Thus,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{1 \cancel{\text{ day}}}\left(\frac{1 \cancel{\text{ day}}}{8.64 \times 10^4 \text{ s}}\right) = \boxed{7.27 \times 10^{-5} \text{ rad/s}}$$

(b) Because of its angular speed, $\boxed{\text{the Earth bulges at the equator}}$.

P10.2 (a) $$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{1.00 \text{ rev/s} - 0}{30.0 \text{ s}} = \left(3.33 \times 10^{-2} \frac{\cancel{\text{rev}}}{\text{s}^2}\right)\left(\frac{2\pi \text{ rad}}{1 \cancel{\text{ rev}}}\right)$$
$$= \boxed{0.209 \text{ rad/s}^2}$$

(b) $\boxed{\text{Yes.}}$ When an object starts from rest, its angular speed is related to the angular acceleration and time by the equation $\omega = \alpha(\Delta t)$.

Thus, the angular speed is directly proportional to both the angular acceleration and the time interval. If the time interval is held constant, doubling the angular acceleration will double the angular speed attained during the interval.

P10.3 (a) $\theta\big|_{t=0} = \boxed{5.00 \text{ rad}}$

$$\omega\big|_{t=0} = \left.\frac{d\theta}{dt}\right|_{t=0} = 10.0 + 4.00t\big|_{t=0} = \boxed{10.0 \text{ rad/s}}$$

$$\alpha\big|_{t=0} = \left.\frac{d\omega}{dt}\right|_{t=0} = \boxed{4.00 \text{ rad/s}^2}$$

(b) $\theta\big|_{t=3.00\text{ s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0 \text{ rad}}$

$$\omega\big|_{t=3.00\text{s}} = \left.\frac{d\theta}{dt}\right|_{t=3.00\text{s}} = 10.0 + 4.00t\big|_{t=3.00\text{s}} = \boxed{22.0 \text{ rad/s}}$$

$$\alpha\big|_{t=3.00\text{s}} = \left.\frac{d\omega}{dt}\right|_{t=3.00\text{s}} = \boxed{4.00 \text{ rad/s}^2}$$

P10.4 $\alpha = \dfrac{d\omega}{dt} = 10 + 6t \quad \rightarrow \quad \int_0^\omega d\omega = \int_0^t (10 + 6t)dt \quad \rightarrow \quad \omega - 0 = 10t + \dfrac{6}{2}t^2$

$$\omega = \frac{d\theta}{dt} = 10t + 3t^2 \quad \rightarrow \quad \int_0^\theta d\theta = \int_0^t (10t + 3t^2)dt \quad \rightarrow \quad \theta - 0 = \frac{10t^2}{2} + \frac{3t^3}{3}$$

$\theta = 5t^2 + t^3$. At $t = 4.00$ s, $\theta = 5(4.00 \text{ s})^2 + (4.00 \text{ s})^3 = \boxed{144 \text{ rad}}$

Section 10.2 Analysis Model: Rigid Object Under Constant Angular Acceleration

P10.5 (a) We start with $\omega_f = \omega_i + \alpha t$ and solve for the angular acceleration α:

$$\alpha = \frac{\omega - \omega_i}{t} = \frac{12.0 \text{ rad/s}}{3.00 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$$

(b) The angular position of a rigid object under constant angular acceleration is given by Equation 10.7:

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2 = \frac{1}{2}(4.00 \text{ rad/s}^2)(3.00 \text{ s})^2 = \boxed{18.0 \text{ rad}}$$

P10.6 $\omega_i = 3\,600 \text{ rev/min} = 3.77 \times 10^2 \text{ rad/s}$

$\theta = 50.0 \text{ rev} = 3.14 \times 10^2 \text{ rad and } \omega_f = 0$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$0 = \left(3.77 \times 10^2 \text{ rad/s}\right)^2 + 2\alpha\left(3.14 \times 10^2 \text{ rad}\right)$

$\alpha = \boxed{-2.26 \times 10^2 \text{ rad/s}^2}$

P10.7 We are given $\alpha = -2.00 \text{ rad/s}^2$, $\omega_f = 0$, and

$$\omega_i = \frac{100 \text{ rev}}{1.00 \text{ min}}\left(\frac{1 \text{ min}}{60.0 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = \frac{10\pi}{3} \text{ rad/s}$$

(a) From $\omega_f - \omega_i = \alpha t$, we have

$$t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - (10\pi/3)}{-2.00} \text{ s} = \boxed{5.24 \text{ s}}$$

(b) Since the motion occurs with constant angular acceleration, we write

$$\theta_f = \bar{\omega}t = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left(\frac{10\pi}{6} \text{ rad/s}\right)\left(\frac{10\pi}{6} \text{ s}\right) = \boxed{27.4 \text{ rad}}$$

P10.8 (a) From $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$, the angular displacement is

$$\Delta\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{(2.2 \text{ rad/s})^2 - (0.06 \text{ rad/s})^2}{2(0.70 \text{ rad/s}^2)} = \boxed{3.5 \text{ rad}}$$

(b) From the equation given above for $\Delta\theta$, observe that when the angular acceleration is constant, the displacement is proportional to the difference in the *squares* of the final and initial angular speeds. Thus, the angular displacement would $\boxed{\text{increase by a factor of 4}}$ if both of these speeds were doubled.

***P10.9** We are given $\omega_f = 2.51 \times 10^4 \text{ rev/min} = 2.63 \times 10^3 \text{ rad/s}$

(a) $\alpha = \dfrac{\omega_f - \omega_i}{t} = \dfrac{2.63 \times 10^3 \text{ rad/s} - 0}{3.20 \text{ s}} = \boxed{8.21 \times 10^2 \text{ rad/s}^2}$

(b) $\theta_f = \omega_i t + \dfrac{1}{2}\alpha t^2 = 0 + \dfrac{1}{2}\left(8.21 \times 10^2 \text{ rad/s}^2\right)(3.20 \text{ s})^2 = \boxed{4.21 \times 10^3 \text{ rad}}$

P10.10 According to the definition of average angular speed (Eq. 10.2), the disk's average angular speed is 50.0 rad/10.0 s = 5.00 rad/s. According to the average angular speed expressed as $(\omega_i + \omega_f)/2$ in the model of a rigid object under constant angular acceleration, the average angular speed of the disk is (0 + 8.00 rad/s)/2 = 4.00 rad/s. Because these two numbers do not match, the angular acceleration of the disk cannot be constant.

P10.11 $\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$ and $\omega_f = \omega_i + \alpha t$ are two equations in two unknowns, ω_i and α.

$$\omega_i = \omega_f - \alpha t\text{:}\quad \theta_f - \theta_i = (\omega_f - \alpha t)t + \frac{1}{2}\alpha t^2 = \omega t - \frac{1}{2}\alpha t^2$$

$$(37.0\text{ rev})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = (98.0\text{ rad/s})(3.00\text{ s}) - \frac{1}{2}\alpha(3.00\text{ s})^2$$

$$232\text{ rad} = 294\text{ rad} - (4.50\text{ s}^2)\alpha\text{:}\quad \alpha = \frac{61.5\text{ rad}}{4.50\text{ s}^2} = \boxed{13.7\text{ rad/s}^2}$$

P10.12 $\omega = 5.00$ rev/s $= 10.0\pi$ rad/s. We will break the motion into two stages: (1) a period during which the tub speeds up and (2) a period during which it slows down.

While speeding up, $\theta_1 = \bar{\omega}t = \frac{0 + 10.0\pi\text{ rad/s}}{2}(8.00\text{ s}) = 40.0\pi\text{ rad}.$

While slowing down, $\theta_2 = \bar{\omega}t = \frac{10.0\pi\text{ rad/s} + 0}{2}(12.0\text{ s}) = 60.0\pi\text{ rad}.$

So, $\theta_{total} = \theta_1 + \theta_2 = 100\pi\text{ rad} = \boxed{50.0\text{ rev}}$.

***P10.13** We use $\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$ and $\omega_f = \omega_i + \alpha t$ to obtain

$$\omega_i = \omega_f - \alpha t \quad\text{and}\quad \theta_f - \theta_i = (\omega_f - \alpha t)t + \frac{1}{2}\alpha t^2 = \omega_f t - \frac{1}{2}\alpha t^2$$

Solving for the final angular speed gives

$$\omega_f = \frac{\theta_f - \theta_i}{t} + \frac{1}{2}\alpha t = \frac{62.4\text{ rad}}{4.20\text{ s}} + \frac{1}{2}(-5.60\text{ rad/s}^2)(4.20\text{ s})$$
$$= \boxed{3.10\text{ rad/s}^2}$$

P10.14 (a) Let R_E represent the radius of the Earth. The base of the building moves east at $v_1 = \omega R_E$, where ω is one revolution per day. The top of the building moves east at $v_2 = \omega(R_E + h)$. Its eastward speed relative to the ground is $v_2 - v_1 = \omega h$. The object's time of fall is given by $\Delta y = 0 + \frac{1}{2}gt^2$, $t = \sqrt{\frac{2h}{g}}$. During its fall the object's eastward motion is unimpeded so its deflection distance is

$$\Delta x = (v_2 - v_1)t = \omega h\sqrt{\frac{2h}{g}} = \boxed{\omega h^{3/2}\left(\frac{2}{g}\right)^{1/2}}$$

(b) $$\left(\frac{2\pi\text{ rad}}{86400\text{ s}}\right)(50.0\text{ m})^{3/2}\left(\frac{2}{9.80\text{ m/s}^2}\right)^{1/2} = \boxed{1.16\text{ cm}}$$

(c) $\boxed{\text{The deflection is only 0.02\% of the original height, so it is negligible in many practical cases.}}$

(d) $\boxed{\text{Decrease.}}$ Because the displacement is proportional to angular speed and the angular acceleration is constant, the displacement decreases linearly in time.

Section 10.3 Angular and Translational Quantities

P10.15 (a) From $v = r\omega$, we have

$$\omega = \frac{v}{r} = \frac{45.0\text{ m/s}}{250\text{ m}} = \boxed{0.180\text{ rad/s}}$$

(b) Traveling at constant speed along a circular track, the car will experience a centripetal acceleration given by

$$a_r = \frac{v^2}{r} = \frac{(45.0\text{ m/s})^2}{250\text{ m}} = \boxed{8.10\text{ m/s}^2\text{ toward the center of track}}$$

P10.16 Estimate the tire's radius at 0.250 m and miles driven as 10 000 per year. Then,

$$\theta = \frac{s}{r} = \left(\frac{1.00\times10^4\text{ mi}}{0.250\text{ m}}\right)\left(\frac{1609\text{ m}}{1\text{ mi}}\right) = 6.44\times10^7\text{ rad/yr}$$

$$\theta = \left(6.44\times10^{7}\ \text{rad/yr}\right)\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = 1.02\times10^{7}\ \text{rev/yr or } \boxed{\sim 10^{7}\ \text{rev/yr}}$$

P10.17 (a) The final angular speed is

$$\omega = \frac{v}{r} = \frac{25.0\ \text{m/s}}{1.00\ \text{m}} = \boxed{25.0\ \text{rad/s}}$$

(b) We solve for the angular acceleration from $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$:

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2(\Delta\theta)} = \frac{(25.0\ \text{rad/s})^2 - 0}{2\left[(1.25\ \text{rev})(2\pi\ \text{rad/rev})\right]} = \boxed{39.8\ \text{rad/s}^2}$$

(c) From the definition of angular acceleration,

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{25.0\ \text{rad/s}}{39.8\ \text{rad/s}^2} = \boxed{0.628\ \text{s}}$$

P10.18 (a) Consider a tooth on the front sprocket. It gives this speed, relative to the frame, to the link of the chain it engages:

$$v = r\omega = \left(\frac{0.152\ \text{m}}{2}\right)(76\ \text{rev/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$
$$= \boxed{0.605\ \text{m/s}}$$

(b) Consider the chain link engaging a tooth on the rear sprocket:

$$\omega = \frac{v}{r} = \frac{0.605\ \text{m/s}}{(0.070\ \text{m})/2} = \boxed{17.3\ \text{rad/s}}$$

(c) Consider the wheel tread and the road. A thread could be unwinding from the tire with this speed relative to the frame:

$$v = r\omega = \left(\frac{0.673\ \text{m}}{2}\right)(17.3\ \text{rad/s}) = \boxed{5.82\ \text{m/s}}$$

(d) $\boxed{\text{We did not need to know the length of the pedal cranks}}$, but we could use that information to find the linear speed of the pedals:

$$v = r\omega = (0.175\ \text{m})(7.96\ \text{rad/s})\left(\frac{1}{1\ \text{rad}}\right) = 1.39\ \text{m/s}$$

P10.19 Given $r = 1.00$ m, $\alpha = 4.00\ \text{rad/s}^2$, $\omega_i = 0$, and $\theta_i = 57.3° = 1.00$ rad:

(a) $\omega_f = \omega_i + \alpha t = 0 + \alpha t$

At $t = 2.00$ s, $\omega_f = 4.00\ \text{rad/s}^2(2.00\ \text{s}) = \boxed{8.00\ \text{rad/s}}$

(b) $v = r\omega = (1.00\text{ m})(8.00\text{ rad/s}) = \boxed{8.00\text{ m/s}}$

(c) $|a_r| = a_c = r\omega^2 = (1.00\text{ m})(8.00\text{ rad/s})^2 = 64.0\text{ m/s}^2$

$a_t = r\alpha = (1.00\text{ m})(4.00\text{ rad/s}^2) = 4.00\text{ m/s}^2$

The magnitude of the total acceleration is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(64.0\text{ m/s}^2)^2 + (4.00\text{ m/s}^2)^2} = \boxed{64.1\text{ m/s}^2}$$

The direction the total acceleration vector makes with respect to the radius to point *P* is

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{4.00}{64.0}\right) = \boxed{3.58°}$$

(d) $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = (1.00\text{ rad}) + \frac{1}{2}(4.00\text{ rad/s}^2)(2.00\text{ s})^2 = \boxed{9.00\text{ rad}}$

P10.20 (a) We first determine the distance travelled by the car during the 9.00-s interval:

$$s = \bar{v}t = \frac{v_i + v_f}{2}t = (11.0\text{ m/s})(9.00\text{ s}) = 99.0\text{ m}$$

the number of revolutions completed by the tire is then

$$\theta = \frac{s}{r} = \frac{99.0\text{ m}}{0.290\text{ m}} = 341\text{ rad} = \boxed{54.3\text{ rev}}$$

(b) $\omega_f = \frac{v_f}{r} = \frac{22.0\text{ m/s}}{0.290\text{ m}} = 75.9\text{ rad/s} = \boxed{12.1\text{ rev/s}}$

P10.21 Every part of this problem is about using radian measure to relate rotation of the whole object to the linear motion of a point on the object.

(a) $\omega = 2\pi f = \frac{2\pi\text{ rad}}{1\text{ rev}}\left(\frac{1\,200\text{ rev}}{60.0\text{ s}}\right) = \boxed{126\text{ rad/s}}$

(b) $v = \omega r = (126\text{ rad/s})(3.00\times10^{-2}\text{ m}) = \boxed{3.77\text{ m/s}}$

(c) $a_c = \omega^2 r = (126\text{ rad/s})^2(8.00\times10^{-2}\text{ m}) = 1\,260\text{ m/s}^2$ so

$\vec{a}_r = \boxed{1.26\text{ km/s}^2\text{ toward the center}}$

(d) $s = r\theta = \omega r t = (126\text{ rad/s})(8.00\times10^{-2}\text{ m})(2.00\text{ s}) = \boxed{20.1\text{ m}}$

P10.22 (a) $\boxed{5.77\text{ cm}}$

(b) Yes. The top of the ladder is displaced

$$\theta = s/r = 0.690\text{ m}/4.90\text{ m} \cong 0.141\text{ rad}$$

from vertical about its right foot. The left foot of the ladder is displaced by the same angle below the horizontal; therefore,

$$\theta = 0.690\text{ m}/4.90\text{ m} = t/0.410\text{ m} \rightarrow t = 5.77\text{ cm}$$

Note that we are approximating the straight-line distance of 0.690 m as an arc length because it is much smaller than the length of the ladder. The thickness of the rock is a cruder approximation of an arc length because the rung of the ladder is much shorter than the length of the ladder.

P10.23 The force of static friction must act forward and then more and more inward on the tires, to produce both tangential and centripetal acceleration. Its tangential component is $m(1.70\text{ m/s}^2)$. Its radially inward component is $ma_c = \frac{mv^2}{r} = m\omega^2 r$, which increases with time: this takes the maximum value

$$m\omega_f^2 r = mr(\omega_i^2 + 2\alpha\Delta\theta) = mr\left(0 + 2\alpha\frac{\pi}{2}\right) = m\pi r\alpha = m\pi a_t$$
$$= m\pi(1.70\text{ m/s}^2)$$

With skidding impending we have $\sum F_y = ma_y$, $+n - mg = 0$, $n = mg$:

$$f_s = \mu_s n = \mu_s mg = \sqrt{m^2(1.70\text{ m/s}^2)^2 + m^2\pi^2(1.70\text{ m/s}^2)^2}$$
$$\mu_s = \frac{1.70\text{ m/s}^2}{g}\sqrt{1+\pi^2} = \boxed{0.572}$$

P10.24 The force of static friction must act forward and then more and more inward on the tires, to produce both tangential and centripetal acceleration. Its tangential component is $ma = mr\alpha$. Its radially inward component is $ma_c = \frac{mv^2}{r} = m\omega^2 r$ which increases with time; this takes the maximum value

$$m\omega_f^2 r = mr(\omega_i^2 + 2\alpha\Delta\theta) = mr\left(0 + 2\alpha\frac{\pi}{2}\right) = m\pi r\alpha = m\pi a$$

With skidding impending we have

$$\sum F_y = ma_y: \quad +n - mg = 0 \quad \rightarrow \quad n = mg$$

$$f_s = \mu_s n = \mu_s mg = \sqrt{(ma_t)^2 + (ma_c)^2} = \sqrt{m^2a^2 + m^2\pi^2a^2}$$

$$\mu_s = \boxed{\frac{a}{g}\sqrt{1+\pi^2}}$$

P10.25 (a) The general expression for angular velocity is

$$\omega = \frac{d\theta}{dt} = \frac{d}{dt}\left(2.50t^2 - 0.600t^3\right) = 5.00t - 1.80t^2$$

where ω is in radians/second and t is in seconds.

The angular velocity will be a maximum when

$$\frac{d\omega}{dt} = \frac{d}{dt}\left(5.00t - 1.80t^2\right) = 5.00 - 3.60t = 0$$

Solving for the time t, we find

$$t = \frac{5.00}{3.60} = 1.39\text{ s}$$

Placing this value for t into the equation for angular velocity, we find

$$\omega_{max} = 5.00t - 1.80t^2 = 5.00(1.39) - 1.80(1.39)^2 = \boxed{3.47\text{ rad/s}}$$

(b) $v_{max} = \omega_{max} r = (3.47\text{ rad/s})(0.500\text{ m}) = \boxed{1.74\text{ m/s}}$

(c) The roller reverses its direction when the angular velocity is zero—recall an object moving vertically upward against gravity reverses its motion when its velocity reaches zero at the maximum height.

$$\omega = 5.00t - 1.80t^2 = t(5.00 - 1.80t) = 0$$

$$\rightarrow 5.00 - 1.80t = 0 \rightarrow t = \frac{5.00}{1.80} = 2.78\text{ s}$$

The driving force should be removed from the roller at $t = \boxed{2.78\text{ s}}$.

(d) Set t = 2.78 s in the expression for angular position:

$$\theta = 2.50t^2 - 0.600t^3 = 2.50(2.78)^2 - 0.600(2.78)^3 = 6.43\text{ rad}$$

or $$(6.43\text{ rad})\left(\frac{1\text{ rotation}}{2\pi\text{ rad}}\right) = \boxed{1.02\text{ rotations}}$$

P10.26 The object starts with $\theta_i = 0$. The location of its final position on the circle is found from $9\,\text{rad} - 2\pi = 2.72\,\text{rad} = 156°$.

(a) Its position vector is

$$3.00\text{ m at }156° = (3.00\text{ m})\cos 156°\hat{\mathbf{i}} + (3.00\text{ m})\sin 156°\hat{\mathbf{j}}$$
$$= \boxed{\left(-2.73\hat{\mathbf{i}} + 1.24\hat{\mathbf{j}}\right)\text{ m}}$$

(b) $\boxed{\text{It is in the second quadrant, at }156°}$

(c) The object's velocity is $v = \omega r = (1.50\text{ rad/s})(3.00\text{ m}) = 4.50\text{ m/s}$ at $90°$. After the displacement, its velocity is

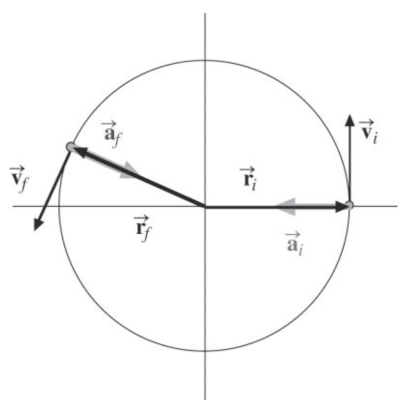
$$4.50\text{ m/s at }90°+156° \quad \text{or}$$
$$4.50\text{ m/s at }246° = (4.50\text{ m/s})\cos 246°\hat{\mathbf{i}} + (4.50\text{ m/s})\sin 246°\hat{\mathbf{j}}$$
$$= \boxed{\left(-1.85\hat{\mathbf{i}} - 4.10\hat{\mathbf{j}}\right)\text{ m/s}}$$

(d) $\boxed{\text{It is moving toward the third quadrant, at }246°}$.

(e) Its acceleration is v^2/r, opposite in direction to its position vector. This is

$$\frac{(4.50\text{ m/s})^2}{3.00\text{ m}}\text{ at }\ 180°+156° \quad \text{or}$$
$$6.75\text{ m/s}^2\text{ at }336° = (6.75\text{ m/s}^2)\cos 336°\hat{\mathbf{i}}$$
$$+ (6.75\text{ m/s}^2)\sin 336°\hat{\mathbf{j}}$$
$$= \boxed{\left(6.15\hat{\mathbf{i}} - 2.78\hat{\mathbf{j}}\right)\text{ m/s}^2}$$

(f) ANS. FIG. P10.26 shows the initial and final position, velocity, and acceleration vectors.



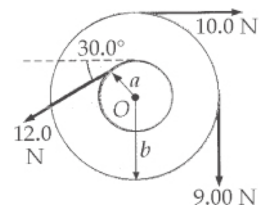
ANS. FIG. P10.26

(g) The total force is given by

$$\mathbf{F} = m\mathbf{a} = (4.00 \text{ kg})(6.15\hat{\mathbf{i}} - 2.78\,\hat{\mathbf{j}}) \text{ m/s}^2 = \boxed{(24.6\,\hat{\mathbf{i}} - 11.1\,\hat{\mathbf{j}}) \text{ N}}$$

Section 10.4 Torque

P10.27 To find the net torque, we add the individual torques, remembering to apply the convention that a torque producing clockwise rotation is negative and a counterclockwise rotation is positive.

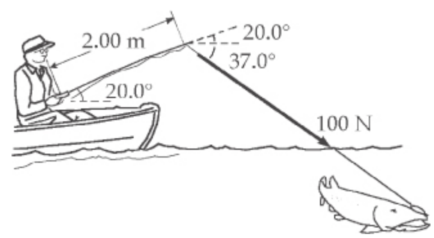


ANS. FIG. P10.27

$$\begin{aligned}\sum \tau &= (0.100 \text{ m})(12.0 \text{ N}) \\ &\quad - (0.250 \text{ m})(9.00 \text{ N}) \\ &\quad - (0.250 \text{ m})(10.0 \text{ N}) \\ &= \boxed{-3.55 \text{ N} \cdot \text{m}}\end{aligned}$$

The thirty-degree angle is unnecessary information.

P10.28 We resolve the 100-N force into components perpendicular to and parallel to the rod, as



ANS. FIG. P10.28

$$F_{\text{par}} = (100 \text{ N})\cos 57.0° = 54.5 \text{ N}$$

and

$$F_{\text{perp}} = (100 \text{ N})\sin 57.0° = 83.9 \text{ N}$$

The torque of F_{par} is zero since its line of action passes through the pivot point.

The torque of F_{perp} is

$$\tau = (83.9 \text{ N})(2.00 \text{ m}) = \boxed{168 \text{ N} \cdot \text{m}} \text{ (clockwise)}$$

Section 10.5 Analysis Model: Rigid Object Under a Net Torque

P10.29 The flywheel is a solid disk of mass M and radius R with axis through its center.

$$\left.\begin{aligned}\sum\tau &= I\alpha\\ I &= \frac{1}{2}MR^2\end{aligned}\right\} -T_u r + T_b r = \frac{1}{2}MR^2\alpha \rightarrow T_b = T_u + \frac{MR^2\alpha}{2r}$$

$$T_b = 135\text{ N} + \frac{(80.0\text{ kg})(0.625\text{ m})^2(-1.67\text{ rad/s}^2)}{2(0.230\text{ m})} = \boxed{21.5\text{ N}}$$

P10.30 (a) The moment of inertia of the wheel, modeled as a disk, is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2.00\text{ kg})(7.00\times10^{-2}\text{ m})^2 = 4.90\times10^{-3}\text{ kg}\cdot\text{m}^2$$

From Newton's second law for rotational motion,

$$\alpha = \frac{\sum\tau}{I} = \frac{0.600}{4.90\times10^{-3}} = 122\text{ rad/s}^2$$

then, from $\alpha = \dfrac{\Delta\omega}{\Delta t}$, we obtain

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{1200(2\pi/60)}{122} = \boxed{1.03\text{ s}}$$

(b) The number of revolutions is determined from

$$\Delta\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(122\text{ rad/s})(1.03\text{ s})^2 = 64.7\text{ rad} = \boxed{10.3\text{ rev}}$$

***P10.31** (a) We first determine the moment of inertia of the merry-go-round:

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(150\text{ kg})(1.50\text{ m})^2 = 169\text{ kg}\cdot\text{m}^2$$

To find the angular acceleration, we use

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \left(\frac{0.500\text{ rev/s} - 0}{2.00\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \frac{\pi}{2}\text{ rad/s}^2$$

From the definition of torque, $\tau = F\cdot r = I\alpha$, we obtain

$$F = \frac{I\alpha}{r} = \frac{(169\text{ kg}\cdot\text{m}^2)\left(\dfrac{\pi}{2}\text{ rad/s}^2\right)}{1.50\text{ m}} = \boxed{177\text{ N}}$$

P10.32 (a) See ANS. FIG. P10.32 below for the force diagrams. For m_1,

$\sum F_y = ma_y$ gives

$$+n - m_1 g = 0$$

$$n_1 = m_1 g$$

with $f_{k1} = \mu_k n_1$.

$\sum F_x = ma_x$ gives

$$-f_{k1} + T_1 = m_1 a \qquad \textbf{[1]}$$

For the pulley, $\sum \tau = I\alpha$ gives

$$-T_1 R + T_2 R = \frac{1}{2} MR^2 \left(\frac{a}{R}\right)$$

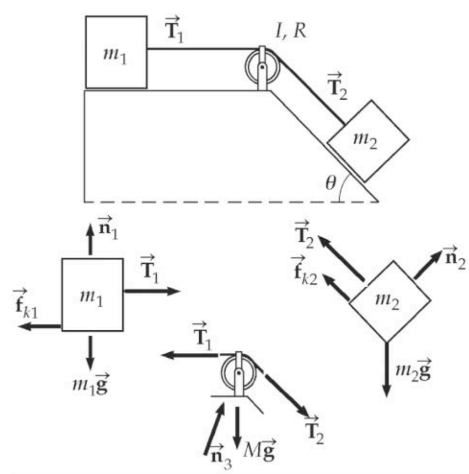
or $$-T_1 + T_2 = \frac{1}{2} MR\left(\frac{a}{R}\right) \rightarrow -T_1 + T_2 = \frac{1}{2} Ma \qquad \textbf{[2]}$$

For m_2,

$$+n_2 - m_2 g\cos\theta = 0 \rightarrow n_2 = m_2 g\cos\theta$$

$$f_{k2} = \mu_k n_2$$

$$-f_{k2} - T_2 + m_2 g\sin\theta = m_2 a \qquad \textbf{[3]}$$



ANS. FIG. P10.32

(b) Add equations [1], [2], and [3] and substitute the expressions for f_{k1} and n_1, and $-f_{k2}$ and n_2:

$$-f_{k1}+T_1+\left(-T_1+T_2\right)-f_{k2}-T_2+m_2 g\sin\theta=m_1 a+\frac{1}{2}Ma+m_2 a$$

$$-f_{k1}-f_{k2}+m_2 g\sin\theta=\left(m_1+m_2+\frac{1}{2}M\right)a$$

$$-\mu_k m_1 g-\mu_k m_2 g\cos\theta+m_2 g\sin\theta=\left(m_1+m_2+\frac{1}{2}M\right)a$$

$$a=\frac{m_2\left(\sin\theta-\mu_k\cos\theta\right)-\mu_k m_1}{m_1+m_2+\frac{1}{2}M}g$$

$$a=\frac{(6.00\text{ kg})\left(\sin 30.0^\circ-0.360\cos 30.0^\circ\right)-0.360(2.00\text{ kg})}{(2.00\text{ kg})+(6.00\text{ kg})+\frac{1}{2}(10.0\text{ kg})}g$$

$$a=\boxed{0.309\text{ m/s}^2}$$

(c) From equation [1]:

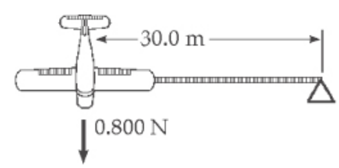
$$-f_{k1}+T_1=m_1 a\rightarrow T_1=2.00\text{ kg}\left(0.309\text{ m/s}^2\right)+7.06\text{ N}=\boxed{7.67\text{ N}}$$

From equation [2]:

$$-T_1+T_2=\frac{1}{2}Ma\rightarrow T_2=7.67\text{ N}+5.00\text{ kg}\left(0.309\text{ m/s}^2\right)$$

$$=\boxed{9.22\text{ N}}$$

P10.33 We use the definition of torque and the relationship between angular and translational acceleration, with $m = 0.750$ kg and $F = 0.800$ N:



ANS. FIG. P10.33

(a) $\tau=rF=(30.0\text{ m})(0.800\text{ N})=\boxed{24.0\text{ N}\cdot\text{m}}$

(b) $$\alpha=\frac{\tau}{I}=\frac{rF}{mr^2}=\frac{24.0\text{ N}\cdot\text{m}}{(0.750\text{ kg})(30.0\text{ m})^2}$$

$$=\boxed{0.035\,6\text{ rad/s}^2}$$

(c) $a_t=\alpha r=\left(0.035\,6\text{ rad/s}^2\right)(30.0\text{ m})=\boxed{1.07\text{ m/s}^2}$

P10.34 (a) The chosen tangential force produces constant torque and therefore constant angular acceleration. Since the disk starts from rest, we write

$$\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\theta_f - 0 = 0 + \frac{1}{2}\alpha t^2$$

$$\theta_f = \frac{1}{2}\alpha t^2$$

Solving for the angular acceleration gives

$$\alpha = \frac{2\theta_f}{t^2} = \frac{2(2.00 \text{ rev})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)}{(10.0 \text{ s})^2} = 0.251 \text{ rad/s}^2$$

We then obtain the required combination of *F* and *R* from the rigid object under a net torque model:

$$\sum \tau = I\alpha: \qquad FR = (100 \text{ kg}\cdot\text{m}^2)(0.251 \text{ rad/s}^2) = 25.1 \text{ N}\cdot\text{m}$$

$$\boxed{\text{For } F = 25.1 \text{ N}, R = 1.00 \text{ m. For } F = 10.0 \text{ N}, R = 2.51 \text{ m.}}$$

(b) $\boxed{\text{No. Infinitely many pairs of values that satisfy this requirement exist: for any } F \le 50.0 \text{ N}, R = 25.1 \text{ N}\cdot\text{m}/F, \text{ as long as } R \le 3.00 \text{ m.}}$

P10.35 (a) From the rigid object under a net torque model, $\sum \tau = I\alpha$ gives

$$I = \frac{\sum \tau}{\alpha} = \frac{\sum \tau}{\Delta\omega}\Delta t = \frac{36.0 \text{ N}\cdot\text{m}}{10.0 \text{ rad/s}}(6.00 \text{ s}) = \boxed{21.6 \text{ kg}\cdot\text{m}^2}$$

(b) For the portion of the motion during which the wheel slows down,

$$\left|\sum \tau\right| = |I\alpha| = \left|I\frac{\Delta\omega}{\Delta t}\right| = \left|(21.6 \text{ kg}\cdot\text{m}^2)\left(\frac{-10.0 \text{ rad/s}}{60.0 \text{ s}}\right)\right|$$
$$= \boxed{3.60 \text{ N}\cdot\text{m}}$$

(c) During the first portion of the motion,

$$\Delta\theta = \omega_{avg}\Delta t = \left(\frac{\omega_i + \omega_f}{2}\right)\Delta t = \left(\frac{0 + 10.0 \text{ rad/s}}{2}\right)(6.00 \text{ s})$$
$$= 30 \text{ rad}$$

During the second portion,

$$\Delta\theta = \omega_{avg}\Delta t = \left(\frac{\omega_i + \omega_f}{2}\right)\Delta t = \left(\frac{10.0\text{ rad/s} + 0}{2}\right)(60.0\text{ s})$$
$$= 300\text{ rad}$$

Therefore, the total angle is 330 rad or $\boxed{52.5\text{ revolutions}}$.

P10.36 (a) Let T_1 represent the tension in the cord above m_1 and T_2 the tension in the cord above the lighter mass. The two blocks move with the same acceleration because the cord does not stretch, and the angular acceleration of the pulley is a/R. For the heavier mass we have

$$\sum F = m_1 a \rightarrow T_1 - m_1 g = m_1(-a) \quad \text{or} \quad -T_1 + m_1 g = m_1 a$$

For the lighter mass,

$$\sum F = m_2 a \quad \rightarrow \quad T_2 - m_2 g = m_2 a$$

We assume the pulley is a uniform disk: $I = (1/2)MR^2$

$$\sum \tau = I\alpha \quad \rightarrow \quad +T_1 R - T_2 R = \frac{1}{2}MR^2(a/R)$$

or $\quad T_1 - T_2 = \frac{1}{2}Ma$

Add up the three equations in a:

$$-T_1 + m_1 g + T_2 - m_2 g + T_1 - T_2 = m_1 a + m_2 a + \frac{1}{2}Ma$$

$$a = \frac{m_1 - m_2}{m_1 + m_2 + \frac{1}{2}M}g$$
$$= \frac{20.0\text{ kg} - 12.5\text{ kg}}{20.0\text{ kg} + 12.5\text{ kg} + \frac{1}{2}(5.00\text{ kg})}(9.80\text{ m/s}^2)$$
$$= 2.10\text{ m/s}^2$$

Next, $x = 0 + 0 + \frac{1}{2}at^2 \rightarrow t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(4.00\text{ m})}{2.10\text{ m/s}^2}} = \boxed{1.95\text{ s}}$

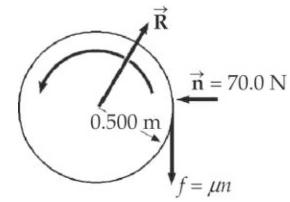
(b) If the pulley were massless, the acceleration would be larger by a factor 35/32.5 and the time shorter by the square root of the factor 32.5/35. That is, the time would be reduced by 3.64%.

P10.37 From the rigid object under a net torque model,

$$\sum \tau = I\alpha$$

$$-f_k R = \mu_k F R = \left(\frac{1}{2}MR^2\right)\frac{\Delta\omega}{\Delta t}$$

$$\mu_k = -\frac{MR\Delta\omega}{2F\Delta t}$$



ANS. FIG. P10.37

Substitute numerical values:

$$\mu_k = -\frac{(100\text{ kg})(0.500\text{ m})(-50.0\text{ rev/min})}{2(70.0\text{ N})(6.00\text{ s})}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)$$

$$= \boxed{0.312}$$

Section 10.6 Calculation of Moments of Inertia

P10.38 Model your body as a cylinder of mass 60.0 kg and a radius of 12.0 cm. Then its moment of inertia is

$$\frac{1}{2}MR^2 = \frac{1}{2}(60.0\text{ kg})(0.120\text{ m})^2 = 0.432\text{ kg}\cdot\text{m}^2$$

$$\sim \boxed{10^0\text{ kg}\cdot\text{m}^2 = 1\text{ kg}\cdot\text{m}^2}$$

P10.39 (a) Every particle in the door could be slid straight down into a high-density rod across its bottom, without changing the particle's distance from the rotation axis of the door. Thus, a rod 0.870 m long with mass 23.0 kg, pivoted about one end, has the same rotational inertia as the door:

$$I = \frac{1}{3}ML^2 = \frac{1}{3}(23.0\text{ kg})(0.870\text{ m})^2 = \boxed{5.80\text{ kg}\cdot\text{m}^2}$$

(b) The $\boxed{\text{height of the door is unnecessary}}$ data.

P10.40 (a) We take a coordinate system with mass M at the origin. The distance from the axis to the origin is also x. The moment of ineria about the axis is

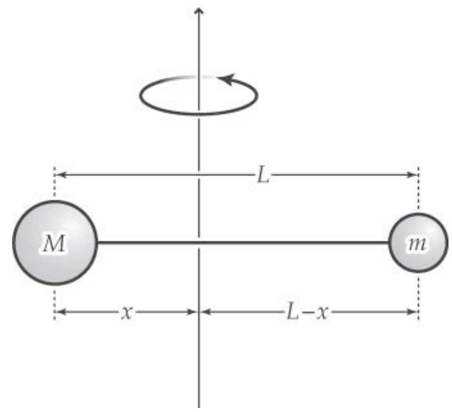
$$I = Mx^2 + m(L-x)^2$$

To find the extrema in the moment of inertia, we differentiate I with respect to x:

$$\frac{dI}{dx} = 2Mx - 2m(L-x) = 0$$

Solving for x then gives

$$x = \frac{mL}{M+m}$$



ANS. FIG. P10.40

Differentiating again gives $\frac{d^2I}{dx^2} = 2m + 2M$; therefore, I is at a minimum when the axis of rotation passes through $x = \frac{mL}{M+m}$, which is also the position of the center of mass of the system if we take mass M to lie at the origin of a coordinate system.

(b) The moment of inertia about an axis passing through x is

$$I_{CM} = M\left[\frac{mL}{M+m}\right]^2 + m\left[1 - \frac{m}{M+m}\right]^2 L^2 = \frac{Mm}{M+m}L^2$$

$$\rightarrow \boxed{I_{CM} = \mu L^2, \text{ where } \mu = \frac{Mm}{M+m}}$$

P10.41 Treat the tire as consisting of three hollow cylinders: two sidewalls and a tread region. The moment of inertia of a hollow cylinder, where $R_2 > R_1$, is $I = \frac{1}{2}M(R_1^2 + R_2^2)$, and the mass of a hollow cylinder of height (or thickness) t is $M = \rho\pi(R_2^2 - R_1^2)t$. Substituting the expression for mass M into the expression for I, we get

$$I = \frac{1}{2}\rho\pi(R_2^2 - R_1^2)t(R_1^2 + R_2^2) = \frac{1}{2}\rho\pi t(R_2^4 - R_1^4)$$

The two sidewalls have inner radius $r_1 = 16.5$ cm, outer radius $r_2 = 30.5$ cm, and height $t_{side} = 0.635$ cm. The tread region has inner radius $r_2 = 30.5$ cm, outer radius $r_3 = 33.0$ cm, and height $t_{tread} = 20.0$ cm. The

density of the rubber is 1.10×10^3 kg/m^3.

For the tire (two sidewalls: $R_1 = r_1$, $R_2 = r_2$; tread region: $R_1 = r_2$, $R_2 = r_3$)

$$I_{\text{total}} = 2\left[\frac{1}{2}\rho\pi t_{\text{side}}\left(R_2^4 - R_1^4\right)\right] + \frac{1}{2}\rho\pi t_{\text{tread}}\left(R_2^4 - R_1^4\right)$$
$$= 2\left[\frac{1}{2}\rho\pi t_{\text{side}}\left(r_2^4 - r_1^4\right)\right] + \frac{1}{2}\rho\pi t_{\text{tread}}\left(r_3^4 - r_2^4\right)$$

Substituting,

$$I_{\text{total}} = 2\left\{\frac{1}{2}\left(1.10\times10^3 \text{ kg/m}^3\right)\pi\left(6.35\times10^{-3} \text{ m}\right) \times\left[\left(0.305 \text{ m}\right)^4 - \left(0.165 \text{ m}\right)^4\right]\right\}$$
$$+\frac{1}{2}\left(1.10\times10^3 \text{ kg/m}^3\right)\pi\left(0.200 \text{ m}\right)\left[\left(0.330 \text{ m}\right)^4 - \left(0.305 \text{ m}\right)^4\right]$$
$$= 2\left(8.68\times10^{-2} \text{ kg}\cdot\text{m}^2\right) + 1.11 \text{ kg}\cdot\text{m}^2 = \boxed{1.28 \text{ kg}\cdot\text{m}^2}$$

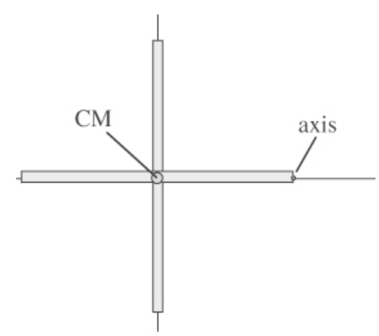
P10.42 We use x as a measure of the distance of each mass element dm in the rod from the y' axis:

$$\boxed{I_{y'} = \int_{\text{all mass}} r^2 dm = \int_0^L x^2 \frac{M}{L} dx = \frac{M}{L}\frac{x^3}{3}\bigg|_0^L = \frac{1}{3}ML^2}$$

P10.43 We assume the rods are thin, with radius much less than L. Note that the center of mass (CM) of the rod combination lies at the origin of the coordinate system. Because the axis of rotation is parallel to the y axis, we can first calculate the moment of inertia of the rods about the y axis, then use the parallel-axis theorem to find the moment about the axis of rotation.

The moment of the rod on the y axis about the y axis itself is essentially zero (axis through center, parallel to rod) because the rod is thin. The moments of the rods on the x and z axes are each

$I = \frac{1}{12}mL^2$ (axis through center, perpendicular to rod) from the table in the chapter.



ANS. FIG. P10.43

The total moment of the three rods about the *y* axis (and about the CM) is

$$I_{CM} = I_{\text{on x axis}} + I_{\text{on y axis}} + I_{\text{on z axis}}$$
$$= \frac{1}{12}mL^2 + 0 + \frac{1}{12}mL^2 = \frac{1}{6}mL^2$$

For the moment of the rod-combination about the axis of rotation, the parallel-axis theorem gives

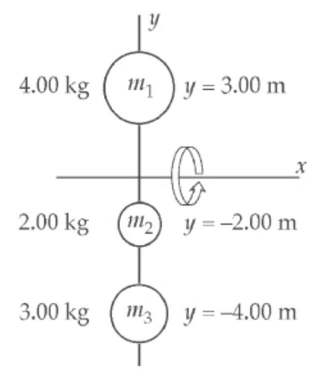
$$I = I_{CM} + 3m\left(\frac{L}{2}\right)^2 = \left[\frac{1}{6} + \frac{3}{4}\right]mL^2 = \left[\frac{2}{12} + \frac{9}{12}\right]mL^2 = \boxed{\frac{11}{12}mL^2}$$

Section 10.7 Rotational Kinetic Energy

P10.44 The masses and distances from the rotation axis for the three particles are:

$$m_1 = 4.00 \text{ kg},\ r_1 = |y_1| = 3.00 \text{ m}$$
$$m_2 = 2.00 \text{ kg},\ r_2 = |y_2| = 2.00 \text{ m}$$
$$m_3 = 3.00 \text{ kg},\ r_3 = |y_3| = 4.00 \text{ m}$$

and $\omega = 2.00 \text{ rad/s}$ about the *x* axis.



ANS. FIG. P10.44

(a) $I_x = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$

$$I_x = (4.00 \text{ kg})(3.00 \text{ m})^2 + (2.00 \text{ kg})(2.00 \text{ m})^2 + (3.00 \text{ kg})(4.00 \text{ m})^2$$
$$= \boxed{92.0 \text{ kg}\cdot\text{m}^2}$$

(b) $K_R = \frac{1}{2}I_x\omega^2 = \frac{1}{2}(92.0 \text{ kg}\cdot\text{m}^2)(2.00 \text{ m})^2 = \boxed{184 \text{ J}}$

(c) $v_1 = r_1\omega = (3.00 \text{ m})(2.00 \text{ rad/s}) = \boxed{6.00 \text{ m/s}}$

$v_2 = r_2\omega = (2.00 \text{ m})(2.00 \text{ rad/s}) = \boxed{4.00 \text{ m/s}}$

$v_3 = r_3\omega = (4.00 \text{ m})(2.00 \text{ rad/s}) = \boxed{8.00 \text{ m/s}}$

(d) $K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}(4.00 \text{ kg})(6.00 \text{ m/s})^2 = 72.0 \text{ J}$

$$K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}(2.00\text{ kg})(4.00\text{ m/s})^2 = 16.0\text{ J}$$

$$K_3 = \frac{1}{2}m_3v_3^2 = \frac{1}{2}(3.00\text{ kg})(8.00\text{ m/s})^2 = 96.0\text{ J}$$

$$K = K_1 + K_2 + K_3 = 72.0\text{ J} + 16.0\text{ J} + 96.0\text{ J} = \boxed{184\text{ J}} = \frac{1}{2}I_x\omega^2$$

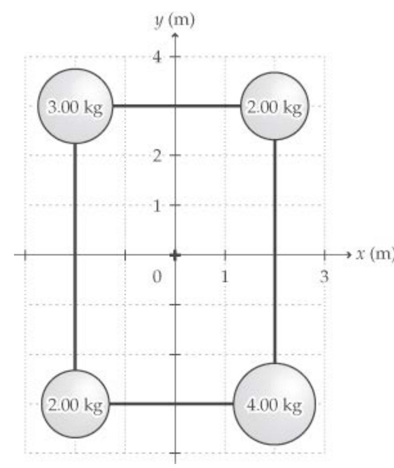
(e) $\boxed{\text{The kinetic energies computed in parts (b) and (d) are the same.}}$
Rotational kinetic energy of an object rotating about a fixed axis can be viewed as the total translational kinetic energy of the particles moving in circular paths.

P10.45 (a) All four particles are at a distance r from the z axis, with

$$r^2 = (3.00\text{ m})^2 + (2.00\text{ m})^2 = 13.0\text{ m}^2$$

Thus the moment of inertia is

$$\begin{aligned} I_z &= \sum m_i r_i^2 \\ &= (3.00\text{ kg})(13.0\text{ m}^2) \\ &\quad + (2.00\text{ kg})(13.0\text{ m}^2) \\ &\quad + (4.00\text{ kg})(13.0\text{ m}^2) \\ &\quad + (2.00\text{ kg})(13.0\text{ m}^2) \\ &= \boxed{143\text{ kg}\cdot\text{m}^2} \end{aligned}$$



ANS. FIG. P10.45

(b) The rotational kinetic energy of the four-particle system is

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(143\text{ kg}\cdot\text{m}^2)(6.00\text{ rad/s})^2 = \boxed{2.57\times 10^3\text{ J}}$$

P10.46 The cam is a solid disk of radius R that has had a small disk of radius $R/2$ cut from it. To find the moment of inertia of the cam, we use the parallel-axis theorem to find the moment of inertia of the solid disk about an axis at distance $R/2$ from its CM, then subtract off the moment of inertia of the small disk of radius $R/2$ with axis through its center.

By the parallel-axis theorem, the moment of inertia of the solid disk about an axis $R/2$ from its CM is

$$I_{\text{disk}} = I_{\text{CM}} + M_{\text{disk}}\left(\frac{R}{2}\right)^2 = \frac{1}{2}M_{\text{disk}}R^2 + \frac{1}{4}M_{\text{disk}}R^2 = \frac{3}{4}M_{\text{disk}}R^2$$

With half the radius, the cut-away small disk has one-quarter the face area and one-quarter the volume and one-quarter the mass M_{disk} of the original solid disk:

$$\frac{M_{\text{small disk}}}{M_{\text{disk}}} = \frac{(R/2)^2}{R^2} = \frac{1}{4}$$

The moment of inertia of the small disk of radius $R/2$ about an axis through its CM is

$$I_{\text{small disk}} = \frac{1}{2} M_{\text{small disk}} \left(\frac{R}{2}\right)^2 = \frac{1}{2}\left[\frac{1}{4} M_{\text{disk}}\right]\frac{R^2}{4} = \frac{1}{32} M_{\text{disk}} R^2$$

Subtracting the moment of the small disk from the solid disk, we find for the cam

$$I_{\text{cam}} = I_{\text{disk}} - I_{\text{small disk}} = \frac{3}{4} M_{\text{disk}} R^2 - \frac{1}{32} M_{\text{disk}} R^2$$

$$I_{\text{cam}} = M_{\text{disk}} R^2 \left[\frac{24}{32} - \frac{1}{32}\right] = \frac{23}{32} M_{\text{disk}} R^2$$

The mass of the cam is $M = M_{\text{disk}} - M_{\text{small disk}} = M_{\text{disk}} - \frac{1}{4} M_{\text{disk}} = \frac{3}{4} M_{\text{disk}}$, therefore

$$I_{\text{cam}} = \frac{23}{32} M_{\text{disk}} R^2 \left(\frac{M}{\frac{3}{4} M_{\text{disk}}}\right) = MR^2 \left(\frac{23}{32}\right)\left(\frac{4}{3}\right) = \frac{23}{24} MR^2$$

The moment of inertia of the cam-shaft is the sum of the moments of the cam and the shaft:

$$I_{\text{cam-shaft}} = I_{\text{cam}} + I_{\text{shaft}} = \frac{23}{24} MR^2 + \frac{1}{2} M \left(\frac{R}{2}\right)^2$$

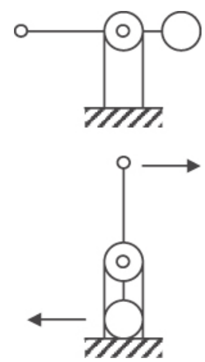
$$= MR^2 \left[\frac{23}{24} + \frac{1}{8}\right] = MR^2 \left[\frac{23}{24} + \frac{3}{24}\right]$$

$$I_{\text{cam-shaft}} = \frac{26}{24} MR^2 = \frac{13}{12} MR^2$$

The kinetic energy of the cam-shaft combination rotating with angular speed ω is

$$K = \frac{1}{2} I_{\text{can-shaft}} \omega^2 = \frac{1}{2}\left(\frac{13}{12} MR^2\right)\omega^2 = \boxed{\frac{13}{24} MR^2 \omega^2}$$

P10.47 (a) Identify the two objects and the Earth as an isolated system. The maximum speed of the lighter object will occur when the rod is in the vertical position so let's define the time interval as from when the system is released from rest to when the rod reaches a vertical orientation. So, for the isolated system,



ANS. FIG. P10.47

$$\Delta K + \Delta U = 0$$

$$\left[\left(\frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2\right) - 0\right] + \left[m_1 g y_1 + m_2 g y_2 - 0\right] = 0$$

$$\omega = \sqrt{\frac{-2g(m_1 y_1 + m_2 y_2)}{I_1 + I_2}} = \sqrt{\frac{-2g(m_1 y_1 + m_2 y_2)}{m_1 r_1^2 + m_2 r_2^2}}$$

$$= \sqrt{\frac{-2(9.80\text{ m/s}^2)\left[(0.120\text{ kg})(2.86\text{ m}) + (60.0\text{ kg})(-0.140\text{ m})\right]}{(0.120\text{ kg})(2.86\text{ m})^2 + (60.0\text{ kg})(0.140\text{ m})^2}}$$

$$= 8.55\text{ rad/s}$$

Then, the tangential speed of the lighter object is,

$$v = r\omega = (2.86\text{ m})(8.55\text{ rad/s}) = \boxed{24.5\text{ m/s}}$$

(b) $\boxed{\text{No.}}$ The overall acceleration is not constant. It has to move either in a straight line or parabolic path to have a chance of being under constant acceleration. The circular path presented here rules out that possibility.

(c) $\boxed{\text{No.}}$ It does not move with constant tangential acceleration, since the angular acceleration is not constant. See explanation in part (d).

(d) $\boxed{\text{No.}}$ The lever arm of the gravitational force acting on the 60-kg mass changes during the motion. As a result, the torque changes, and so does the angular acceleration.

(e) $\boxed{\text{No.}}$ The angular velocity changes, therefore the angular momentum of the trebuchet changes.

(f) $\boxed{\text{Yes.}}$ The mechanical energy stays constant because the system is isolated—that is how we solved the problem in (a).

Section 10.8 Energy Considerations in Rotational Motion

P10.48 From the rigid object under a net torque model,

$$\sum\tau = I\alpha \rightarrow \alpha = \frac{\sum\tau}{I} = \frac{FR}{\frac{1}{2}MR^2} = \frac{2F}{MR}$$

From the definition of rotational kinetic energy and the rigid object under constant angular acceleration model,

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}I(\omega_i + \alpha t)^2 = \frac{1}{2}I\alpha^2t^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{2F}{MR}\right)^2 t^2$$
$$= \frac{F^2t^2}{M}$$

Substituting,

$$K = \frac{(50.0\text{ N})^2(3.00\text{ s})^2}{800\text{ N}/9.80\text{ m/s}^2} = \boxed{276\text{ J}}$$

P10.49 The moment of inertia of a thin rod about an axis through one end is $I = \frac{1}{3}ML^2$. The total rotational kinetic energy is given as

$$K_R = \frac{1}{2}I_h\omega_h^2 + \frac{1}{2}I_m\omega_m^2$$

with $$I_h = \frac{m_hL_h^2}{3} = \frac{60.0\text{ kg}(2.70\text{ m})^2}{3} = 146\text{ kg}\cdot\text{m}^2$$

and $$I_m = \frac{m_mL_m^2}{3} = \frac{100\text{ kg}(4.50\text{ m})^2}{3} = 675\text{ kg}\cdot\text{m}^2$$

In addition,

$$\omega_h = \frac{2\pi\text{ rad}}{12\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.45\times10^{-4}\text{ rad/s}$$

while $$\omega_m = \frac{2\pi\text{ rad}}{1\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.75\times10^{-3}\text{ rad/s}$$

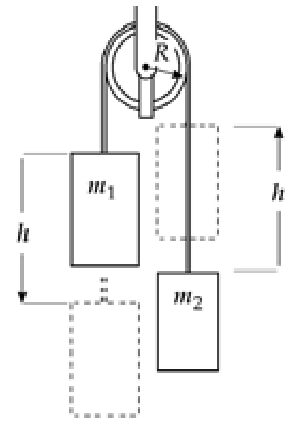
Therefore,

$$K_R = \frac{1}{2}(146\text{ kg}\cdot\text{m}^2)(1.45\times10^{-4}\text{ rad/s})^2$$
$$+\frac{1}{2}(675\text{ kg}\cdot\text{m}^2)(1.75\times10^{-3}\text{ rad/s})^2$$
$$= \boxed{1.04\times10^{-3}\text{ J}}$$

***P10.50** Take the two objects, pulley, and Earth as the system. If we neglect friction in the system, then mechanical energy is conserved and we can state that the increase in kinetic energy of the system equals the decrease in potential energy. Since $K_i = 0$ (the system is initially at rest), we have

$$\Delta K = K_f - K_i$$
$$= \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2$$

where m_1 and m_2 have a common speed. But $v = R\omega$ so that $\Delta K = \frac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)v^2$.



ANS. FIG. P10.50

From ANS. FIG. P10.50, we see that the system loses potential energy because of the motion of m_1 and gains potential energy because of the motion of m_2. Applying the law of conservation of energy, $\Delta K + \Delta U = 0$, gives

$$\frac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)v^2 + m_2gh - m_1gh = 0$$

$$\boxed{v = \sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2 + \frac{I}{R^2}}}}$$

Since $v = R\omega$, the angular speed of the pulley at this instant is given by

$$\boxed{\omega = \frac{v}{R} = \sqrt{\frac{2(m_1 - m_2)gh}{m_1R^2 + m_2R^2 + I}}}$$

P10.51 For the nonisolated system of the top,

$$W = \Delta K \quad \to \quad F\Delta x = \left(\frac{1}{2}I\omega^2 - 0\right)$$

$$\to \quad \omega = \sqrt{\frac{2F\Delta x}{I}} = \sqrt{\frac{2(5.57\text{ N})(0.800\text{ m})}{4 \times 10^{-4}\text{ kg}\cdot\text{m}^2}} = \boxed{149\text{ rad/s}}$$

P10.52 The power output of the bus is $P = \frac{E}{\Delta t}$, where

$$E = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\omega^2\right) = \frac{1}{4}MR^2\omega^2$$

is the stored energy and $\Delta t = \dfrac{d}{v}$ is the time it can roll. Then

$\dfrac{1}{4}MR^2\omega^2 = P\Delta t = \dfrac{Pd}{v}$. The maximum range of the bus is then

$d = \dfrac{MR^2\omega^2 v}{4P}$.

For average $P = (25.0 \text{ hp})\left(\dfrac{746 \text{ W}}{1 \text{ hp}}\right) = 18\ 650 \text{ W}$ and average

v =35.0 km/h = 9.72 m/s, the maximum range is

$$\begin{aligned}
d &= \frac{MR^2\omega^2 v}{4P} \\
&= \frac{(1\,200 \text{ kg})(0.500 \text{ m})^2(3\ 000\cdot 2\pi/60 \text{ s})^2(9.72 \text{ m/s})}{4(18\ 650 \text{ W})} \\
&= 3.86 \text{ km}
\end{aligned}$$

$\boxed{\text{The situation is impossible because the range is only 3.86 km, not city-wide.}}$

P10.53 (a) Apply $\Delta K + \Delta U + \Delta E_{int} = 0$, where $\Delta E_{int} = f_k d$, $\mu = 0.250$, and $f_k = \mu n_2 = \mu m_2 g$. Both translational and rotational kinetic energy are present in the system. $v_i = 0.820$ m/s. Find v. The angular speed of the pulley is $\omega_i = v_i/R_2$, and $\omega = v/R_2$. Mass m_1 drops by $h = d$ when mass m_2 moves distance $d = 0.700$ m.

$I = \dfrac{1}{2}M\left(R_1^2 + R_2^2\right)$, where $R_1 = 0.020\ 0$ m, $R_2 = 0.030\ 0$ m, and $M = 0.350$ kg.

$$(K_f - K_i) + (U_f - U_f) + \Delta E_{int} = 0$$

$$\left(\frac{1}{2}m_2 v^2 - \frac{1}{2}m_2 v_i^2\right) + \left(\frac{1}{2}m_1 v^2 - \frac{1}{2}m_1 v_i^2\right) + \left(\frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_i^2\right) + (m_1 g y - m_1 g y_i) + f_k d = 0$$

$$\frac{1}{2}(m_1 + m_2)(v^2 - v_i^2) + \frac{1}{2}I\left[\left(\frac{v}{R_2}\right)^2 - \left(\frac{v_i}{R_2}\right)^2\right] + m_1 g(y - y_i) + \mu m_2 g d = 0$$

$$\frac{1}{2}(m_1+m_2)(v^2-v_i^2)$$
$$+\frac{1}{2}\left[\frac{1}{2}M(R_1^2+R_2^2)\right]\left(\frac{1}{R_2}\right)^2\left[v^2-v_i^2\right]$$
$$+m_1g(-d)+\mu m_2gd=0$$

$$\frac{1}{2}(m_1+m_2)(v^2-v_i^2)$$
$$+\frac{1}{2}\left[\frac{1}{2}M\left(1+\frac{R_1^2}{R_2^2}\right)\right]\left[v^2-v_i^2\right]=gd(m_1-\mu m_2)$$

$$\frac{1}{2}\left[(m_1+m_2)+\frac{1}{2}M\left(1+\frac{R_1^2}{R_2^2}\right)\right](v^2-v_i^2)=gd(m_1-\mu m_2)$$

$$v=\left\{v_i^{\,2}+\frac{4gd(m_1-\mu_k m_2)}{2(m_1+m_2)+M\left(1+\frac{R_1^{\,2}}{R_2^{\,2}}\right)}\right\}^{1/2}$$

Suppressing units,

$$v=\left\{(0.820)^2+\frac{4(9.80)(0.700)\left[0.420-(0.250)(0.850)\right]}{\left[2(0.420+0.850)+0.350\left(1+\frac{(0.020\ 0)^2}{(0.030\ 0)^2}\right)\right]}\right\}^{1/2}$$
$$=\boxed{1.59\ \text{m/s}}$$

(b) $$\omega=\frac{v}{r}=\frac{1.59\ \text{m/s}}{0.030\ 0\ \text{m}}=\boxed{53.1\ \text{rad/s}}$$

P10.54 (a) For the isolated rod-ball-Earth system,

$$\Delta K+\Delta U=0\ \rightarrow\ (K_f-0)+(0-U_i)=0\ \rightarrow\ K_f=U_i$$
$$K_f=m_{\text{rod}}gy_{\text{CM, rod}}+m_{\text{ball}}gy_{\text{CM, ball}}$$
$$=(m_{\text{rod}}y_{\text{CM, rod}}+m_{\text{ball}}y_{\text{CM, ball}})g$$
$$=\left[(1.20\ \text{kg})(0.120\ \text{m})+(2.00\ \text{kg})(0.280\ \text{m})\right](9.80\ \text{m/s}^2)$$
$$=\boxed{6.90\ \text{J}}$$

(b) We assume the rod is thin. For the compound object

$$I = \frac{1}{3}M_{rod}L^2 + \left[\frac{2}{5}m_{ball}R^2 + M_{ball}D^2\right]$$

$$= \frac{1}{3}(1.20 \text{ kg})(0.240 \text{ m})^2$$

$$+\frac{2}{5}(2.00 \text{ kg})(4.00\times10^{-2} \text{ m})^2 + (2.00 \text{ kg})(0.280 \text{ m})^2$$

$$I = 0.181 \text{ kg}\cdot\text{m}^2$$

$$K_f = \frac{1}{2}\omega^2 \quad\rightarrow\quad \omega = \sqrt{\frac{2K_f}{I}} = \sqrt{\frac{2(6.90 \text{ J})}{0.181 \text{ kg}\cdot\text{m}^2}} = \boxed{8.73 \text{ rad/s}}$$

(c) $v = r\omega = (0.280 \text{ m})(8.73 \text{ rad/s}) = \boxed{2.44 \text{ m/s}}$

(d) $v_f^2 = v_i^2 + 2a(y_f - y_i)$

$$v_f = \sqrt{0 + 2(9.80 \text{ m/s}^2)(0.280 \text{ m})} = 2.34 \text{ m/s}$$

$\boxed{\text{The speed it attains in swinging is greater by}}$

$$\frac{2.44}{2.34} = \boxed{1.0432 \text{ times}}$$

P10.55 The gravitational force exerted on the reel is

$$mg = (5.10 \text{ kg})(9.80 \text{ m/s}^2) = 50.0 \text{ N down}$$

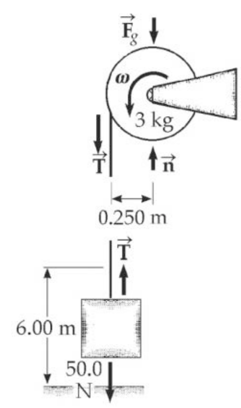
We use $\sum\tau = I\alpha$ to find T and a.

First find I for the reel, which we know is a uniform disk.

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(3.00 \text{ kg})(0.250 \text{ m})^2$$

$$= 0.093\ 8 \text{ kg}\cdot\text{m}^2$$

The forces on the reel are shown in ANS. FIG. P10.55, including a normal force exerted by its axle. From the diagram, we can see that the tension is the only force that produces a torque causing the reel to rotate.



ANS. FIG. P10.55

$\sum\tau = I\alpha$ becomes

$$n(0) + F_{gp}(0) + T(0.250 \text{ m}) = (0.093\ 8 \text{ kg}\cdot\text{m}^2)(a/0.250 \text{ m}) \qquad \textbf{[1]}$$

where we have applied $a_t = r\alpha$ to the point of contact between string

and reel. For the object that moves down,

$$\sum F_y = ma_y \qquad \text{becomes} \qquad 50.0\ \text{N} - T = (5.10\ \text{kg})a \qquad \textbf{[2]}$$

Note that we have defined downwards to be positive, so that positive linear acceleration of the object corresponds to positive angular acceleration of the reel. We now have our two equations in the unknowns T and a for the two connected objects. Substituting T from equation [2] into equation [1], we have

$$\left[50.0\ \text{N} - (5.10\ \text{kg})a\right](0.250\ \text{m}) = \left(0.093\ 8\ \text{kg}\cdot\text{m}^2\right)\left(\frac{a}{0.250\ \text{m}}\right)$$

(b) Solving for a from above gives

$$50.0\ \text{N} - (5.10\ \text{kg})a = (1.50\ \text{kg})a$$

$$a = \frac{50.0\ \text{N}}{6.60\ \text{kg}} = \boxed{7.57\ \text{m/s}^2}$$

Because we eliminated T in solving the simultaneous equations, the answer for a, required for part (b), emerged first. No matter—we can now substitute back to get the answer to part (a).

(a) $T = 50.0\ \text{N} - 5.10\ \text{kg}\ (7.57\ \text{m/s}^2) = \boxed{11.4\ \text{N}}$

(c) For the motion of the hanging weight,

$$v_f^2 = v_i^2 + 2a(y_f - y_i) = 0^2 + 2(7.57\ \text{m/s}^2)(6.00\ \text{m})$$

$$v_f = 9.53\ \text{m/s (down)}$$

(d) The isolated-system energy model can take account of multiple objects more easily than Newton's second law. Like your bratty cousins, the equation for conservation of energy grows between visits. Now it reads for the counterweight-reel-Earth system:

$$(K_1 + K_2 + U_g)_i = (K_1 + K_2 + U_g)_f$$

where K_1 is the translational kinetic energy of the falling object and K_2 is the rotational kinetic energy of the reel.

$$0 + 0 + m_1 g y_{1i} = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}I_2\omega_{2f}^2 + 0$$

Now note that $\omega = v/r$ as the string unwinds from the reel.

$$mgy_i = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$2mgy_i = mv^2 + I\left(\frac{v^2}{R^2}\right) = v^2\left(m + \frac{I}{R^2}\right)$$

$$v = \sqrt{\frac{2mgy_i}{m + \left(I/R^2\right)}} = \sqrt{\frac{2(5.10\text{ kg})(9.80\text{ m/s}^2)(6.00\text{ m})}{5.10\text{ kg} + \dfrac{0.093\ 8\text{ kg}\cdot\text{m}^2}{(0.250\text{ m})^2}}}$$

$$= \boxed{9.53\text{ m/s}}$$

P10.56 Each point on the cord moves at a linear speed of $v = \omega r$, where r is the radius of the spool. The energy conservation equation for the counterweight-turntable-Earth system is:

$$(K_1 + K_2 + U_g)_i + W_{\text{other}} = (K_1 + K_2 + U_g)_f$$

Specializing, we have

$$0 + 0 + mgh + 0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + 0$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}$$

$$2mgh - mv^2 = I\frac{v^2}{r^2}$$

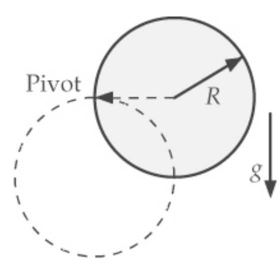
and finally,

$$I = \boxed{mr^2\left(\frac{2gh}{v^2} - 1\right)}$$

P10.57 To identify the change in gravitational energy, think of the height through which the center of mass falls. From the parallel-axis theorem, the moment of inertia of the disk about the pivot point on the circumference is

$$I = I_{\text{CM}} + MD^2 = \frac{1}{2}MR^2 + MR^2$$

$$= \frac{3}{2}MR^2$$



ANS. FIG. P10.57

The pivot point is fixed, so the kinetic energy is entirely rotational around the pivot. The equation for the isolated system (energy) model

$$(K + U)_i = (K + U)_f$$

for the disk-Earth system becomes

$$0 + \mathsf{MgR} = \frac{1}{2}\left(\frac{3}{2}\mathsf{MR}^2\right)\omega^2 + 0$$

Solving for ω, $$\omega = \sqrt{\frac{4\mathsf{g}}{3\mathsf{R}}}$$

(a) At the center of mass, $\mathsf{v} = \mathsf{R}\omega = \boxed{2\sqrt{\frac{\mathsf{Rg}}{3}}}$

(b) At the lowest point on the rim, $\mathsf{v} = 2\mathsf{R}\omega = \boxed{4\sqrt{\frac{\mathsf{Rg}}{3}}}$

(c) For a hoop,

$$\mathsf{I}_{\text{CM}} = \mathsf{MR}^2 \quad \text{and} \quad \mathsf{I}_{\min} = 2\mathsf{MR}^2$$

By conservation of energy for the hoop-Earth system, then

$$\mathsf{MgR} = \frac{1}{2}\left(2\mathsf{MR}^2\right)\omega^2 + 0$$

so $$\omega = \sqrt{\frac{\mathsf{g}}{\mathsf{R}}}$$

and the center of mass moves at $\mathsf{v}_{\text{CM}} = \mathsf{R}\omega = \boxed{\sqrt{\mathsf{gR}}}$, slower than the disk.

P10.58 (a) The moment of inertia of the cord on the spool is

$$\frac{1}{2}\mathsf{M}\left(\mathsf{R}_1^2 + \mathsf{R}_2^2\right) = \frac{1}{2}(0.100\text{ kg})\left[(0.015\ 0\text{ m})^2 + (0.090\ 0\text{ m})^2\right]$$
$$= 4.16 \times 10^{-4}\text{ kg}\cdot\text{m}^2$$

The protruding strand has mass

$$\left(1.00 \times 10^{-2}\text{ kg/m}\right)(0.160\text{ m}) = 1.60 \times 10^{-3}\text{ kg}$$

and moment of inertia

$$\mathsf{I} = \mathsf{I}_{\text{CM}} + \mathsf{Md}^2 = \frac{1}{12}\mathsf{ML}^2 + \mathsf{Md}^2$$
$$= \left(1.60 \times 10^{-3}\text{ kg}\right)\left[\frac{1}{12}(0.160\text{ m})^2 + (0.090\,0\text{ m} + 0.080\,0\text{ m})^2\right]$$
$$= 4.97 \times 10^{-5}\text{ kg}\cdot\text{m}^2$$

For the whole cord, $I = 4.66 \times 10^{-4}\ \text{kg}\cdot\text{m}^2$. In speeding up, the average power is

$$P = \frac{E}{\Delta t} = \frac{\frac{1}{2}I\omega^2}{\Delta t} = \left[\frac{4.66\times 10^{-4}\ \text{kg}\cdot\text{m}^2}{2(0.215\ \text{s})}\right]\left(\frac{2\,500\cdot 2\pi}{60\ \text{s}}\right)^2 = \boxed{74.3\ \text{W}}$$

(b) $$P = \tau\omega = (7.65\ \text{N})(0.160\ \text{m} + 0.090\,0\ \text{m})\left(\frac{2\,000\cdot 2\pi}{60\ \text{s}}\right) = \boxed{401\ \text{W}}$$

Section 10.9 Rolling Motion of a Rigid Object

P10.59 (a) The kinetic energy of translation is

$$K_{\text{trans}} = \frac{1}{2}mv^2 = \frac{1}{2}(10.0\ \text{kg})(10.0\ \text{m/s})^2 = \boxed{500\ \text{J}}$$

(b) Call the radius of the cylinder *r*. An observer at the center sees the rough surface and the circumference of the cylinder moving at 10.0 m/s, so the angular speed of the cylinder is

$$\omega = \frac{v_{\text{CM}}}{r} = \frac{10.0\ \text{m/s}}{r}$$

The moment of inertia about an axis through the center of mass is $I_{\text{CM}} = \frac{1}{2}mr^2$, so

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{4}(10.0\ \text{kg})(10.0\ \text{m/s})^2$$
$$= \boxed{250\ \text{J}}$$

(c) We can now add up the total energy:

$$K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \boxed{750\ \text{J}}$$

P10.60 Conservation of energy for the sphere rolling without slipping is

$$U_i = K_{\text{translation, f}} + K_{\text{rotation, f}}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{7}{10}mv^2$$

which gives $\boxed{v_f = \sqrt{\frac{10}{7}gh}}$

Conservation of energy for the sphere sliding without friction, with $\omega = 0$, is

$$mgh = \frac{1}{2}mv^2$$

which gives $\boxed{v_f = \sqrt{2gh}}$

The time intervals required for the trips follow from $x = 0 + v_{avg}t$:

$$\frac{h}{\sin\theta} = \left(\frac{0 + v_f}{2}\right)t \quad \rightarrow \quad t = \frac{2h}{v_f \sin\theta}$$

For rolling we have $t = \left(\dfrac{2h}{\sin\theta}\right)\sqrt{\dfrac{10}{7}gh}$

and for sliding, $t = \left(\dfrac{2h}{\sin\theta}\right)\sqrt{\dfrac{1}{2}gh}$

$\boxed{\text{The time to roll is longer by a factor of } (0.7/0.5)^{1/2} = 1.18.}$

P10.61 (a) We can consider the weight force acting at the center of mass (gravity) to exert a torque about the point of contact (the axis, in this case) between the disk and the incline. Then, from the particle under a net torque model, we have

$$\tau = I\alpha \quad \text{and} \quad a = R\alpha$$

$$mgR\sin\theta = \left(I_{CM} + mR^2\right)\alpha$$

$$a = \frac{mgR^2\sin\theta}{I_{CM} + mR^2}$$

$$a_{disk} = \frac{mgR^2\sin\theta}{\frac{3}{2}mR^2} = \boxed{\frac{2}{3}g\sin\theta}$$

(b) By the same method,

$$a = \frac{mgR^2\sin\theta}{I_{CM} + mR^2}$$

$\boxed{a_{hoop} = \dfrac{mgR^2\sin\theta}{2mR^2} = \dfrac{1}{2}g\sin\theta\text{. The acceleration of the hoop is smaller than that of the disk.}}$

(c) Torque about the CM is caused by friction because the lever arm of the weight force is zero:

$$\tau = fR = I\alpha$$

$$f = \mu n = \mu mg\cos\theta$$

$$\mu = \frac{f}{mg\cos\theta} = \frac{I\alpha/R}{mg\cos\theta} = \frac{\left(\frac{2}{3}g\sin\theta\right)\left(\frac{1}{2}mR^2\right)}{R^2 mg\cos\theta} = \boxed{\frac{1}{3}\tan\theta}$$

P10.62 (a) Both systems of cube-Earth and cylinder-Earth are isolated; therefore, mechanical energy is conserved in both. $\boxed{\text{The cylinder}}$ has extra kinetic energy, in the form of rotational kinetic energy, that is available to be transformed into potential energy, so it travels farther up the incline.

(b) The system of cube-Earth is isolated, so mechanical energy is conserved:

$$K_i = U_f \quad \rightarrow \quad \frac{1}{2}mv^2 = mgd\sin\theta \quad \rightarrow \quad d = \frac{v^2}{2g\sin\theta}$$

Static friction does no work on the cylinder because it acts at the point of contact and not through a distance; therefore, mechanical energy is conserved in the cylinder-Earth system:

$$K_{\text{translation, i}} + K_{\text{rotation, i}} = U_f \rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\left[\frac{1}{2}mr^2\right]\left(\frac{v}{r}\right)^2 = mgd\sin\theta$$

which gives $d = \dfrac{3v^2}{4g\sin\theta}$.

The difference in distance is

$$\frac{3v^2}{4g\sin\theta} - \frac{v^2}{2g\sin\theta} = \boxed{\frac{v^2}{4g\sin\theta}}$$

or, the cylinder travels 50% farther.

(c) $\boxed{\text{The cylinder does not lose mechanical energy because static friction does no work on it. Its rotation means that it has 50\% more kinetic energy than the cube at the start, and so it travels 50\% farther up the incline.}}$

P10.63 (a) $\boxed{\text{The disk reaches the bottom first}}$ because the ratio of its moment of inertia to its mass is smaller than for the hoop; this result is independent of the radius.

(b) Both systems of disk-Earth and hoop-Earth are isolated because static friction does no work because it acts at the point of contact and not through a distance. Mechanical energy is conserved in both systems:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2$$

where $\omega = \dfrac{v}{R}$ since no slipping occurs.

Also, $U_i = mgh, U_f = 0,$ and $v_i = 0$

Therefore, $\dfrac{1}{2}\left[m + \dfrac{I}{R^2}\right]v^2 = mgh$

Thus, $v^2 = \dfrac{2gh}{\left[1 + \left(I/mR^2\right)\right]}$

For a disk, $I = \dfrac{1}{2}mR^2$, so $v^2 = \dfrac{2gh}{1+\frac{1}{2}}$ or $\boxed{v_{\text{disk}} = \sqrt{\dfrac{4gh}{3}}}$

For a hoop, $I = mR^2$ so $v^2 = \dfrac{2gh}{2}$ or $\boxed{v_{\text{hoop}} = \sqrt{gh}}$

Since $v_{\text{disk}} > v_{\text{hoop}}$, $\boxed{\text{the disk}}$ reaches the bottom first.

P10.64 (a) Energy conservation for the system of the ball and the Earth between the horizontal section and top of loop:

$$\frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2$$

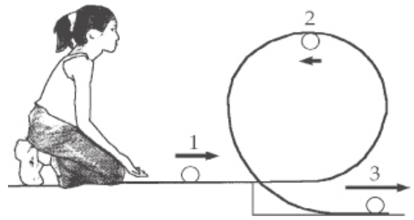
$$\frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_2}{r}\right)^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$\frac{5}{6}v_2^2 + gy_2 = \frac{5}{6}v_1^2$$

$$v_2 = \sqrt{v_1^2 - \frac{6}{5}gy_2}$$

$$= \sqrt{(4.03 \text{ m/s})^2 - \frac{6}{5}(9.80 \text{ m/s}^2)(0.900 \text{ m})}$$

$$= \boxed{2.38 \text{ m/s}}$$



ANS. FIG. P10.64

(b) The centripetal acceleration at the top is

$$\frac{v_2^2}{r} = \frac{(2.38\text{ m/s})^2}{0.450\text{ m}} = 12.6\text{ m/s}^2 > g$$

Thus, the ball must be in contact with the track, with the track pushing downward on it.

(c) $$\frac{1}{2}mv_3^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_3}{r}\right)^2 + mgy_3 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$v_3 = \sqrt{v_1^2 - \frac{6}{5}gy_3}$$
$$= \sqrt{(4.03\text{ m/s})^2 - \frac{6}{5}(9.80\text{ m/s}^2)(-0.200\text{ m})}$$
$$= \boxed{4.31\text{ m/s}}$$

(d) $$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2$$

$$v_2 = \sqrt{v_1^2 - 2gy_2} = \sqrt{(4.03\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(0.900\text{ m})}$$
$$= \boxed{\sqrt{-1.40\text{ m}^2/\text{s}^2}}\,!$$

(e) This result is imaginary. In the case where the ball does not roll, the ball starts with less kinetic energy than in part (a) and $\boxed{\text{never makes it to the top of the loop}}$.

P10.65 (a) For the isolated can-Earth system,

$$\Delta K + \Delta U = 0 \quad\rightarrow\quad \left(\frac{1}{2}mv_{CM}^2 + \frac{1}{2}I\omega^2 - 0\right) + (0 - mgh) = 0$$

which gives

$$I = \frac{2mgh - mv_{CM}^2}{\omega^2} = \left(2mgh - mv_{CM}^2\right)\left(\frac{r^2}{v_{CM}^2}\right) = mr^2\left(\frac{2gh}{v_{CM}^2} - 1\right)$$

From the particle under constant acceleration model,

$$v_{\text{CM, avg}} = \frac{0 + v_{\text{CM}}}{2} \rightarrow v_{\text{CM}} = 2v_{\text{CM, avg}} = \frac{2d}{\Delta t}$$

Therefore, the moment of inertia is

$$I = mr^2\left(\frac{2gh(\Delta t)^2}{4d^2} - 1\right) = mr^2\left(\frac{2g(d\sin\theta)(\Delta t)^2}{4d^2} - 1\right)$$

$$= mr^2\left(\frac{g(\sin\theta)(\Delta t)^2}{2d} - 1\right)$$

Substitute numerical values:

$$I = (0.215\text{ kg})(0.031\ 9\text{ m})^2 \times \left(\frac{(9.80\text{ m/s}^2)(\sin 25.0°)(1.50\text{ s})^2}{2(3.00\text{ m})} - 1\right)$$

$$= \boxed{1.21 \times 10^{-4}\text{ kg}\cdot\text{m}^2}$$

(b) The $\boxed{\text{height of the can}}$ is unnecessary data.

(c) $\boxed{\text{The mass is not uniformly distributed; the density of the metal can is larger than that of the soup.}}$

Additional Problems

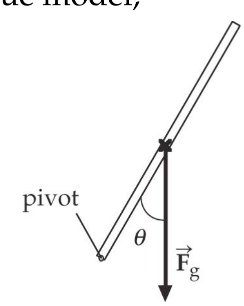
P10.66 When the rod is at angle θ from the vertical, the vertical weight force *mg* is at the same angle from the vertical so that its torque about the pivot is $mg\frac{\ell}{2}\sin\theta$. From the particle under a net torque model,

$$\sum \tau = I\alpha$$

$$mg\frac{\ell}{2}\sin\theta = \frac{1}{3}m\ell^2\alpha$$

$$\alpha = \frac{3}{2}\frac{g}{\ell}\sin\theta \rightarrow a_t = \left(\frac{3}{2}\frac{g}{\ell}\sin\theta\right)r$$

For $\left(\frac{3}{2}\frac{g}{\ell}\sin\theta\right)r > g\sin\theta \rightarrow r > \frac{2}{3}\ell$



ANS. FIG. P10.66

$\therefore$ About $\boxed{\frac{1}{3}\text{ the length of the chimney}}$ will have a tangential acceleration greater than $g\sin\theta$.

P10.67 (a) The spool starts from rest, with zero rotational kinetic energy, and accelerates to 8.00 rad/s. The work done to accomplish this is given by the work-kinetic energy theorem:

$$W = \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = \frac{1}{2}I\left(\omega_f^2 - \omega_i^2\right), \quad \text{where} \quad I = \frac{1}{2}mR^2$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1.00\text{ kg})(0.500\text{ m})^2\left[(8.00\text{ rad/s})^2 - 0\right] = \boxed{4.00\text{ J}}$$

(b) The time interval can be found from

$$\omega_f = \omega_i + \alpha t, \quad \text{where} \quad \alpha = \frac{a}{r} = \frac{2.50\text{ m/s}^2}{0.500\text{ m}} = 5.00\text{ rad/s}^2$$

Therefore,

$$t = \frac{\omega_f - \omega_i}{\alpha} = \frac{8.00\text{ rad/s} - 0}{5.00\text{ rad/s}^2} = \boxed{1.60\text{ s}}$$

(c) The spool turns through angular displacement

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$= 0 + 0 + \frac{1}{2}(5.00\text{ rad/s}^2)(1.60\text{ s})^2 = 6.40\text{ rad}$$

The length pulled from the spool is

$$s = r\theta = (0.500\text{ m})(6.40\text{ rad}) = 3.20\text{ m}$$

When the spool reaches an angular velocity of 8.00 rad/s, 1.60 s will have elapsed and 3.20 m of cord will have been removed from the spool. Remaining on the spool will be $\boxed{0.800\text{ m}}$.

P10.68 (a) We consider the elevator-sheave-counterweight-Earth system, including n passengers, as an isolated system and apply the conservation of mechanical energy. We take the initial configuration, at the moment the drive mechanism switches off, as representing zero gravitational potential energy of the system.

Therefore, the initial mechanical energy of the system [elevator (e), counterweight (c), sheave (s)] is

$$E_i = K_i + U_i = \frac{1}{2}m_e v^2 + \frac{1}{2}m_c v^2 + \frac{1}{2}I_s\omega^2 + 0$$
$$= \frac{1}{2}m_e v^2 + \frac{1}{2}m_c v^2 + \frac{1}{2}\left[\frac{1}{2}m_s r^2\right]\left(\frac{v}{r}\right)^2$$
$$= \frac{1}{2}\left[m_e + m_c + \frac{1}{2}m_s\right]v^2$$

The final mechanical energy of the system is entirely gravitational because the system is momentarily at rest:

$$E_f = K_f + U_f = 0 + m_e gd - m_c gd$$

where we have recognized that the elevator car goes up by the same distance d that the counterweight goes down. Setting the initial and final energies of the system equal to each other, we have

$$\frac{1}{2}\left[m_e + m_c + \frac{1}{2}m_s\right]v^2 = (m_e - m_c)gd$$

$$\frac{1}{2}\{[800\text{ kg} + n(80.0\text{ kg})] + 950\text{ kg} + 140\text{ kg}\}(3.00\text{ m/s})^2$$
$$= [800\text{ kg} + n(80.0\text{ kg}) - 950\text{ kg}](9.80\text{ m/s}^2)d$$

$$\boxed{d = (1890 + 80n)\left(\frac{0.459\text{ m}}{80n - 150}\right)}$$

(b) For $n = 2$: $d = (1890 + 80.0\times 2)\dfrac{0.459\text{ m}}{(80.0\times 2 - 150)} = \boxed{94.1\text{ m}}$

(c) For $n = 12$: $d = (1890 + 80.0\times 12)\dfrac{0.459\text{ m}}{(80.0\times 12 - 150)} = \boxed{1.62\text{ m}}$

(d) For $n = 0$: $d = (1890 + 80.0\times 0)\dfrac{0.459\text{ m}}{(80.0\times 0 - 150)} = \boxed{-5.79\text{ m}}$

(e) The raising car will coast to a stop only for $n \geq 2$.

(f) For $n = 0$ or $n = 1$, the mass of the elevator is less than the counterweight, so the car would accelerate upward if released.

(g) For $n \to \infty$, $d \to 80n(0.459\text{ m})/(80n) = \boxed{0.459\text{ m}}$

P10.69 (a) We find the angular speed by integrating the angular acceleration, which is given as $\alpha = -10.0 - 5.00t = \dfrac{d\omega}{dt}$, where α is in rad/s^2 and *t* is in seconds:

$$\Delta\omega = \int_{65.0}^{\omega} d\omega = \int_0^t [-10.0 - 5.00t]\,dt$$

$$\omega - 65.0 = -10.0t - 2.50t^2 \rightarrow \omega = 65.0 - 10.0t - 2.50t^2$$

where ω is in rad/s and *t* is in seconds.

For *t* = 3.00 s: $\omega = 65.0 - 10.0(3.00) - 2.50(3.00)^2 = \boxed{12.5\text{ rad/s.}}$

(b) $\omega = \dfrac{d\theta}{dt} = 65.0\text{ rad/s} - (10.0\text{ rad/s}^2)t - (2.50\text{ rad/s}^3)t^2$

Suppressing units,

$$\Delta\theta = \int_0^t \omega\,dt = \int_0^t [65.0 - 10.0t - 2.50t^2]\,dt$$

$$\Delta\theta = 65.0t - 5.00t^2 - (2.50/3)t^3$$

$$\Delta\theta = 65.0t - 5.00t^2 - 0.833t^3$$

At *t* = 3.00 s,

$$\Delta\theta = (65.0\text{ rad/s})(3.00\text{ s}) - (5.00\text{ rad/s}^2)(9.00\text{ s}^2) - (0.833\text{ rad/s}^3)(27.0\text{ s}^3)$$

$$\Delta\theta = \boxed{128\text{ rad}}$$

P10.70 (a) We find the angular speed by integrating the angular acceleration, which is given as $\alpha(t) = A + Bt = \dfrac{d\omega}{dt}$, where the shaft is turning at angular speed ω at time *t* = 0.

$$\Delta\omega = \int_{\omega(0)}^{\omega(t)} d\omega = \int_0^t [A + Bt]\,dt$$

$$\omega(t) - \omega(0) = At + \frac{1}{2}Bt^2,\text{ and } \omega(0) = \omega \rightarrow \boxed{\omega(t) = \omega + At + \frac{1}{2}Bt^2}$$

(b) $\dfrac{d\theta}{dt} = \omega + At + \dfrac{1}{2}Bt^2$

$$\Delta\theta = \int_0^t \omega(t)\,dt = \int_0^t \left[\omega + At + \frac{1}{2}Bt^2\right]dt$$

$$\Delta\theta = \boxed{\omega t + \frac{1}{2}At^2 + \frac{1}{6}Bt^3}$$

***P10.71** The resistive force on each ball is $R = D\rho A v^2$. Here $v = r\omega$, where *r* is the radius of each ball's path. The resistive torque on each ball is $\tau = rR$, so the total resistive torque on the three-ball system is $\tau_{\text{total}} = 3rR$.

The power required to maintain a constant rotation rate is $P = \tau_{\text{total}}\omega = 3rR\omega$. This required power may be written as

$$P = \tau_{\text{total}}\omega = 3r\left[D\rho A(r\omega)^2\right]\omega = \left(3r^3 DA\omega^3\right)\rho$$

with

$$\omega = \frac{2\pi\ \text{rad}}{1\ \text{rev}}\left(\frac{10^3\ \text{rev}}{1\ \text{min}}\right)\left(\frac{1\ \text{min}}{60.0\ \text{s}}\right) = \frac{1\,000\pi}{30.0}\ \text{rad/s}$$

Then

$$P = 3(0.100\ \text{m})^3(0.600)\left(4.00\times10^{-4}\ \text{m}^2\right)\left(\frac{1\,000\pi}{30.0\ \text{s}}\right)^3\rho$$

or $\quad P = \left(0.827\ \text{m}^5/\text{s}^3\right)\rho$, where ρ is the density of the resisting medium.

(a) In air, $\rho = 1.20\ \text{kg/m}^3$, and

$$P = \left(0.827\ \text{m}^5/\text{s}^3\right)\left(1.20\ \text{kg/m}^3\right) = 0.992\ \text{N}\cdot\text{m/s} = \boxed{0.992\ \text{W}}$$

(b) In water, $\rho = 1\,000\ \text{kg/m}^3$ and $P = \boxed{827\ \text{W}}$.

***P10.72** Consider the total weight of each hand to act at the center of gravity (midpoint) of that hand. Then the total torque (taking CCW as positive) of these hands about the center of the clock is given by

$$\tau = -m_h g\left(\frac{L_h}{2}\right)\sin\theta_h - m_m g\left(\frac{L_m}{2}\right)\sin\theta_m$$
$$= -\frac{g}{2}\left(m_h L_h \sin\theta_h + m_m L_m \sin\theta_m\right)$$

If we take *t* = 0 at 12 o'clock, then the angular positions of the hands at time *t* are $\theta_h = \omega_h t$, where $\omega_h = \frac{\pi}{6}$ rad/h and $\theta_m = \omega_m t$, where $\omega_m = 2\pi$ rad/h. Therefore,

$$\tau = \left(-4.90\ \text{m/s}^2\right)$$
$$\times\left[\left(60.0\ \text{kg}\right)\left(2.70\ \text{m}\right)\sin\left(\frac{\pi t}{6}\right)+\left(100\ \text{kg}\right)\left(4.50\ \text{m}\right)\sin 2\pi t\right]$$

or $\tau = \left(-794\ \text{N}\cdot\text{m}\right)\left[\sin\left(\frac{\pi t}{6}\right)+2.78\sin 2\pi t\right]$, where t is in hours.

(a) (i) At 3:00, $t = 3.00$ h, so

$$\tau = \left(-794\ \text{N}\cdot\text{m}\right)\left[\sin\left(\frac{\pi}{2}\right)+2.78\sin 6\pi\right] = \boxed{-794\ \text{N}\cdot\text{m}}$$

(ii) At 5:15, $t = 5\ \text{h} + \frac{15}{60}\ \text{h} = 5.25\ \text{h}$, and substitution gives:

$$\tau = \boxed{-2\,510\ \text{N}\cdot\text{m}}$$

(iii) At 6:00, $\tau = \boxed{0\ \text{N}\cdot\text{m}}$

(iv) At 8:20, $\tau = \boxed{-1\,160\ \text{N}\cdot\text{m}}$

(v) At 9:45, $\tau = \boxed{2\,940\ \text{N}\cdot\text{m}}$

(b) The total torque is zero at those times when

$$\sin\left(\frac{\pi t}{6}\right)+2.78\sin 2\pi t = 0$$

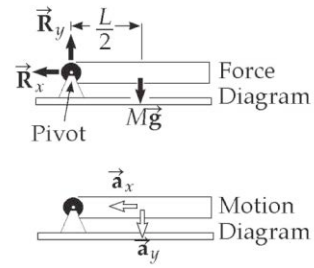
We proceed numerically, to find 0, 0.515 295 5, ..., corresponding to the times

12:00:00	12:30:55	12:58:19	1:32:31	1:57:01
2:33:25	2:56:29	3:33:22	3:56:55	4:32:24
4:58:14	5:30:52	6:00:00	6:29:08	7:01:46
7:27:36	8:03:05	8:26:38	9:03:31	9:26:35
10:02:59	10:27:29	11:01:41	11:29:05	

P10.73 (a) Since only conservative forces are acting on the bar, we have conservation of energy of the bar-Earth system:

$$K_i + U_i = K_f + U_f$$

For evaluation of the gravitational energy of the system, a rigid body can be modeled as a particle at its center of mass. Take the zero configuration for potential energy for the bar-Earth system with the bar horizontal.



ANS. FIG. P10.73

Under these conditions, $U_f = 0$ and $U_i = MgL/2$.

Using the conservation of energy equation above,

$$0 + \frac{1}{2}MgL = \frac{1}{2}I\omega_f^2 \quad \text{and} \quad \omega_f = \sqrt{MgL/I}$$

For a bar rotating about an axis through one end, $I = ML^2/3$.

Therefore,

$$\omega_f = \sqrt{\frac{MgL}{\frac{1}{3}ML^2}} = \boxed{\sqrt{\frac{3g}{L}}}$$

Note that we have chosen clockwise rotation as positive.

(b) $\sum \tau = I\alpha$: $\quad Mg\left(\frac{L}{2}\right) = \left(\frac{1}{3}ML^2\right)\alpha$ and $\alpha = \boxed{\frac{3g}{2L}}$

(c) $a_x = -a_c = -r\omega_f^2 = -\left(\frac{L}{2}\right)\left(\frac{3g}{L}\right) = -\frac{3g}{2}$

Since this is **centripetal** acceleration, it is directed along the **negative** horizontal.

$$a_y = -a_t = -r\alpha = \frac{L}{2}\alpha = -\frac{3g}{4}$$

$$\boxed{\vec{\mathbf{a}} = -\frac{3}{2}g\hat{\mathbf{i}} - \frac{3}{4}g\hat{\mathbf{j}}}$$

(d) The pivot exerts a force $\vec{\mathbf{F}}$ on the rod. Using Newton's second law, we find

$$F_x = Ma_x = -\frac{3}{2}Mg$$

$$F_y - Mg = Ma_y = -\frac{3}{4}Mg \to F_y = Mg - \frac{3}{4}Mg = \frac{1}{4}Mg$$

$$\boxed{\vec{\mathbf{F}} = M\vec{\mathbf{a}} = -\frac{3}{2}Mg\hat{\mathbf{i}} + \frac{1}{4}Mg\hat{\mathbf{j}}}$$

P10.74 We assume that air resistance has a negligible effect on a drop so that mechanical energy is conserved in the drop-Earth system. The first drop leaving the wheel has a velocity v_1 directed upward. The magnitude of this velocity is found from

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh_1$$

so $$v_1 = \sqrt{2gh_1} = \sqrt{2(9.80\text{ m/s}^2)(0.540\text{ m})} = 3.25\text{ m/s}$$

Similarly, the second drop has a velocity given by

$$v_2 = \sqrt{2gh_2} = \sqrt{2(9.80\text{ m/s}^2)(0.510\text{ m})} = 3.16\text{ m/s}$$

From $\omega = \frac{v}{r}$, we find

$$\omega_1 = \frac{v_1}{r} = \frac{3.25\text{ m/s}}{0.381\text{ m}} = 8.53\text{ rad/s}$$

and $$\omega_2 = \frac{v_2}{r} = \frac{3.16\text{ m/s}}{0.381\text{ m}} = 8.29\text{ rad/s}$$

or $$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\Delta\theta} = \frac{(8.29\text{ rad/s})^2 - (8.53\text{ rad/s})^2}{4\pi} = \boxed{-0.322\text{ rad/s}^2}$$

P10.75 We assume that air resistance has a negligible effect on a drop so that mechanical energy is conserved in the drop-Earth system. At the instant it comes off the wheel, the first drop has a velocity v_1 directed upward. The magnitude of this velocity is found from

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh_1 \text{ or } v_1 = \sqrt{2gh_1}$$

The angular velocity of the wheel at the instant the first drop leaves is

$$\omega_1 = \frac{v_1}{R} = \sqrt{\frac{2gh_1}{R^2}}$$

Similarly for the second drop: $v_2 = \sqrt{2gh_2}$ and $\omega_2 = \frac{v_2}{R} = \sqrt{\frac{2gh_2}{R^2}}$

The angular acceleration of the wheel is then

$$a = \frac{\omega_2^2 - \omega_1^2}{2\Delta\theta} = \frac{2gh_2/R^2 - 2gh_1/R^2}{2(2\pi)} = \boxed{\frac{g(h_2 - h_1)}{2\pi R^2}}$$

P10.76 (a) Modeling the Earth as a sphere, its rotational kinetic energy is

$$K = \frac{1}{2}\left(\frac{2}{5}MR^2\right)(\omega^2)$$

$$= \frac{1}{2}\left[\frac{2}{5}(5.98\times10^{24}\text{ kg})(6.37\times10^6\text{ m})^2\right]\left(\frac{2\pi}{86\,400\text{ s}}\right)^2$$

$$= \boxed{2.57\times10^{29}\text{ J}}$$

(b) The change in rotational kinetic energy is found by differentiating the equation for rotational kinetic energy with respect to time:

$$\frac{dK}{dt} = \frac{d}{dt}\left[\frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right)^2\right]$$

$$= \frac{1}{5}MR^2(2\pi)^2(-2T^{-3})\frac{dT}{dt}$$

$$= \frac{1}{5}MR^2\left(\frac{2\pi}{T}\right)^2\left(\frac{-2}{T}\right)\frac{dT}{dt} = K\left(\frac{-2}{T}\right)\frac{dT}{dt}$$

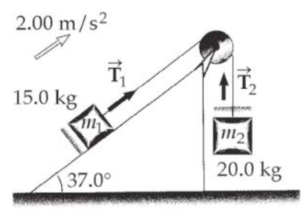
Substituting,

$$\frac{dK}{dt} = (2.57\times10^{29}\text{ J})\left(\frac{-2}{86\,400\text{ s}}\right)\left(\frac{10\times10^{-6}\text{ s}}{3.16\times10^7\text{ s}}\right)(86\,400\text{ s/day})$$

$$= \boxed{-1.63\times10^{17}\text{ J/day}}$$

P10.77 (a) We apply the $\boxed{\text{particle under a net force}}$ model to each block.

(b) We apply the $\boxed{\text{rigid object under a net torque}}$ model to the pulley.



ANS. FIG. P10.77

(c) We use $\sum F = ma$ for each block to find each string tension. The forces acting on the 15-kg block are its weight, the normal support from the incline, and T_1. Taking the positive x axis as directed up the incline,

$$\sum F_x = ma_x \quad \text{yields:} \quad -(m_1 g)_x + T_1 = m_1(+a)$$

Solving and substituting known values, we have

$$\begin{aligned} T_1 &= m_1(+a) + (m_1 g)_x \\ &= (15.0 \text{ kg})(2.00 \text{ m/s}^2) + (15.0 \text{ kg})(9.80 \text{ m/s}^2)\sin 37.0° \\ &= \boxed{118 \text{ N}} \end{aligned}$$

(d) Similarly, for the counterweight, we have

$$\sum F_y = ma_y \quad \text{or} \quad T_2 - m_2 g = m_2(-a)$$

$$\begin{aligned} T_2 &= m_2 g + m_2(-a) \\ &= (20.0 \text{ kg})(9.80 \text{ m/s}^2) + (20.0 \text{ kg})(-2.00 \text{ m/s}^2) \\ &= \boxed{156 \text{ N}} \end{aligned}$$

(e) Now for the pulley,

$$\sum \tau = r(T_2 - T_1) = I\alpha = I\,a/r$$

so $$I = \frac{r^2}{a}(T_2 - T_1)$$

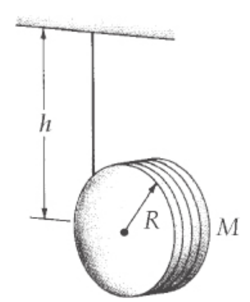
where we have chosen to call clockwise positive.

(f) Computing from above, the pulley's rotational inertia is

$$I = \frac{r^2}{a}(T_2 - T_1) = \frac{(156 \text{ N} - 118 \text{ N})(0.250 \text{ m})^2}{2.00 \text{ m/s}^2} = \boxed{1.17 \text{ kg} \cdot \text{m}^2}$$

P10.78 Choosing positive linear quantities to be downwards and positive angular quantities to be clockwise, $\sum F_y = ma_y$ yields

$$\sum F = Mg - TM = a \quad \text{or} \quad a = \frac{Mg - T}{M}$$



ANS. FIG. P10.78

$\sum \tau = I\alpha$ then becomes

$$\sum \tau = TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right) \quad \text{so} \quad a = \frac{2T}{M}$$

(a) Setting these two expressions equal,

$$\frac{Mg - T}{M} = \frac{2T}{M} \quad \text{and} \quad T = \boxed{Mg/3}$$

(b) Substituting back,

$$a = \frac{2T}{M} = \frac{2Mg}{3M} \quad \text{or} \quad a = \boxed{\frac{2}{3}g}$$

(c) Since $v_i = 0$ and $a = \frac{2}{3}g$, $v_f^2 = v_i^2 + 2ah$ gives us $v_f^2 = 0 + 2\left(\frac{2}{3}g\right)h$,

or $v_f = \boxed{\sqrt{4gh/3}}$

(d) Now we verify this answer. Requiring conservation of mechanical energy for the disk-Earth system, we have

$$U_i + K_{\text{rot},\,i} + K_{\text{trans},\,i} = U_f + K_{\text{rot},\,f} + K_{\text{trans},\,f}$$

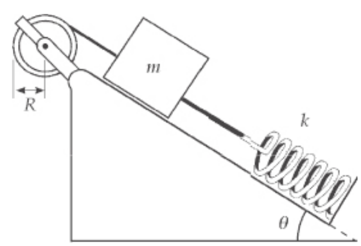
$$mgh + 0 + 0 = 0 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 + \frac{1}{2}Mv^2$$

When there is no slipping, $\omega = \frac{v}{R}$ and $v = \sqrt{\frac{4gh}{3}}$.

$\boxed{\text{The answer is the same.}}$

P10.79 The block and end of the spring are pulled a distance d up the incline and then released. The angular speed of the reel and the speed of the block are related by $v = \omega R$. The block-reel-Earth system is isolated, so



ANS. FIG. P10.79

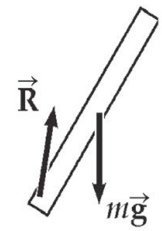
$$\Delta K + \Delta U = 0 \rightarrow K_f - K_i + U_f - U_i = 0$$

$$\left(\frac{1}{2}mv^2 - 0\right) + \left(\frac{1}{2}I\omega^2 - 0\right) + \left(0 - mgd\sin\theta\right) + \left(0 - \frac{1}{2}kd^2\right) = 0$$

$$\frac{1}{2}\omega^2\left(I + mR^2\right) = mgd\sin\theta + \frac{1}{2}kd^2$$

$$\boxed{\omega = \sqrt{\frac{2mgd\sin\theta + kd^2}{I + mR^2}}}$$

P10.80 The center of gravity of the uniform board is at its middle. For the board just starting to move,



ANS. FIG. P10.80

$$\sum \tau = I\alpha:$$

$$mg\left(\frac{\ell}{2}\right)\cos\theta = \left(\frac{1}{3}m\ell^2\right)\alpha$$

$$\alpha = \frac{3}{2}\left(\frac{g}{\ell}\right)\cos\theta$$

The tangential acceleration of the end is $a_t = \ell\alpha = \frac{3}{2}g\cos\theta$

and its vertical component is $a_y = a_t\cos\theta = \frac{3}{2}g\cos^2\theta$.

If this is greater than g, the board will pull ahead of the falling ball:

(a) $\frac{3}{2}g\cos^2\theta \ge g$ gives $\cos^2\theta \ge \frac{2}{3}$ so $\cos\theta \ge \sqrt{\frac{2}{3}}$ and $\boxed{\theta \le 35.3°}$

(b) When $\theta = 35.3°$, the cup will land underneath the release point of the ball if $r_c = \ell\cos\theta$.

When $\ell = 1.00$ m and $\theta = 35.3°$,

$$r_c = 1.00\text{ m}\sqrt{\frac{2}{3}} = 0.816\text{ m}$$

so the cup should be

$$\ell - r_c = 1.00\text{ m} - 0.816\text{ m} = \boxed{0.184\text{ m from the moving end}}$$

P10.81 For the isolated sphere-Earth system, energy is conserved, so

$$\Delta U + \Delta K_{rot} + \Delta K_{trans} = 0$$

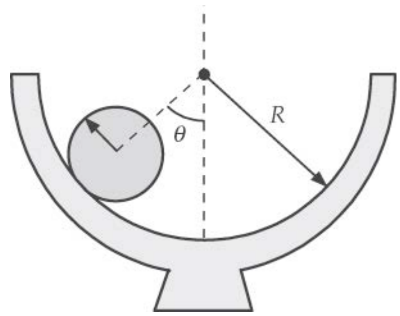
$$mg(R - r)(\cos\theta - 1) + \left[\frac{1}{2}mv^2 - 0\right] + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2 = 0$$

Substituting $v = r\omega$, we obtain

$$mg(R-r)(\cos\theta-1)+\left[\frac{1}{2}m(r\omega)^2-0\right]+\frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2=0$$

$$mg(R-r)(\cos\theta-1)+\left[\frac{1}{2}+\frac{1}{5}\right]mr^2\omega^2=0$$

$$\boxed{\omega=\sqrt{\left(\frac{10}{7}\right)\frac{(R-r)(1-\cos\theta)g}{r^2}}}$$



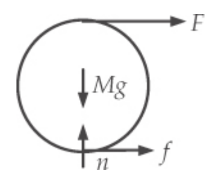
ANS. FIG. P10.81

P10.82 (a) From the particle under a net force model in the *x* direction, we have

$$\sum F_x = F + f = Ma_{CM}$$

From the particle under a net torque model,

$$\sum \tau = FR - fR = I\alpha$$



ANS. FIG. P10.82

Combining the two equations, and noting that $I=\frac{1}{2}MR^2$, gives

$$FR-(Ma_{CM}-F)R=\frac{Ia_{CM}}{R} \qquad \boxed{a_{CM}=\frac{4F}{3M}}$$

(b) Assuming friction is to the right, then

$$f + F = Ma_{CM} = M\left(\frac{4F}{3M}\right)$$

$$\rightarrow f = M\left(\frac{4F}{3M}\right)-F=\boxed{\frac{1}{3}F}$$

The facts that (1) we assumed that friction is to the right in Figure P10.82 and (2) our value for *f* comes out positive indicate that the friction force must indeed be to the right.

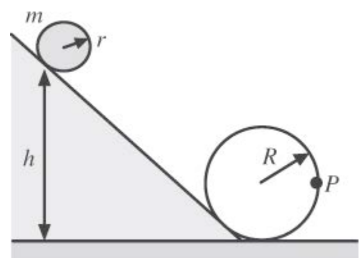
(c) From the kinematic equations,

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$
$$= 0 + 2ad$$

or

$$v_f = \sqrt{2ad} = \boxed{\sqrt{\frac{8Fd}{3M}}}$$

P10.83 (a) $\Delta K_{rot} + \Delta K_{trans} + \Delta U = 0$



ANS. FIG. P10.83

Note that initially the center of mass of the sphere is slightly higher than the distance *h* above the bottom of the loop; and as the mass reaches the top of the loop, this distance above the reference level is $2R - r$, but we are told that $r << R$, so we ignore *r* when considering heights for the gravitational potential energy of the sphere-Earth system. The conservation of energy requirement gives

$$mgh = mg(2R) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For the sphere $I = \frac{2}{5}mr^2$ and $v = r\omega$, so that the expression becomes

$$gh = 2gR + \frac{7}{10}v^2 \qquad \textbf{[1]}$$

Note that $h = h_{min}$ when the speed of the sphere at the top of the loop satisfies the condition

$$\sum F = mg = \frac{mv^2}{R} \quad \text{or} \quad v^2 = gR$$

Substituting this into equation [1] gives

$$h_{min} = 2R + 0.700R \text{ or } \boxed{h_{min} = 2.70R}$$

(b) When the sphere is initially at $h = 3R$ and finally at point *P*, the conservation of energy equation gives

$$mg3R = mgR + \frac{1}{2}mv^2 + \frac{1}{5}mv^2\text{, or } v^2 = \frac{20}{7}Rg$$

Turning clockwise as it rolls without slipping past point *P*, the sphere is slowing down with counterclockwise angular acceleration caused by the torque of an upward force *f* of static friction. We have

$$\sum F_y = ma_y \quad \rightarrow \quad f - mg = -m\alpha r$$

and $\sum \tau = I\alpha \rightarrow fr = \left(\frac{2}{5}\right)mr^2\alpha.$

Eliminating *f* by substitution yields

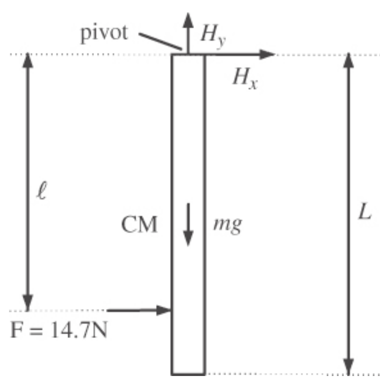
$$\alpha = \frac{5g}{7r} \text{ so that } \sum F_y = \boxed{-\frac{5}{7}mg}$$

P10.84 The length of the rod is *L*, and the horizontal force is applied the vertical distance *L* from the hinge. Consider the free-body diagram shown. The sum of torques about the chosen pivot is

$$\sum \tau = I\alpha \Rightarrow F = \left(\frac{1}{3}mL^2\right)\left(\frac{a_{CM}}{L/2}\right) = \left(\frac{2}{3}mL\right)a_{CM} \qquad [1]$$

(a) $\ell = L = 1.24$ m: In this case, equation [1] becomes

$$a_{CM} = \frac{3F}{2m} = \frac{3(14.7 \text{ N})}{2(0.630 \text{ kg})} = \boxed{35.0 \text{ m/s}^2}$$



ANS. FIG. P10.84

(b) We apply the particle under a net force model in the horizontal direction (see ANS. FIG. P10.84 for the labelling of forces):

$$\sum F_x = ma_{CM} \Rightarrow F + H_x = ma_{CM}$$

or $\quad H_x = ma_{CM} - F$

Thus,

$$H_x = (0.630 \text{ kg})(35.0 \text{ m/s}^2) - 14.7 \text{ N} = +7.35 \text{ N}$$

or $\vec{\mathbf{H}}_x = \boxed{7.35\hat{\mathbf{i}}\text{ N}}$

(c) With $\ell = \frac{1}{2}L = 0.620\text{ m}$, equation [1] yields

$$a_{CM} = \frac{3F}{4m} = \frac{3(14.7\text{ N})}{4(0.630\text{ kg})} = \boxed{17.5\text{ m/s}^2}$$

(d) Again, $\sum F_x = ma_{CM} \Rightarrow H_x = ma_{CM} - F$, so

$$H_x = (0.630\text{ kg})(17.5\text{ m/s}^2) - 14.7\text{ N} = -3.68\text{ N}$$

or $\mathbf{H}_x = \boxed{-3.68\hat{\mathbf{i}}\text{ N}}$

(e) If $H_x = 0$, then $\sum F_x = ma_{CM} \Rightarrow F = ma_{CM}$, or $a_{CM} = \frac{F}{m}$.

Thus, equation [1] becomes

$$F\ell = \left(\frac{2}{3}mL\right)\left(\frac{F}{m}\right)$$

so $\ell = \frac{2}{3}L = \frac{2}{3}(1.24\text{ m}) = \boxed{0.827\text{ m (from the top)}}$

P10.85 Note that when the CM of the falling rod is very near the surface, the velocity of the end of the rod in contact with the surface is a combination of the downward motion of the CM and the upward motion of the rotating end: $v_{end} = v_{CM} - \omega r$. Because the velocity of this end relative to the surface is zero,

$$v_{end} = v_{CM} - \omega(h/2) = 0 \rightarrow v_{CM} = \omega(h/2)t$$

(a) There are no horizontal forces acting on the rod, so the center of mass (CM) will not move horizontally. Rather, the center of mass drops straight downward (distance $h/2$) with the rod rotating about the center of mass as it falls.

From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}Mv_{CM}^2 + \frac{1}{2}\left(\frac{1}{12}Mh^2\right)\left(\frac{v_{CM}}{h/2}\right)^2 = Mg\left(\frac{h}{2}\right)$$

which reduces to

$$v_{CM} = \boxed{\sqrt{\frac{3gh}{4}}}$$

(b) In this case, the motion is a pure rotation about a fixed pivot point (the lower end of the rod) with the center of mass moving in a circular path of radius $h/2$. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

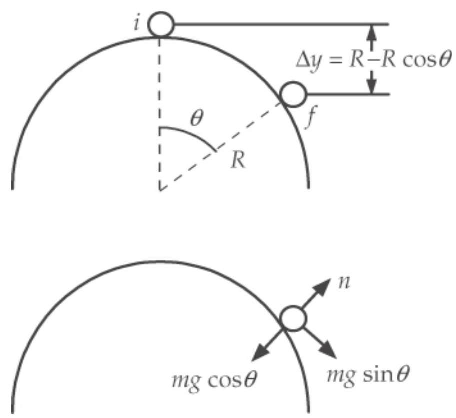
$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}\left(\frac{1}{3}Mh^2\right)\left(\frac{v_{CM}}{h/2}\right)^2 = Mg\left(\frac{h}{2}\right)$$

which reduces to

$$v_{CM} = \boxed{\sqrt{\frac{3gh}{4}}}$$

P10.86 The grape-Earth system is isolated, so mechanical energy in that system is conserved. Between top of the clown's head and the point where the grape leaves the surface:



ANS. FIG. P10.86

$$K_i + U_i = K_f + U_f$$

$$0 + mg\Delta y = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + 0$$

$$mgR(1-\cos\theta) = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_f}{R}\right)^2$$

which gives

$$g(1-\cos\theta) = \frac{7}{10}\left(\frac{v_f^2}{R}\right) \qquad \textbf{[1]}$$

Consider the radial forces acting on the grape:

$$mg\cos\theta - n = \frac{mv_f^2}{R}$$

At the point where the grape leaves the surface, $n \to 0$. Thus,

$$mg\cos\theta = \frac{mv_f^2}{R} \quad \text{or} \quad \frac{v_f^2}{R} = g\cos\theta$$

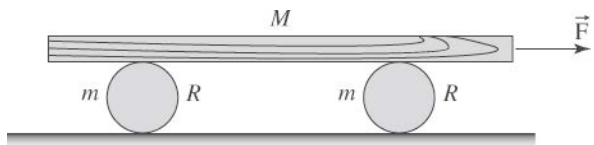
Substituting this into equation [1] gives

$$g - g\cos\theta = \frac{7}{10} g\cos\theta$$

or $$\theta = \cos^{-1}\left(\frac{10}{17}\right) = \boxed{54.0°}$$

Challenge Problems

P10.87 Refer to the force diagrams for the plank and rollers in ANS. FIG. P10.87(b) below. Call f_t the frictional force exerted by each roller backward on the plank. Name as f_b the rolling resistance exerted backward by the ground on each roller.



ANS. FIG. P10.87(a)

For the plank,

$$\sum F_x = ma_x: \quad 6.00\text{ N} - 2f_t = (6.00\text{ kg})a_p \qquad \textbf{[1]}$$

If we think of the motion of a roller as a small rotation about its point of contact with the surface, we see that the center of each roller moves forward only half as far as the plank.

Each roller has acceleration $\frac{a_p}{2}$ and angular acceleration

$$\frac{a_p/2}{5.00\text{ cm}} = \frac{a_p}{0.100\text{ m}}$$

Then for each,

$$\sum F_x = ma_x: \quad +f_t - f_b = (2.00\text{ kg})\frac{a_p}{2} \qquad \textbf{[2]}$$

$$\sum \tau = I\alpha:$$

$$f_t(5.00\text{ cm}) + f_b(5.00\text{ cm}) = \frac{1}{2}(2.00\text{ kg})(5.00\text{ cm})^2 \frac{a_p}{10.0\text{ cm}}$$

So $$f_t + f_b = \left(\frac{1}{2}\text{ kg}\right)a_p \qquad \textbf{[3]}$$

Add equations [2] and [3] to eliminate f_b: $\quad 2f_t = (1.50\text{ kg})a_p$

(a) Substituting the value for $2f_t$ into equation [1] gives

$$6.00\text{ N} - (1.50\text{ kg})a_p = (6.00\text{ kg})a_p$$

$$\rightarrow a_p = \frac{6.00\text{ N}}{7.50\text{ kg}} = \boxed{0.800\text{ m/s}^2}$$

(b) For each roller, $a = \dfrac{a_p}{2} = \boxed{0.400\text{ m/s}^2}$

(c) Substituting back,

$$2f_t = (1.50\text{ kg})(0.800\text{ m/s}^2)$$

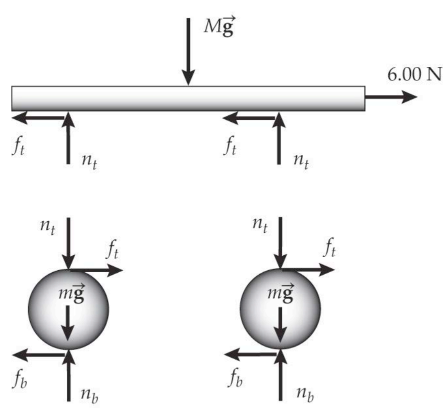
$$f_t = \boxed{0.600\text{ N}}$$

then, from equation [3],

$$0.600\text{ N} + f_b = \left(\frac{1}{2}\text{ kg}\right)(0.800\text{ m/s}^2)$$

$$f_b = -0.200\text{ N}$$

The negative sign means that the horizontal force of ground on each roller is $\boxed{\text{0.200 N forward}}$ rather than backward as we assumed.



ANS. FIG. P10.87(b)

P10.88 For large energy storage at a particular rotation rate, we want a large moment of inertia. To combine this requirement with small mass, we place the mass as far away from the axis as possible.

ANS. FIG. P10.88

We choose to make the flywheel as a hollow cylinder 18.0 cm in diameter and 8.00 cm long. To support this rim, we place a disk across its center. We assume that a disk 2.00 cm thick will be sturdy enough to support the hollow cylinder securely.

The one remaining adjustable parameter is the thickness of the wall of the hollow cylinder. From Table 10.2, the moment of inertia can be written as

$$\begin{aligned} I_{\text{disk}} + I_{\text{hollow cylinder}} &= \frac{1}{2}M_{\text{disk}}R_{\text{disk}}^2 + \frac{1}{2}M_{\text{wall}}\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right) \\ &= \frac{1}{2}\rho V_{\text{disk}}R_{\text{outer}}^2 + \frac{1}{2}\rho V_{\text{wall}}\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right) \end{aligned}$$

$$\begin{aligned} &= \frac{\rho}{2}\pi R_{\text{outer}}^2(2.00\text{ cm})R_{\text{outer}}^2 + \frac{\rho}{2}\left[\pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2\right] \\ &\qquad \times(6.00\text{ cm})\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right) \\ &= \frac{\rho\pi}{2}\Big[(9.00\text{ cm})^4(2.00\text{ cm}) \\ &\qquad +(6.00\text{ cm})\left[(9.00\text{ cm})^2 - R_{\text{inner}}^2\right]\left[(9.00\text{ cm})^2 + R_{\text{inner}}^2\right]\Big] \\ &= \rho\pi\left[6\,561\text{ cm}^5 + (3.00\text{ cm})\left((9.00\text{ cm})^4 - R_{\text{inner}}^4\right)\right] \\ &= \rho\pi\left[26\,244\text{ cm}^5 - (3.00\text{ cm})R_{\text{inner}}^4\right] \end{aligned}$$

For the required energy storage,

$$\frac{1}{2}I\omega_1^2 = \frac{1}{2}I\omega_2^2 + W_{\text{out}}$$

$$\begin{aligned} &\frac{1}{2}I\left[(800\text{ rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2 \\ &\qquad -\frac{1}{2}I\left[(600\text{ rev/min})\left(\frac{2\pi\text{ rad}}{60\text{ s}}\right)\right]^2 \\ &= 60.0\text{ J} \end{aligned}$$

$$\begin{aligned}
I &= \frac{60.0\text{ J}}{1\,535/\text{s}^2} \\
&= \left(7.85\times10^3\ \text{kg/m}^3\right)\pi\left[26\,244\text{ cm}^5-(3.00\text{ cm})R_{\text{inner}}^4\right]
\end{aligned}$$

$$1.58\times10^{-5}\ \text{m}^5\left(\frac{100\text{ cm}}{1\text{ m}}\right)^5 = 26\,244\text{ cm}^5-(3.00\text{ cm})R_{\text{inner}}^4$$

$$R_{\text{inner}} = \left(\frac{26\,244\text{ cm}^4-15\,827\text{ cm}^4}{3.00}\right)^{1/4} = 7.68\text{ cm}$$

The inner radius of the flywheel is 7.68 cm. The mass of the flywheel is then 7.27 kg, found as follows:

$$\begin{aligned}
M_{\text{disk}}+M_{\text{wall}} &= \rho\pi R_{\text{outer}}^2(2.00\text{ cm}) \\
&\quad+\rho\left[\pi R_{\text{outer}}^2-\pi R_{\text{inner}}^2\right](6.00\text{ cm}) \\
&= \left(7.86\times10^3\ \text{kg/m}^3\right)\pi \\
&\quad\left[(0.090\text{ m})^2(0.020\text{ m})\right. \\
&\quad\left.+\left[(0.090\text{ m})^2-(0.076\,8\text{ m})^2\right](0.060\text{ m})\right] \\
&= 7.27\text{ kg}
\end{aligned}$$

If we made the thickness of the disk somewhat less than 2.00 cm and the inner radius of the cylindrical wall less than 7.68 cm to compensate, the mass could be a bit less than 7.27 kg.

The flywheel can be shaped like a cup or open barrel, 9.00 cm in outer radius and 7.68 cm in inner radius, with its wall 6 cm high, and with its bottom forming a disk 2.00 cm thick and 9.00 cm in radius. It is mounted to the crankshaft at the center of this disk and turns about its axis of symmetry. Its mass is 7.27 kg. If the disk were made somewhat thinner and the barrel wall thicker, the mass could be smaller.

P10.89 (a) At $t = 0$, $\omega = 3.50\text{ rad/s} = \omega_0 e^0$. Thus, $\boxed{\omega_0 = 3.50\text{ rad/s}}$.

At $t = 9.30$ s, $\omega = 2.00\text{ rad/s} = \omega_0 e^{-\sigma(9.30\text{s})}$.

We now calculate σ: To solve $\omega = \omega_0 e^{-\sigma t}$ for σ, we recall that the natural logarithm function is the inverse of the exponential function.

$$\omega/\omega_0 = e^{-\sigma t} \quad \text{becomes} \quad \ln(\omega/\omega_0) = -\sigma t \quad \text{or} \quad \ln(\omega_0/\omega) = +\sigma t$$

so $\sigma = \left(\frac{1}{t}\right)\ln(\omega_0/\omega) = \left(\frac{1}{9.30\text{ s}}\right)\ln\left(\frac{3.50}{2.00}\right) = \frac{0.560}{9.30\text{ s}} = \boxed{6.02\times10^{-2}\text{s}^{-1}}$

(b) At all times,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left[\omega_0 e^{-\sigma t}\right] = -\sigma\omega_0 e^{-\sigma t}$$

At t = 3.00 s,

$$\alpha = -(0.060\ 2\text{ s}^{-1})(3.50\text{ rad/s})e^{-0.181} = \boxed{-0.176\text{ rad/s}^2}$$

(c) From the given equation, we have $d\theta = \omega_0^{e^{-\sigma t}} dt$

and

$$\theta = \int_{0\text{ s}}^{2.50\text{ s}} \omega_0 e^{-\sigma t} dt = \frac{\omega_0}{-\sigma} e^{-\sigma t}\Big|_{0\text{ s}}^{2.50\text{ s}} = \frac{\omega_0}{-\sigma}\left(e^{-2.50\sigma} - 1\right)$$

Substituting and solving,

$$\theta = -58.2(0.860 - 1)\text{ rad} = 8.12\text{ rad}$$

or $\theta = (8.12\text{ rad})\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{1.29\text{ rev}}$

(d) The motion continues to a finite limit, as ω approaches zero and t goes to infinity. From part (c), the total angular displacement is

$$\theta = \int_0^\infty \omega_0 e^{-\sigma t} dt = \frac{\omega_0}{-\sigma} e^{-\sigma t}\Big|_0^\infty = \frac{\omega_0}{-\sigma}(0 - 1) = \frac{\omega_0}{\sigma}$$

Substituting,

$$\theta = 58.2\text{ rad} \quad \text{or} \quad \theta = \left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)(58.2\text{ rad}) = \boxed{9.26\text{ rev}}$$

P10.90 (a) If we number the loops of the spiral track with an index n, with the innermost loop having $n = 0$, the radii of subsequent loops as we move outward on the disc is given by $r = r_i + hn$. Along a given radial line, each new loop is reached by rotating the disc through 2π rad. Therefore, the ratio $\theta/2\pi$ is the number of revolutions of the disc to get to a certain loop. This is also the number of that loop, so $n = \theta/2\pi$. Therefore, $r = r_i + h\theta/2\pi$.

(b) Starting from $\omega = v/r$, we substitute the definition of angular speed on the left and the result for r from part (a) on the right:

$$\omega = \frac{v}{r} \rightarrow \frac{d\theta}{dt} = \frac{v}{r_i + (h\theta/2\pi)}$$

(c) Rearrange terms in preparation for integrating both sides:

$$\left(r_i + \frac{h}{2\pi}\theta\right)d\theta = v\,dt$$

and integrate from $\theta = 0$ to $\theta = \theta$ and from $t = 0$ to $t = t$:

$$r_i\theta + \frac{h}{4\pi}\theta^2 = vt$$

We rearrange this equation to form a standard quadratic equation in θ:

$$\frac{h}{4\pi}\theta^2 + r_i\theta - vt = 0$$

The solution to this equation is

$$\theta = \frac{-r_i \pm \sqrt{r_i^2 + \frac{h}{\pi}vt}}{\frac{h}{2\pi}} = \boxed{\frac{2\pi r_i}{h}\left(\sqrt{1 + \frac{vh}{\pi r_i^2}t} - 1\right)}$$

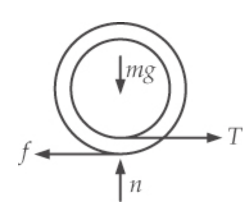
where we have chosen the positive root in order to make the angle θ positive.

(d) We differentiate the result in (c) twice with respect to time to find the angular acceleration, resulting in

$$\boxed{\alpha = -\frac{hv^2}{2\pi r_i^3\left(1 + \frac{vh}{\pi r_i^2}t\right)^{3/2}}}$$

Where we have used $\frac{d}{dx}\sqrt{u} = \frac{1}{2\sqrt{u}}\frac{du}{dx}$. Because this expression involves the time t, the angular acceleration is not constant.

P10.91 (a) $\sum F_x = ma_x$ reads $-f + T = ma$. If we take torques around the center of mass, we can use $\sum \tau = I\alpha$, which reads $+fR_2 - TR_1 = I\alpha$. For rolling without slipping, $\alpha = \frac{a}{R_2}$. By substitution,



ANS. FIG. P10.91

$$fR_2 - TR_1 = \frac{Ia}{R_2} = \frac{I}{R_2 m}(T - f)$$

$$fR_2^2m - TR_1R_2m = IT - If$$
$$f\left(I + mR_2^2\right) = T\left(I + mR_1R_2\right)$$
$$f = \left(\frac{I + mR_1R_2}{I + mR_2^2}\right)T$$

(b) Since the answer is positive, the friction force is confirmed to be $\boxed{\text{to the left}}$.

P10.92 (a) From the isolated system model for the block-pulley-Earth system,

$$\Delta K + \Delta U + \Delta E_{int} = 0$$
$$\left(\frac{1}{2}Mv^2 - 0\right) + \left(\frac{1}{2}I\omega^2 - 0\right) + \left(0 - Mgd\sin\theta\right) + fd = 0$$
$$\frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 - Mgd\sin\theta + \left(\mu Mg\cos\theta\right)d = 0$$
$$v = \sqrt{\frac{4Mgd\left(\sin\theta - \mu\cos\theta\right)}{2M + m}}$$

(b) From the particle under constant acceleration model for the block,

$$v_f^2 = v_i^2 + 2ad$$
$$a = \frac{v_f^2 - v_i^2}{2d} = \frac{v^2}{2d} = \boxed{\frac{2Mg\left(\sin\theta - \mu\cos\theta\right)}{2M + m}}$$

P10.93 The location of the dog is described by $\theta_d = \left(0.750\text{ rad/s}\right)t$. For the bone,

$$\theta_b = \frac{1}{3}2\pi\text{ rad} + \frac{1}{2}0.015\text{ rad/s}^2\ t^2$$

(a) We look for a solution to (suppressing units)

$$0.75t = \frac{2\pi}{3} + 0.007\,5t^2$$
$$0 = 0.007\,5t^2 - 0.75t + 2.09 = 0$$
$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4\left(0.007\,5\right)2.09}}{0.015} = 2.88\text{ s or } 97.1\text{ s}$$

The first time the dog reaches the bone is $\boxed{2.88\text{ s.}}$

(b) If the dog passes the bone, he must run around the merry-go-round again. The dog will draw even with the bone when

$$0.75t = \frac{2\pi}{3} + 2\pi + 0.0075t^2.$$

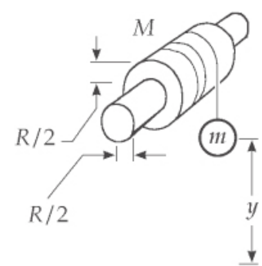
Solving this equation, we find (suppressing units)

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.0075)8.38}}{0.015} = 12.8 \text{ s} \quad \text{or} \quad 87.2 \text{ s}$$

The dog draws even with the bone again at the time of $\boxed{12.8 \text{ s.}}$

P10.94 τ_f will oppose the torque due to the hanging object:

$$\sum \tau = I\alpha = TR - \tau_f: \quad \tau_f = TR - I\alpha \qquad \textbf{[1]}$$



ANS. FIG. P10.94

Now find T, I, and α in given or known terms and substitute into equation [1].

$$\sum F_y = T - mg = -ma: \quad T = m(g - a) \qquad \textbf{[2]}$$

also, $$\Delta y = v_i t + \frac{at^2}{2} \quad a = \frac{2y}{t^2} \qquad \textbf{[3]}$$

and $$\alpha = \frac{a}{R} = \frac{2y}{Rt^2} \qquad \textbf{[4]}$$

with $$I = \frac{1}{2}M\left[R^2 + \left(\frac{R}{2}\right)^2\right] = \frac{5}{8}MR^2 \qquad \textbf{[5]}$$

Substituting [2], [3], [4], and [5] into [1], we find

$$\tau_f = m\left(g - \frac{2y}{t^2}\right)R - \frac{5}{8}\frac{MR^2(2y)}{Rt^2} = \boxed{R\left[m\left(g - \frac{2y}{t^2}\right) - \frac{5}{4}\frac{My}{t^2}\right]}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P10.2 (a) 0.209 rad/s^2; (b) yes

P10.4 144 rad

P10.6 -2.26×10^2 rad/s^2

P10.8 (a) 3.5 rad; (b) increase by a factor of 4

P10.10 Because the disk's average angular speed does not match the average angular speed expressed as $(\omega_i + \omega_f)/2$ in the model of a rigid object under constant angular acceleration, the angular acceleration of the disk cannot be constant.

P10.12 50.0 rev

P10.14 (a) $\boxed{\omega h^{3/2}\left(\frac{2}{g}\right)^{1/2}}$; (b) 1.16 cm; (c) The deflection is only 0.02% of the original height, so it is negligible in many practical cases; (d) Decrease

P10.16 ~10^7 rev/yr

P10.18 (a) 0.605 m/s; (b) 17.3 rad/s; (c) 5.82 m/s; (d) We did not need to know the length of the pedal cranks.

P10.20 (a) 54.3 rev; (b) 12.1 rev/s

P10.22 (a) 5.77 cm; (b) Yes. See P10.20 for full explanation.

P10.24 $\frac{a}{g}\sqrt{1+\pi^2}$

P10.26 (a) $\left(-2.73\hat{\mathbf{i}} + 1.24\hat{\mathbf{j}}\right)$ m; (b) It is in the second quadrant, at 156°; (c) $\left(-1.85\hat{\mathbf{i}} - 4.10\hat{\mathbf{j}}\right)$ m/s; (d) It is moving toward the third quadrant, at 246°; (e) $\left(6.15\hat{\mathbf{i}} - 2.78\hat{\mathbf{j}}\right)$ m/s^2; (f) See ANS. FIG. P10.26; (g) $\left(24.6\hat{\mathbf{i}} - 11.1\hat{\mathbf{j}}\right)$ N

P10.28 168 N · m

P10.30 (a) 1.03 s; (b) 10.3 rev

P10.32 (a) See ANS. FIG. P10.32; (b) 0.309 m/s^2; (c) T_1 = 7.67 N, T_2 = 9.22 N

P10.34 (a) For F = 25.1 N, R = 1.00 m. For F = 10.0 N, R = 25.1 m; (b) No. Infinitely many pairs of values that satisfy this requirement may exist: for any $F \le 50.0$ N, R = 25.1 N · m/F, as long as $R \le 3.00$ m.

P10.36 (a) 1.95 s; (b) If the pulley were massless, the acceleration would be larger by a factor 35/32.5 and the time short by the square root of the factor 32.5/35. That is, the time would be reduced by 3.64%.

P10.38 $10^0\ \text{kg}\cdot\text{m}^2 = 1\ \text{kg}\cdot\text{m}^2$

P10.40 (a) See P10.40(a) for full description; (b) See P10.40(b) for full description

P10.42 $I_{y'} = \int_{\text{all mass}} r^2 dm = \int_0^L x^2 \frac{M}{L} dx = \frac{M}{L}\frac{x^3}{3}\Big|_0^L = \frac{1}{3}ML^2$

P10.44 (a) 92.0 kg·m^2; (b) 184 J; (c) 6.00 m/s, 4.00 m/s, 8.00 m/s; (d) 184 J; (e) The kinetic energies computed in parts (b) and (d) are the same.

P10.46 $\frac{13}{24}MR^2\omega^2$

P10.48 276 J

P10.50 $v = \sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2 + \frac{I}{R^2}}}$ and $\omega = \sqrt{\frac{2(m_1 - m_2)gh}{m_1R^2 + m_2R^2 + I}}$

P10.52 The situation is impossible because the range is only 3.86 km, not city-wide.

P10.54 (a) 6.90 J; (b) 8.73 rad/s; (c) 2.44 m/s; (d) The speed it attains in swinging is greater by 1.043 2 times

P10.56 $mr^2\left(\frac{2gh}{v^2} - 1\right)$

P10.58 (a) 74.3 W; (b) 401 W

P10.60 rolling: $v_f = \sqrt{10gh/7}$; sliding: $v_f = \sqrt{2gh}$; The time to roll is longer by a factor of $(0.7/0.5)^{1/2} = 1.18$

P10.62 (a) the cylinder; (b) $v^2/4g\sin\theta$; (c) The cylinder does not lose mechanical energy because static friction does not work on it. Its rotation means that it has 50% more kinetic energy than the cube at the start, and so it travels 50% farther up the incline.

P10.64 (a) 2.38 m/s; (b) The centripetal acceleration at the top is $\frac{v_2^2}{r} = \frac{(2.38\ \text{m/s})^2}{0.450\ \text{m}} = 12.6\ \text{m/s}^2 > g$. Thus, the ball must be in contact with the track, with the track pushing downward on it; (c) 4.31 m/s; (d) $\sqrt{-1.40\,\text{m}^2/\text{s}^2}$; (e) never makes it to the top of the loop

P10.66 $\frac{1}{3}$ the length of the chimney

P10.68 (a) $d = (1890 + 80n)\left(\frac{0.459\,\text{m}}{80n - 150}\right)$; (b) 94.1 m; (c) 1.62 m; (d) –5.79 m; (e) The rising car will coast to a stop only for $n \geq 2$; (f) For $n = 0$ or $n = 1$, the mass of the elevator is less than the counterweight, so the car would accelerate upward if released; (g) 0.459 m

P10.70 $\omega(t) = \omega + At + \frac{1}{2}Bt^2$; (b) $\omega t + \frac{1}{2}At^2 + \frac{1}{6}Bt^3$

P10.72 (a) (i) –794 N·m, (ii) –2 510 N·m, (iii) 0 N· m, (iv) –1 160 N·m, (v) 2 940 N·m; (b) See P10.72(b) for full description.

P10.74 $-0.322\ \text{rad/s}^2$

P10.76 (a) 2.57×10^{29} J; (b) -1.63×10^{17} J/day

P10.78 (a) $Mg/3$; (b) $2g/3$; (c) $\sqrt{4gh/3}$; (d) The answer is the same.

P10.80 (a) $\theta \leq 35.5°$; (b) 0.184 m from the moving end

P10.82 (a) $a_{CM} = \frac{4F}{3M}$; (b) $\frac{1}{3}F$; (c) $\sqrt{\frac{8Fd}{3M}}$

P10.84 (a) $35.0\ \text{m/s}^2$; (b) $7.35\hat{\mathbf{i}}$ N; (c) $17.5\ \text{m/s}^2$; (d) $-3.68\hat{\mathbf{i}}$ N; (e) 0.827 m (from the top)

P10.86 54.0°

P10.88 See P10.88 for full design and specifications of flywheel.

P10.90 (a) See P10.90(a) for full solution; (b) See P10.90(g) for full solution; (c) $\frac{2\pi r_i}{h}\left(\sqrt{1 + \frac{vh}{\pi r_i^2}t} - 1\right)$; (d) $\alpha = -\frac{hv^2}{2\pi r_i^2\left(1 + \frac{vh}{\pi r_i^2}t\right)^{3/2}}$

P10.92 (a) See P10.92(a) for full explanation; (b) $\frac{2Mg(\sin\theta - \mu\cos\theta)}{2M + m}$

P10.94 $R\left[m\left(g - \frac{2y}{t^2}\right) - \frac{5}{4}\frac{My}{t^2}\right]$